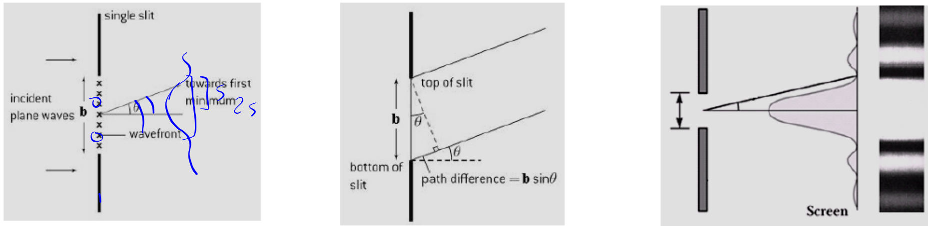
Single slit diffraction formula



QLD

$\frac{s}{D} = \frac{\frac{1}{2}\lambda}{d}$ ← $b/2$

Position of first minimum (angular half-width of central maximum): $\theta = \lambda / b$

Note: θ must be in radians

Half-width of central maximum: location of first dark fringe

Angular half-width: θ

Angular width: 2θ

Linear half-width: s

Linear width: $2s$

Ratio: $\theta = \frac{\lambda}{b} = \frac{s}{D}$

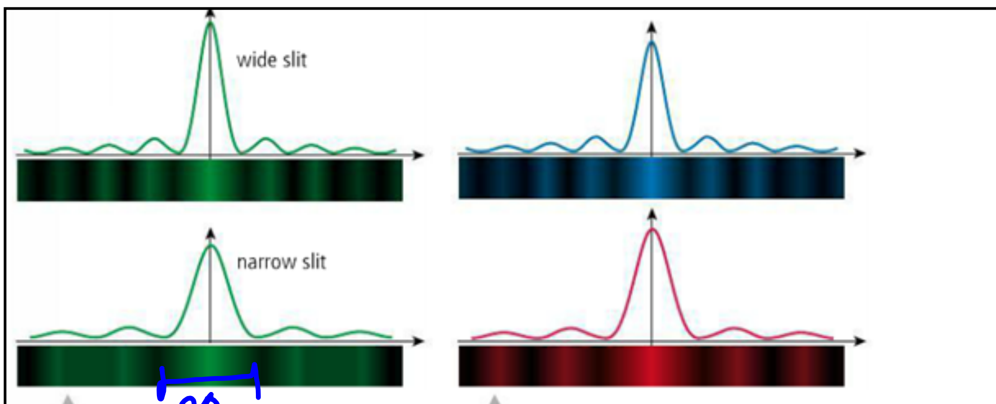


Figure 5.94 Notice that with a narrower slit the pattern is wider but less intense.

Figure 5.95 Same size slit but different wavelength light, longer wavelength gives a wider pattern.

1. A diffraction pattern forms on a screen 2.5 meters away when light of wavelength 675 nm passes through a single slit.

   a) Determine the angle that locates the first dark fringe if the width of the slit is 18 μm.

   $\theta = \frac{\lambda}{b} = \frac{675\times10^{-9}\,m}{18\times10^{-6}\,m} = .0375\,rad = 2.1°$

   $\sin\theta = \tan\theta \sim \theta$

   b) How wide is the central maximum?

   $2\theta = .075\,rad = 4.2°$

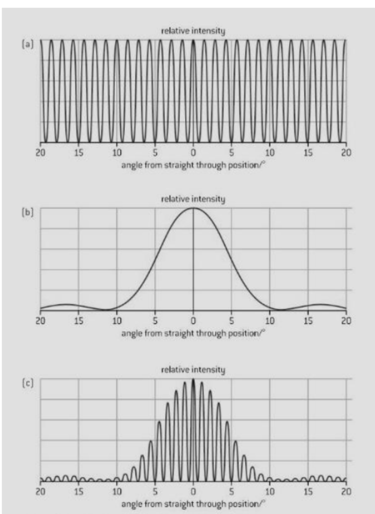
   $s = \frac{\lambda D}{b} \qquad 2s = 2\left(\frac{\lambda D}{b}\right) = .1875\,m$

2. Light passes through a single slit and shines on a flat screen that is located 0.40 m away. The width of the slit is 4.0×10^{-6} m. Determine the width of the central bright fringe when the wavelength of the light in a vacuum is λ = 690 nm (red).

   $2\theta = .35\,rad$

   $2s = .14\,m$

Modulation of double-slit interference pattern by single-slit diffraction effect



▲ Figure 2 Combination of diffraction and interference.

Ideal double-slit interference pattern:

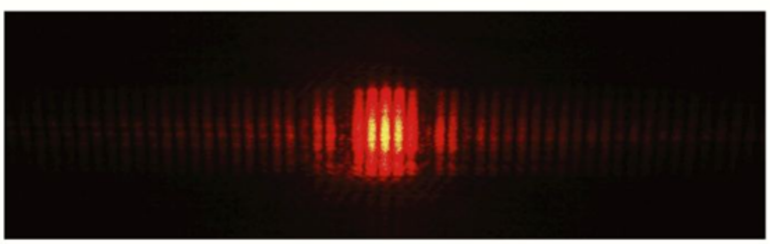
Fringes of equal spacing and intensity

Double-slit diffraction effect:

Wide and intense central maximum

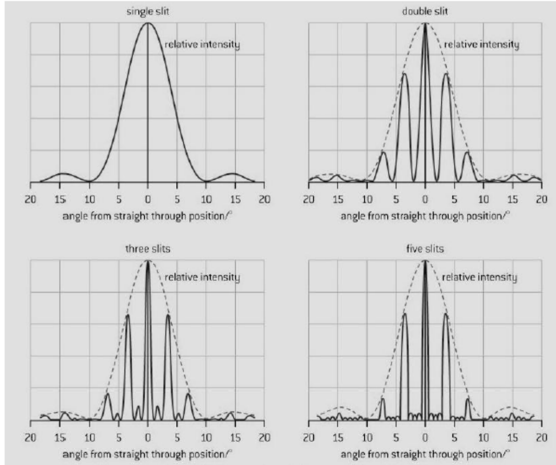
Modulation of double-slit interference pattern by single-slit diffraction effect:

Amplitude of double-slit pattern modulated by single-slit pattern

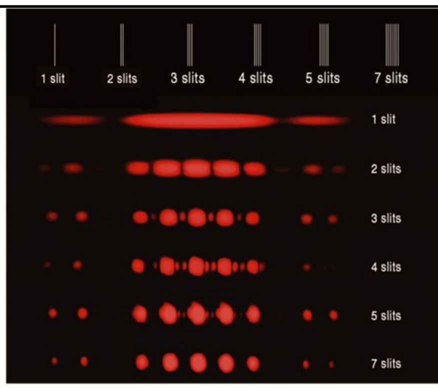


▲ Figure 1 Double-slit diffraction pattern for light from He–Ne laser.

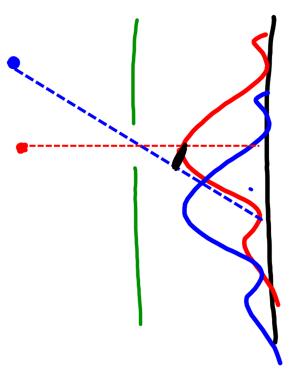
Comparison of intensity patterns showing the effect of increasing the number of slits on intensity



The effect of increasing the number of slits on variation of intensity.



▲ Figure 3 The effect of increasing the number of slits on an interference pattern.



Resolution

IB 12

Resolution (resolving power): the ability to distinguish between two sources of light

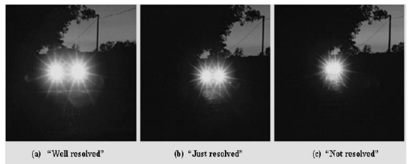
The ability to **resolve** two sources of light depends on . . .

a) distance away from aperture

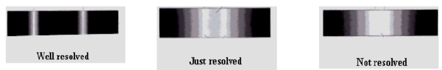
b) distance between two sources

c) wavelength

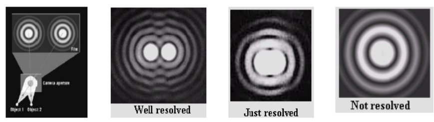
d) size of aperture



Resolution of two sources through a single slit



Resolution of two sources through a circular aperture

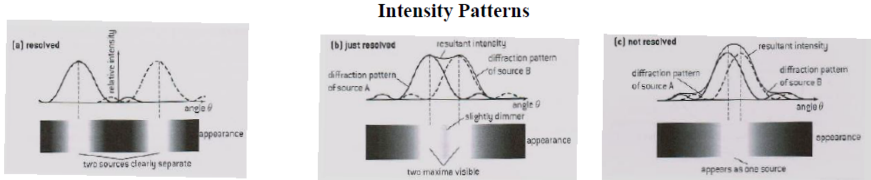


Examples: pupil of eye, telescope

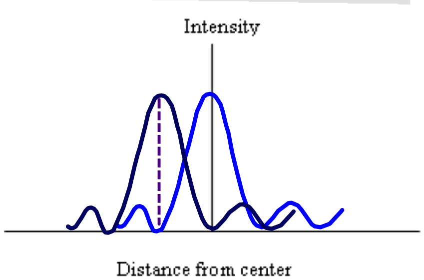
Rayleigh Criterion:

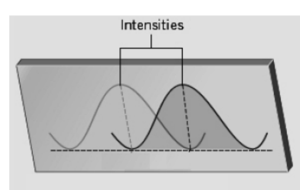
For two sources to be "just resolved," the central maximum of one diffraction pattern is located on top of the first minimum of the other diffraction pattern.

Intensity Patterns



1. Sketch an intensity pattern for two sources of light that are just resolved.





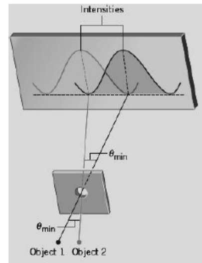
Just resolved

The minimum angle (θ_{min}) between the sources for them to be "just resolved" is . . .

the angular half-width of one diffraction pattern

Single Slit $\theta_{min} = \frac{\lambda}{b}$

Circular Aperture $\theta_{min} = \frac{1.22\lambda}{b}$



Examples of "b":
width of pupil, diameter of opening of telescope or satellite/radar dish

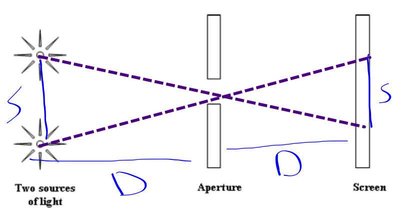
When θ_{min} decreases, the resolution (or resolving power) of the optical instrument is said to . . .
increase

2. To increase the resolution of an optical instrument . . .

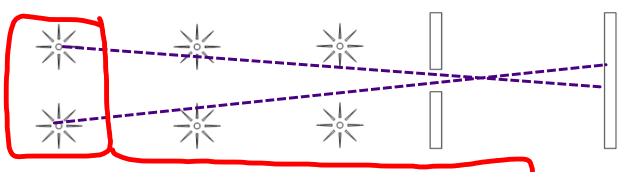
a) decrease the wavelength (increase frequency) of the light used

b) increase the size of the opening or diameter of aperture

Calculating the angle between two sources



	θ Actual Angle	Minimum Angle
Single Slit	S/D	λ/b
Circular Aperture	S/D	$\frac{1.22\lambda}{b}$



Two sources of light can be resolved when . . .
$\theta > \theta_{min}$

Two sources of light can be **just** resolved when . . .
$\theta = \theta_{min}$

Two sources of light are not resolved when . . .
$\theta < \theta_{min}$

1. The brightest star in the winter sky in the Northern Hemisphere is Sirius. In reality, Sirius is a system of two stars that orbit each other with an average separation of 3×10^{12} m. The Hubble Space Telescope (diameter 2.4 m) is pointed at the Sirius system which is 8×10^{16} meters from Earth. Can the Hubble Space Telescope resolve the two stars or does it see them as a single point of light?

Assumption: average light has a wavelength of 500 nm

$\theta = s/D$

$3\times10^{12}\,m / 8\times10^{6}\,m$

3.8×10^{-5} rad $>$

θ_{min}

$1.22\lambda/b$

$1.22\cdot500\times10^{-9}\,m/2.4\,m$

2.5×10^{-7} rad

Conclusion: Since the angular separation between the two stars is greater than the minimum angle, they can be resolved.

2. A spy satellite is orbiting 180 km above the surface of the Earth. If the diameter of the lens is 45 cm, what is the smallest distance on the ground that can be resolved by the camera on the satellite?

$1.22\lambda/b$

$\rightarrow .24\,m$

3. The Galaxy Cygnus A can be resolved optically as an elliptically shaped galaxy. However, it is also a strong emitter of radio waves of wavelength 0.15 m. The Galaxy is estimated to be 5.0×10^{24} m from Earth. Use of a radio telescope shows that the radio emission is from two sources separated by a distance of 3.0×10^{21} m. Estimate the diameter of the dish required to just resolve the sources.

World's largest single-aperture (filled-dish) telescope (305 m) in Arecibo, Puerto Rico