

1. The brightest star in the winter sky in the Northern Hemisphere is Sirius. In reality, Sirius is a system of two stars that orbit each other with an average separation of 3×10^{12} m. The Hubble Space Telescope (diameter 2.4 m) is pointed at the Sirius system which is 8×10^{16} meters from Earth. Can the Hubble Space Telescope resolve the two stars or does it see them as a single point of light?

Assumption: average light has a wavelength of 500 nm

$$\theta = s/D$$

$$3 \times 10^{12} \text{ m} / 8 \times 10^{16} \text{ m}$$

$$3.8 \times 10^{-5} \text{ rad} >$$

$$\theta_{\min} = 1.22 \lambda / b$$

$$1.22 \cdot 500 \times 10^{-9} \text{ m} / 2.4 \text{ m}$$

$$2.5 \times 10^{-7} \text{ rad}$$

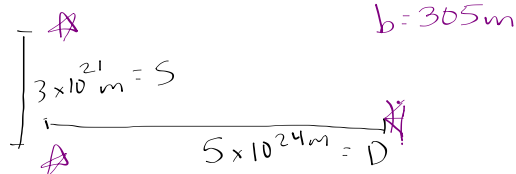
Conclusion: Since the angular separation between the two stars is greater than the minimum angle, they can be resolved.

2. A spy satellite is orbiting 180 km above the surface of the Earth. If the diameter of the lens is 45 cm, what is the smallest distance on the ground that can be resolved by the camera on the satellite?

$$\underline{\underline{1.22 \lambda / b}}$$

$$\rightarrow .24 \text{ m}$$

3. The Galaxy Cygnus A can be resolved optically as an elliptically shaped galaxy. However, it is also a strong emitter of radio waves of wavelength 0.15 m. The Galaxy is estimated to be 5.0×10^{24} m from Earth. Use of a radio telescope shows that the radio emission is from two sources separated by a distance of 3.0×10^{21} m. Estimate the diameter of the dish required to just resolve the sources.



World's largest single-aperture (filled-dish) telescope (305 m) in Arecibo, Puerto Rico

$$\frac{1.22 \lambda}{b} \leq \frac{s}{D}$$

Applications of Resolution

IB 12

Due to diffraction effects, all devices have a limit on their ability to perceive and to resolve between sources of light. For instance, our eyes can never see atoms since atoms are smaller than the wavelength of visible light so light waves will just diffract around them. Here are some cases where diffraction and resolution are important.

1. CDs and DVDs

CDs and DVDs store digital information as “bumps” and “pits” etched into a plastic surface. Music CDs have data tracks approximately 5×10^{-7} m wide with the bumps and pits just over 1×10^{-7} m high. The bumps and pits on a DVD are much smaller so that more data can be stored. The data is read by reflecting a laser beam off the surface. The wavelength of laser light used to read the data and the size of the aperture of the lens used to receive the laser light places a limit on how close together the bumps and pits can be placed, that is, places a limit on the resolution of the data.



CD laser light:

700nm

DVD laser light:

640nm

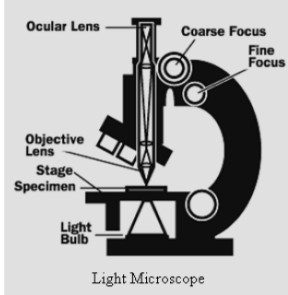
Blu-ray laser light:

405nm

Why are lasers with smaller wavelengths used?

Greater resolving power, greater resolution, tracks can be placed closer together, more data stored on disk, better clarity of sound and picture

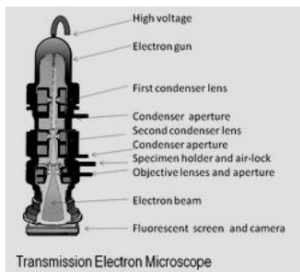
used in electron microscopes. The de Broglie wavelength of an electron can be 100,000 times smaller than the wavelength of a photon of visible light so a microscope using an electron beam can resolve objects that are much smaller than those of a light microscope.



Uses: light

Resolution: ~200 nm

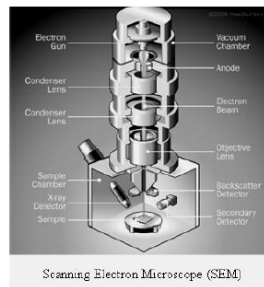
Magnification: ~2000x



Uses: e-

Resolution: 50 pm

Magnification: 50,000,000x



Uses: e-

Resolution: ~1 nm

Magnification: 50,000x

Why are electrons used instead of light?

Smaller wavelengths mean greater resolving power, greater resolution

4. Resolvance of a diffraction grating

The width of the interference fringes produced from a diffraction grating is related to the number of lines on the grating: more lines means narrower fringes.

As a result of Rayleigh's criterion, there is a limit placed on a grating's ability to resolve different wavelengths. This means that if two spectral lines are close together they will be easier to resolve if the grating has many lines.

Resolvance (R) of a diffraction grating: the ratio between a wavelength being investigated (λ) and the smallest possible resolvable wavelength difference ($\Delta\lambda$)

Formula: $R = \frac{\lambda}{\Delta\lambda} = n m$

where:

- R = **resolvance**
- m = **1,2,3... (order)**
- N = **total number of slits**

Adapted from Honolulu Community College Science 122 site

In the sodium emission spectrum there are two wavelengths that are close to one another (the sodium D-lines). These are 589.00 nm and 589.59 nm. For these two lines to be resolved by a diffraction grating, determine the:

a) resolvance

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589 \text{ nm}}{.59 \text{ nm}} \approx 1000 =$$

b) number of slits that must be illuminated for the first order maxima ($m=1$)

$$R = \overset{(1)}{m} n \quad n \approx 1000$$

c) number of slits that must be illuminated for the second order maxima

$$\overset{(2)}{m} n \quad n \approx 500$$