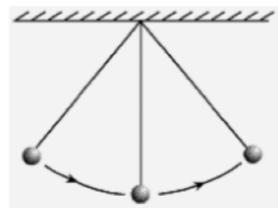
Oscillations and Waves

IB 12

Oscillation: the vibration of an object



Examples of oscillations:

a) a mass on a spring (eg. bungee jumping),

b) a pendulum (eg. swing),

c) an object bobbing in water (eg. buoy, boat),

d) a vibrating cantilever (eg. diving board),

e) musical instruments (eg. strings, percussion, brass, woodwinds, vocal chords)

f) an earthquake (eg. P waves and S waves),

Wave: a transfer of energy without a transfer of matter

Connection: Waves are caused by oscillating objects

Quantity	Symbol	Units	Definition	Formula
Period	T	$[s]$	time taken for one complete oscillation	$\frac{sec}{cycle}$
Frequency	f	$[hz] = [s^{-1}]$	number of oscillations per unit time	$\frac{cycles}{sec}$
Angular Frequency (Angular speed)	ω	$\left[\frac{rad}{sec}\right]$	Angular displacement per unit time (measures rotation rate)	$\omega = 2\pi f$

1. A pendulum completes 10 swings in 8.0 seconds.

a) Calculate its period.

$T = 0.8 s$

b) Calculate its frequency.

$1.25 hz$

c) Calculate its angular frequency.

$2\pi f = 7.8 s^{-1}$

IB 12

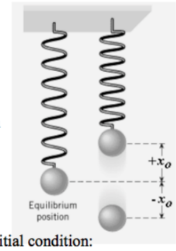
Mean Position (Equilibrium Position) – position of object at rest

Displacement (x, meters) – distance in a particular direction of a particle from its mean position

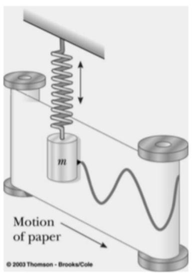
Sign: + or −

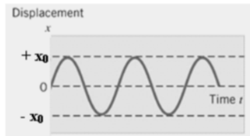
Amplitude (A or x_o, meters) – maximum displacement from the mean position

Sign: +



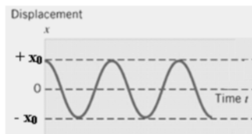
The Displacement Function





Initial condition:
starts at mean position

Function: $x = x_o \sin(\omega t)$

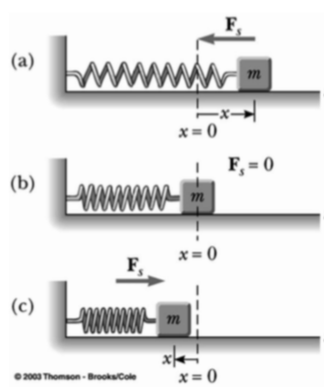


Initial condition:
starts at amplitude position

Function: $x = x_o \cos(\omega t)$

Restoring Force: A force that tends to restore the system to its equilibrium position.

Therefore, the restoring force is . . .
always opposite in direction to the displacement and points toward the equilibrium position



In general:

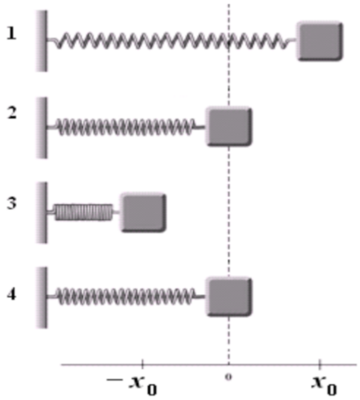
$\vec{a} \propto -\vec{x}$

For a spring:

$\vec{F}_s = -k\vec{x}$

$\vec{a} = -\frac{k}{m}\vec{x}$

A mass oscillates on a horizontal spring without friction. At each position, analyze the restoring force, displacement, velocity and acceleration.



	Displacement	Velocity	Restoring Force	Acceleration
1	$+X_o$	0	$-F_o$	$-a_o$
2	0	$-V_o$	0	0
3	$-X_o$	0	$+F_o$	$+a_o$
4	0	$+V_o$	0	0

1. When is the velocity of the mass at its maximum value?

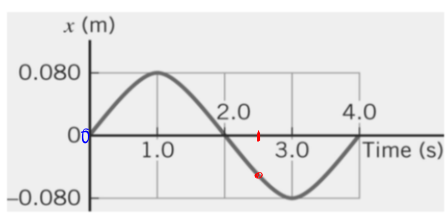
When the displacement = 0
at equilibrium position

2. When is the acceleration of the mass at its maximum value?

When the displacement and force = max
at extreme positions

4. The graph shows the displacement as a function of time for an oscillating object.

a) What is the amplitude of the motion?



b) Calculate the period, frequency, and angular frequency of the system.

$X_o = 0.08m$

$T = 4s$

$f = .25Hz$

$\omega = \pi/2 \ s^{-1}$

c) At what time(s) is the displacement of the object at a maximum? 1s 3s

d) At what time(s) is the velocity of the object at a maximum? 0s 2s 4s

e) At what time(s) is the acceleration of the object at a maximum? 1s 3s

f) Write a function for the displacement of the object versus time.

$$\vec{X} = 0.08m \sin\left(\frac{\pi}{2} t\right)$$

g) Use this function to determine the displacement of the mass at each of the times listed below. Then, check your answer by using the graph.

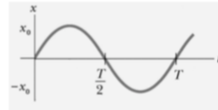
i) t = 1.0 s?

ii) t = 2.5 s? $-0.057m$

Equations of Motion for Simple Harmonic Motion IB 12

a) Displacement Function

$\vec{x} = x_0 \sin(\omega t)$



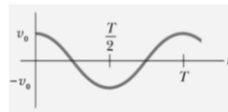
Maximum displacement: x_0

Occurs at t = $\frac{T}{4}, \frac{3T}{4}$

b) Velocity Function

$\vec{V} = \underbrace{\omega x_0}\cos(\omega t)$

$V = V_0 \cos(\omega t)$



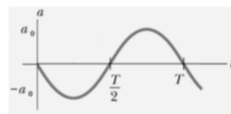
Maximum velocity: $V_0 = \omega x_0$

Occurs at t = $0, T/2, T$

c) Acceleration Function

$\vec{a} = -\underbrace{\omega^2 x_0}\sin(\omega t)$

$a = -a_0 \sin(\omega t)$



Maximum acceleration: $a_0 = \omega^2 x_0$

Occurs at t = $\frac{T}{4}, \frac{3T}{4}$

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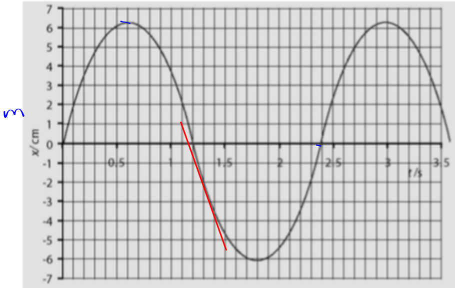
1. What is the phase difference between velocity and displacement?

   Velocity "leads" displacement by 90° or π/2 or T/4

2. What is the phase difference between acceleration and displacement?

   Acceleration "leads" displacement by 180° or π or T/2

3. The graph shown at right shows the displacement of an object in SHM. Determine the:



a) angular frequency

$T = 2.4\,s$

$X_o = 6.2\,cm$

b) maximum velocity and the times it occurs

$16\,cm/s$

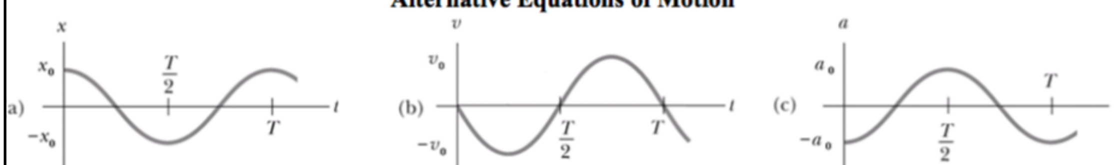
c) maximum acceleration and the times it occurs

$42\,cm/s^2$

d) Determine the velocity of the object at 1.3 seconds.

$-15.5\,cm/s$

Alternative Equations of Motion



$x = X_o \cos \omega t$ $V = -V_o \sin \omega t$ $a = -a_o \cos \omega t$

When would these equations be used?

When the object starts at its maximum position

Alternate Velocity Function

Derivation:

$v = \omega x_o \cos(\omega t)$

$\sin^2(\omega t) + \cos^2(\omega t) = 1$

$\cos^2(\omega t) = 1 - \sin^2(\omega t)$

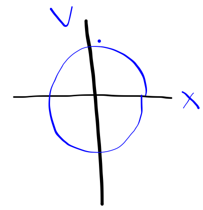
$\cos(\omega t) = \pm\sqrt{1 - \sin^2(\omega t)}$

$v = \omega x_o \left(\pm\sqrt{1 - \sin^2(\omega t)}\right)$

$v = \omega \left(\pm\sqrt{x_o^2 - \underbrace{x_o^2 \sin^2(\omega t)}_{x}}\right)$

$x = x_o \sin(\omega t)$

$x^2 = x_o^2 \sin^2(\omega t)$

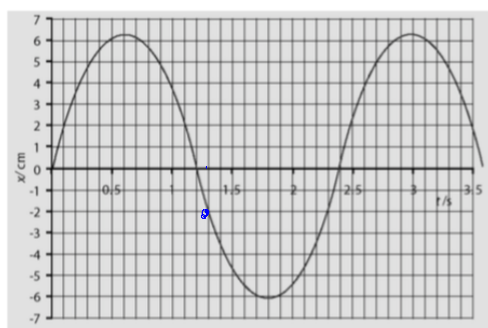


Formula:

$V = \pm \omega \sqrt{X_o^2 - X^2}$

variable

4. (Same graph as question #3) Determine the velocity of the object at 1.3 seconds by this new formula. Compare this to your answer for question #3 part (d) above.

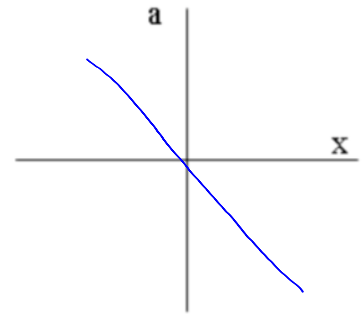


-2 cm

$V = \pm \omega \sqrt{X_o^2 - X^2}$

variable

Simple Harmonic Motion (SHM) – motion that takes place when the acceleration of an object is proportional to its displacement from its equilibrium position **and** is always directed toward its equilibrium position



Relationship

$a \propto -x$

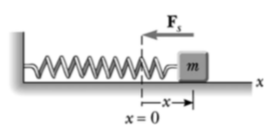
Defining Equation for SHM:

$\vec{a} = -\omega^2 \vec{x}$

Two Simple Harmonic Systems

I. Mass on a spring

i) Show that this is simple harmonic motion.



ii) Determine the frequency, angular frequency, and period of oscillation.

iii) What factors influence the period of a mass – spring system?