1. A person wants to build an enclosure for their garden. They have 400 feet of fence to enclose two areas as indicated below. What dimensions maximize the area?

\[
\begin{align*}
400 &= 2y + 3x \\
\frac{400 - 3x}{2} &= y
\end{align*}
\]

\[A = x \left( \frac{400 - 3x}{2} \right)\]

\[A = \frac{1}{2} x (400 - 3x)\]

2. Sketch \(y = -2x^2(x-3)^3(x+2)(x-6)^4\)
   a. Describe end behavior
   b. Check with grapher
   c. How many turning points does it have?
   d. At most, how many turning points could a 10th degree equation have?
   e. How many zeros does it have?
   f. At most, how many zeros could a 10th degree equation have?
2. Sketch \( y = -2x^2(x-3)^3(x+2)(x-6)^4 \)
   
   a. Describe end behavior
   \[ \lim_{x \to \infty} f(x) = -\infty \]
   \[ \lim_{x \to -\infty} f(x) = -\infty \]
   
   b. Check with grapher
   
   c. How many turning points does it have? 5
   
   d. At most, how many turning points could a 10th degree equation have? 9
   
   e. How many zeros does it have? 4
   
   f. At most, how many zeros does a 10th degree equation have? 10
HW Questions: p. 191
Matching

1. \( f(x) = -3x + 5 \)
2. \( f(x) = x^2 - 2x \)
3. \( f(x) = -2x^2 - 8x - 9 \)
4. \( f(x) = 3x^3 - 9x + 1 \)
5. \( f(x) = -\frac{1}{3}x^3 + x - \frac{2}{3} \)
6. \( f(x) = -\frac{1}{4}x^4 + 2x^2 \)
7. \( f(x) = 3x^4 + 4x^3 \)
8. \( f(x) = x^5 - 5x^3 + 4x \)
In Exercises 9–18, determine the right-hand and left-hand behavior of the graph of the polynomial function.

11. \( g(x) = 5 - \frac{7}{2}x - 3x^2 \)  

15. \( f(x) = 6 - 2x + 4x^3 - 5x^3 \)

\( \lim_{x \to \infty} f(x) = \infty \)

\( \lim_{x \to -\infty} f(x) = -\infty \)

In Exercises 19–34, find all the real zeros of the polynomial function.

21. \( h(t) = t^2 - 6t + 9 \)

25. \( f(x) = 3x^2 - 12x + 3 \)

33. \( f(x) = 5x^4 + 15x^2 + 10 \)

\( 0 = 5(x^4 + 3x^2 + 2) \)

\( 0 = 5(x^2 + 2)(x^2 + 1) \)

\( x^2 + 2 = 0 \)  

\( \sqrt{x^2} = \sqrt{-2} \)

11. No real zeros
In Exercises 35–44, find a polynomial function that has the given zeros.

35. 0, 10
   \[ f(x) = (x - 0)(x - 10) \]
   \[ f(x) = x(x - 10) \]
   \[ f(x) = x^2 - 10x \]

41. 4, -3, 3, 0
   \[ f(x) = (x - 4)(x + 3)(x - 3)(x + 0) \]
   \[ f(x) = (x - 4)(x + 3)(x - 3)(x) \]
   \[ f(x) = (x^2 - 10x) \]

43. 1 + \sqrt{3}, 1 - \sqrt{3}
   \[ f(x) = (x - (1 + \sqrt{3}))(x - (1 - \sqrt{3})) \]
   \[ f(x) = (x - 1 - \sqrt{3})(x - 1 + \sqrt{3}) \]

Notice sum & difference pattern gives you the difference of squares.

\[ f(x) = (x - 1)^2 - (\sqrt{3})^2 \]
\[ f(x) = x^2 - 2x + 1 - 3 \]
\[ f(x) = x^2 - 2x - 2 \]
In Exercises 47–58, sketch the graph of the given function.

47. \( f(x) = -\frac{3}{2} \)

51. \( f(x) = x^3 - 3x^2 \)
   - Opposite end behavior:
   - Rises \( B+ \)
   - \( f(x) = x^2(x-3) \)
   - Zeros \( @ 0 \& 3 \)
   - Repeat zero: touches \( x \)-axis without passing through.

55. \( g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2 \)
Use long division to divide the dividend 30489 by the divisor 22:

\[
\begin{array}{r}
22) & 30489 \\
-22 & \\
\hline
8489 \\
-6600 & \\
\hline
1889 \\
-1760 & \\
\hline
129 \\
-110 & \\
\hline
19 \\
\end{array}
\]

Remainder: 19
Divide the polynomial \( f(x) = 8x^3 - 20x^2 + 3x - 1 \) by \( x-5 \).

Is \( x - 5 \) a factor of \( f(x) \)? **No.**

\[
\begin{array}{r|ccc}
\multicolumn{2}{c}{x-5} & 8x^2 + 20x + 103 \\
\hline
8x^3 & -20x^2 & +3x & -1 \\
\hline
& & -(8x^3 - 40x^2) & \\
& & 20x^2 & +3x & -1 \\
& & -(20x^2 - 100x) & \\
& & 103x & -1 \\
& & -(103x - 515) & \\
& & 514 & \\
\end{array}
\]

\( 8x^2(x-5) \)

Answer:
\( 8x^2 + 20x + 103 + \frac{514}{x-5} \)

Do this with synthetic division

\[
\begin{array}{r|cccc}
& & 5 & & \\
\hline
8 & -20 & 3 & -1 & 514 \\
\hline
& & 40 & 100 & 515 \\
& & 8 & 20 & 103 & 514 \\
\end{array}
\]

\( 8x^2 + 20x + 103 + \frac{514}{x-5} \)
Divide the polynomial \( f(x) = 8x^3 - 20x^2 + 3x - 1 \) by \( x-5 \).

Is \( x - 5 \) a factor of \( f(x) \)?

Do this with synthetic division.
Synthetic division is easier, but when the divisor is not linear you MUST use long division:

Ex: Divide $2x^4 + 3x - 4$ by $x^2 - x + 5$

\[
\begin{align*}
2x^2(x^2-x+5) & \quad \frac{2x^2+2x-8}{x^2-x+5) \ 2x^4+0x^3+0x^2+3x-4} \\
& \quad - (2x^4-2x^3+10x^2) \\
& \quad \frac{2x^3-10x^2+3x-4}{- (2x^3-2x^2+10x)} \\
& \quad - (-8x^2+7x-4) \\
& \quad \frac{-15x+36}{-8x^2+8x-40} \\
& \quad - (-15x+36)
\end{align*}
\]
Synthetic division is easier, but when the divisor is not linear you MUST use long division:

Ex: Divide $2x^4 + 3x - 4$ by $x^2 - x + 5$

\[
\begin{array}{c|ccccc}
& 2x^2 & 2x & -8 & \xi & -15x + 30 \\
\hline
x^2 & & & & & \\
x & 2x^4 & 2x^3 & -8x^2 & \xi & -15x + 15 \\
-x & & -2x^3 & -2x^2 & +8x & \\
5 & & 10x^2 & 10x & -40 & \\
\end{array}
\]
Divide \( f(x) = x^4 - 3x^3 + 10x^2 - 4x + 8 \) by \( x - 3 \).

\[
\begin{array}{c|cccc}
3 & 1 & -3 & 10 & -4 & 8 \\
\hline
 & 3 & 0 & 30 & 78 \\
 & 1 & 10 & 26 & 86 \\
\hline
 & 1 & 10 & 26 & 86 & R
\end{array}
\]

\[\text{answer: } x^3 + 10x + 26 + \frac{86}{x-3}\]

What is the zero of \( x - 3 \)? \( \rightarrow 3 \)

Find \( f(3) = 3^4 - 3(3)^3 + 10(3)^2 - 4(3) + 8 \)

\[
= 81 - 81 + 90 - 12 + 8
\]

\[= 86\]

What is the remainder to the original problem? \(86\)
REMAINDER THEOREM:

If \( f(x) \) is divided by \((x-k)\), then \( f(k) \) is the remainder \( r \) when you do syn. \( \div \).

\[ f(x) = x^3 - 4x + 2 \quad \text{Find } f(-8) \text{ without a calculator.} \]

\[
\begin{array}{c|cccc}
  -8 & 1 & 0 & -4 & 2 \\
  \hline 
   & -8 & 64 & -480 \\
  \hline 
   & 1 & -8 & 60 & -478 \\
\end{array}
\]

\[ f(-8) = -478 \]
Divide \(6x^3 - 19x^2 + 16x - 4\) by \(x - 2\).

Is \((x - 2)\) a factor? \(\text{Yes}\) \hspace{1cm} f(2) = ? \(0\)

\[
\begin{array}{c|cccc}
& 6 & -19 & 16 & -4 \\
\hline
2 & & 12 & -14 & 4 \\
\hline
& 6 & -7 & 2 & 0
\end{array}
\]

Factor \(6x^3 - 19x^2 + 16x - 4\) completely.

\[\begin{align*}
(x - 2)(6x^2 - 7x + 2) \\
(x - 2)(2x - 1)(3x - 2)
\end{align*}\]
FACTOR THEOREM

\[ f(x) \text{ has a FACTOR } (x-k) \text{ if and only if } f(k) = 0 \]

remainder is 0 when you do \( \div \)
Use the remainder theorem to evaluate:

\[ f(x) = 4x^5 + x^4 - 10x^3 + x^2 - 4 \]  at  \( x = -3 \)

\[
\begin{array}{c|cccccc}
  -3 & 4 & 1 & -10 & 1 & 0 & -4 \\
    &   & -12 & 33 & -69 & 204 & -612 \\
  \hline
    & 4 & -11 & 23 & -68 & 204 & -616 \\
\end{array}
\]

\[ f(-3) = -616 \]
Use synthetic division to show that \(-4\) is a solution to

\[ x^3 - 28x - 48 = 0 \]

and use the result to factor the polynomial completely.

\[
\begin{array}{c|cccc}
-4 & 1 & 0 & -28 & -48 \\
 & & -4 & 16 & 48 \\
\hline
 & 1 & -4 & -12 & 0 \\
\end{array}
\]

\[ (x+4)(x^2-4x-12) \]

\[ (x+4)(x-6)(x+2) \]

Give all solutions. Sketch the graph.

Solutions to \(x^3 - 28x - 48 = 0\)
are \(x = -4, 6, -2\)

end behavior
Rises Rt (Positive leading coefficient)
Falls Left (Odd power, so opposite end behavior.)
Short quiz Friday:
* Maximize area problem
* Work problem
* Factoring!