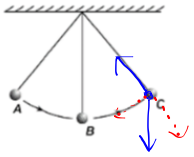
II. Pendulum

i) What factors influence the period of a pendulum?

Acceleration of gravity
Length of string

Initial angle (to some extent)
Not mass



ii) Determine the period, frequency, and angular frequency of oscillation.

$\omega = 2\pi f$ $\omega = \sqrt{g/L}$

$f = \frac{1}{2\pi}\sqrt{g/L}$ $T = 2\pi\sqrt{L/g}$

$a \propto \sin\theta$

$\sin\theta \propto \theta$ for small θ (in Rad)

Isochronous: same period for any amplitude of swing or oscillation

Natural frequency: the frequency at which an object naturally vibrates due to its physical properties

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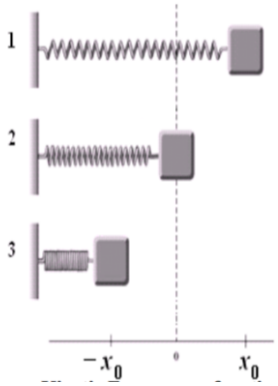
A mass oscillates back and forth on a spring between its extreme positions at 1 and 3.

1. Analyze the energy in the system at each location and derive expressions for each type of energy.



	Kinetic Energy	Potential Energy	Total Energy
1	0	$E_{p\,max}$	$E_T = E_{p\,max}$
2	$E_{k\,max}$	0	$E_T = E_{k\,max}$
3			

Kinetic Energy as a function of displacement

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

$V = V_0\cos(\omega t)$

$V = \pm\omega\sqrt{x_0^2 - x^2}$

Maximum Kinetic Energy **Total Energy**

$E_{p\,max} = E_{k\,max} = E_T$

$\frac{1}{2}m\omega^2x_0^2$ $\frac{1}{2}m\omega^2x_0^2$ $\frac{1}{2}m\omega^2x_0^2$

Potential Energy as a function of displacement

$E_p = \frac{1}{2}m\omega^2x^2$

$\omega = \sqrt{\frac{k}{m}}$

$k = \omega^2 m$

Special Note: The total energy of a system in simple harmonic motion is proportional to . . .

the square of the amplitude of oscillation

$E \propto x^2$

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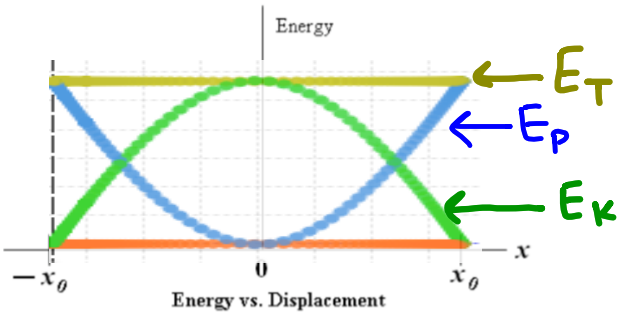
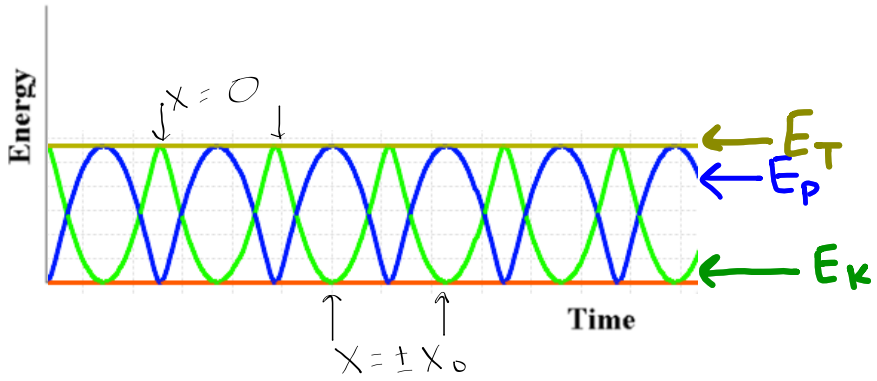
2. Sketch the energies as a function of displacement from equilibrium.



Energy vs. Displacement

3. Sketch the energies as a function of time.



4. A mass M is put on a spring, pulled back a distance A from its equilibrium position and set in simple harmonic motion with period T.

a) If the mass is pulled back to 2A and then released, what is the effect on the mass's:

i) period

ii) total energy

b) If the mass is doubled and then released from the original position, what is the effect on the mass's:

i) period

ii) angular frequency

iii) total energy