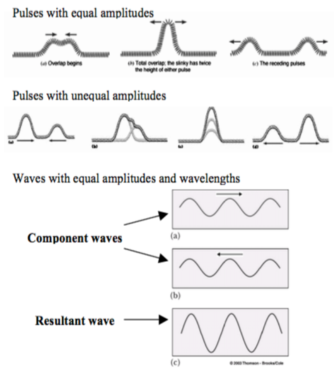
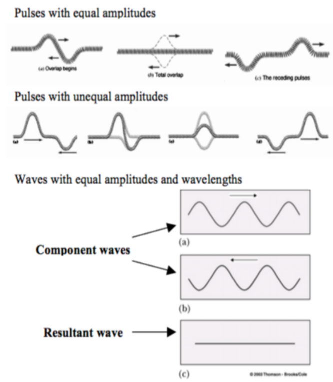
**Principle of Linear Superposition**: When two or more waves meet, the displacement of the resultant wave is the vector sum of the displacements of the component waves.

**Constructive Interference**: superposition of two or more pulses (or waves) in phase




**Destructive Interference**: superposition of two or more pulses (or waves) out of phase




**Standing (stationary) wave** - resultant wave formed when two waves of equal amplitude and frequency traveling in opposite directions in the same medium interfere

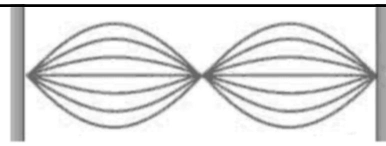
How is a standing wave on a string formed?

A traveling wave moving in one direction in the string is reflected off the end of the string. This sends a reflected wave traveling in the opposite direction in the string which is nearly identical with the first traveling wave. It has the same frequency, the same wavelength, and almost the same amplitude. These two traveling waves moving in opposite directions in the string are the component waves. These component waves interfere with each other creating the standing wave whose amplitude at any point is the superposition of the components' amplitudes. This standing wave is the resultant wave of the two component traveling waves.

**Node**: location of constant complete destructive interference

**Anti-Node**: location of maximum constructive interference

Traveling Wave



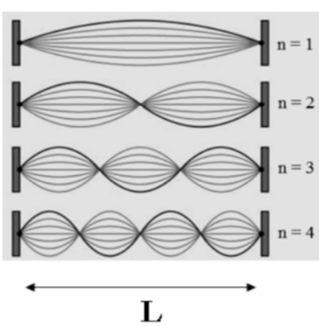
Standing Wave

## Comparison of travelling waves and stationary waves

| Property | Travelling wave | Standing wave |
|---|---|---|
| energy transfer | energy is transferred in the direction of propagation | no energy is transferred by the wave although there is interchange of kinetic and potential energy within the standing wave |
| amplitude | all particles have the same amplitude | amplitude varies within a loop – maximum occurs at an antinode and zero at a node |
| phase | within a wavelength the phase is different for each particle | all particles within a "loop" are in phase and are antiphase (180° out of phase) with the particles in adjacent "loops" |
| wave profile (shape) | propagates in the direction of the wave at the speed of the wave | stays in the same position |
| wavelength | the distance between adjacent particles which are in phase | twice the distance between adjacent nodes (or adjacent antinodes) |
| frequency | all particles vibrate with same frequency. | all particles vibrate with same frequency except at nodes (which are stationary) |

**I. Transverse Standing Wave: string fixed at both ends**

**Boundary conditions:** 2 fixed ends – node at each end




| Name | Frequency | Wavelength |
|---|---|---|
| 1st harmonics | $f_1$ | $\frac{1}{2}\lambda = L$<br>$\lambda = 2L$ |
| 2nd harmonic | $f_2 = 2f_1$ | $\lambda = \frac{2}{2}L$ |
| 3rd harmonic | $f_3 = 3f_1$ | $\lambda = \frac{2}{3}L$ |
| 4th harmonic | $f_4 = 4f_1$ | $\lambda = \frac{2}{4}L$ |

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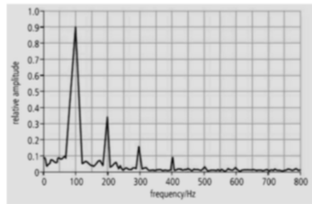
$f_n = n f_1$ $\quad \lambda_n = \frac{2}{n} L$

Notice that a stable standing wave will only occur at certain discrete frequencies or certain discrete lengths of the string. It is only at these frequencies or lengths that the wave resonates so they are called *resonant frequencies* or *harmonics* or *resonant modes of vibration* of the string. We say that these resonant frequencies are *quantized*.

**Harmonics:** $\lambda_n = \frac{2L}{n}$ $\qquad f_n = \frac{nv}{2L}$

What do you hear when a guitar (or piano, etc.) string is played?

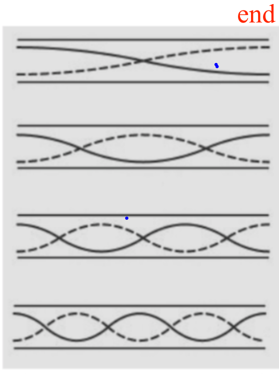
A combination of resonant frequencies in different amounts (different amplitudes)




## II. Longitudinal Standing Wave: pipe open at both ends

**How is this standing wave formed?** Vibrations of air at one end produce a traveling (longitudinal) wave that reflects off the open end of the pipe which causes a second traveling wave in the opposite direction. These two component waves interfere to produce a standing wave if they have a frequency which is one of the resonant frequencies of the pipe.

**Boundary conditions:** 2 free ends – anti-node at each end



**L**

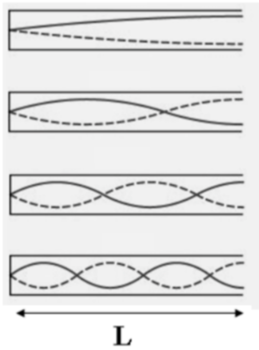
| Name | Frequency | Wavelength |
|---|---|---|
| 1st harmonics | $f_1$ | $\frac{1}{2}\lambda = L$ <br> $\lambda = 2L$ |
| 2nd harmonic | $f_2 = 2f_1$ | $\lambda = \frac{2}{2}L$ |
| 3rd harmonic | $f_3 = 3f_1$ | $\lambda = \frac{2}{3}L$ |
| 4th harmonic | $f_4 = 4f_1$ | $\lambda = \frac{2}{4}L$ |

**Note**: An anti-node does not represent an up/down motion of air particles but a left/right motion of air particles.

$f_n = nf_1 \qquad \lambda_n = \frac{2}{n}L$

**III. Longitudinal Standing Wave: pipe open at only one end**

**Boundary conditions:** 1 fixed and one free end – one node and one anti-node




| Name | Frequency | Wavelength |
|---|---|---|
| 1st harmonic | $f_1$ | $\lambda = 4L$ |
| 3rd | $f_3 = 3f_1$ | $\lambda = \frac{4}{3}L$ |
| 5th | $f_5 = 5f_1$ | $\lambda = \frac{4}{5}L$ |
| 7th | $f_7 = 7f_1$ | $\lambda = \frac{4}{7}L$ |

$\lambda_n = \frac{4L}{n}$ $n = 1, 3, 5 \ldots$

**Harmonics:**

**Summary:**

| | Boundary conditions | First Harmonic (fundamental) | Resonant Wavelengths (higher harmonics) | Resonant Frequencies (higher harmonics) |
|---|---|---|---|---|
| String of length L | Both ends fixed or both ends free | $L = \frac{1}{2}\lambda_1$ <br> $\lambda_1 = 2L$ | $\lambda_n = \frac{2L}{n}$ <br> where n = 1,2,3,4... | $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$ <br> where n = 1,2,3,4... |
| Pipe of length L | Both ends open or both ends closed | | | |
| | | | | |
| String of length L | One end fixed, the other end free | $L = \frac{1}{4}\lambda_1$ <br> $\lambda_1 = 4L$ | $\lambda_n = \frac{4L}{n}$ <br> where n = 1,3,5,7... | $f_n = \frac{v}{\lambda_n} = \frac{nv}{4L}$ <br> where n = 1,3,5,7... |
| Pipe of length L | One end open, the other end closed | | | |