

Charging a Capacitor

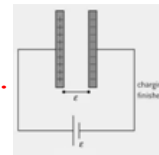
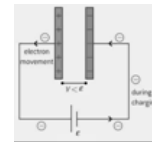
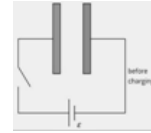
A capacitor which is initially uncharged is connected to a cell in order to be charged, as shown.

a) Charge on capacitor increases at a decreasing rate (exponential)

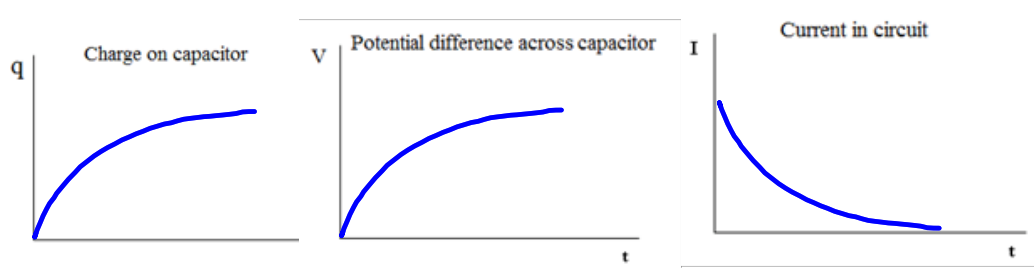
b) Potential difference across capacitor increases at a decreasing rate.

c) Current in circuit decreases at a decreasing rate.

d) After a “long time” the circuit reaches a **steady state**.



1. Sketch the following graphs for the charging circuit shown.



Type of relationships: **exponential (growth or decay)**

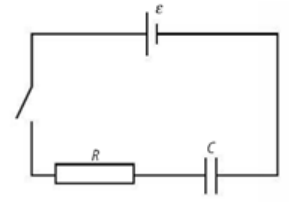
R-C circuit: circuit with capacitor and resistor used to control charging rate of capacitor

$$I = \frac{\Delta q}{\Delta t} \quad C = \frac{q}{V}$$

$$R = \frac{V}{I}$$

Effect of adding resistor: **slow down the rate of charging**

Common use: **timing circuit**



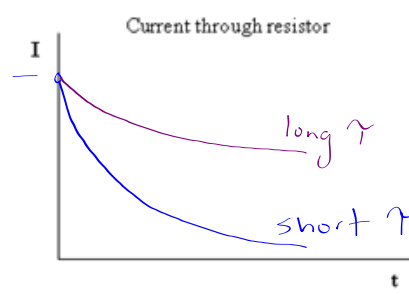
2. What determines the amount of time it takes the capacitor to charge? **Both resistance and capacitance**

Time constant: $\tau = RC$ $[\Omega \cdot F] = \left[\frac{V}{A} \cdot \frac{C}{V} \right] = \left[\frac{C}{C/s} \right] = [s]$

Units: **seconds**

3. On the axes at right, sketch a graph of the charging current for a circuit with a large time constant and one with a small time constant. Label each.

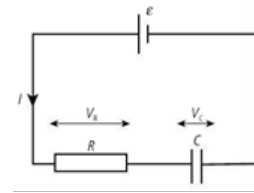
$$I_0 = \frac{\mathcal{E}}{R}$$



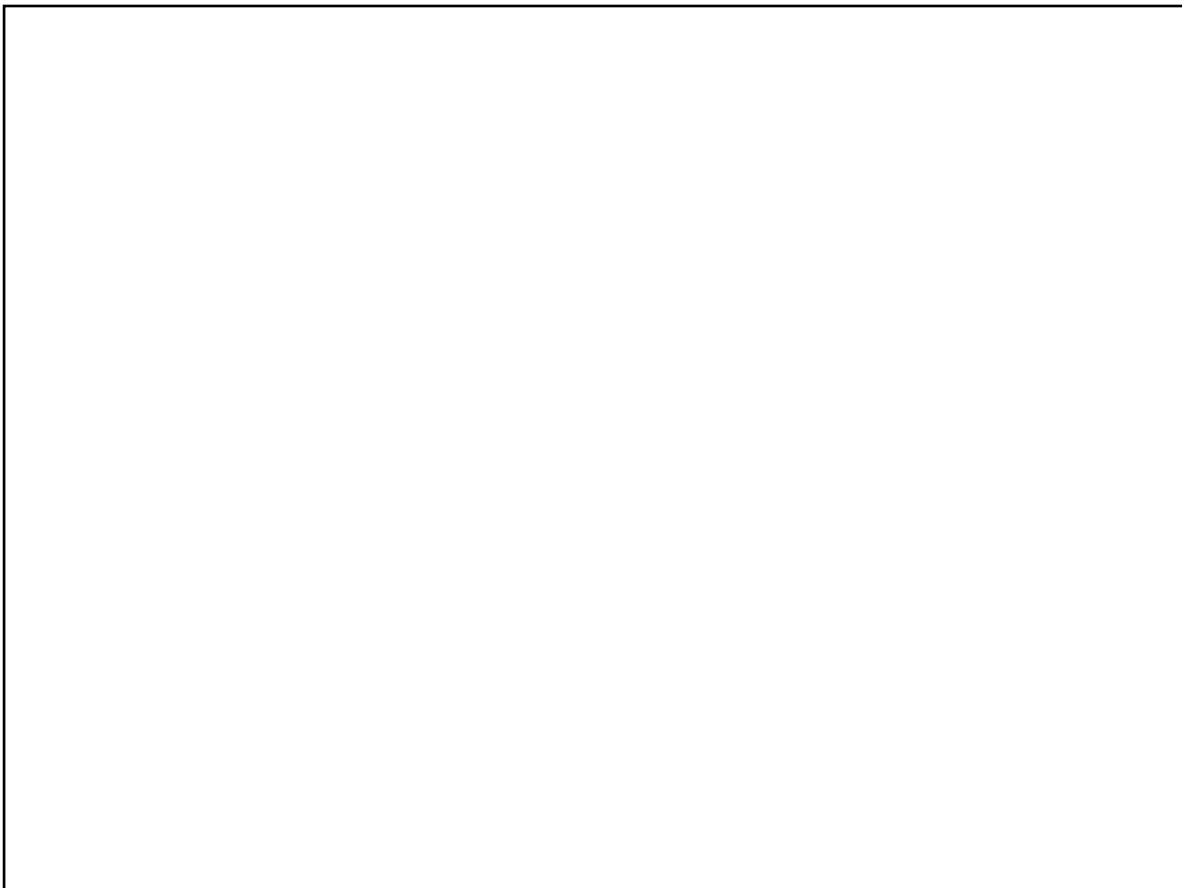
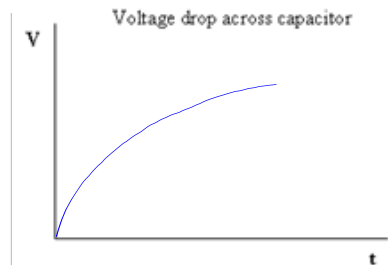
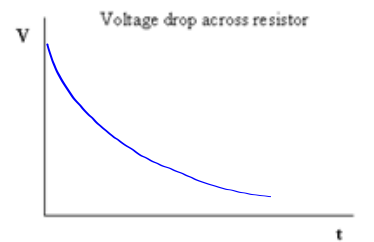
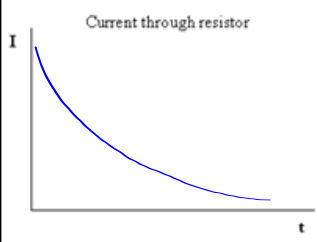
Maximum current: $I_0 = \mathcal{E}/R$

4. What are the steady state values?

Steady State: $I = 0$ $V_c = \epsilon$ $V_R = 0$



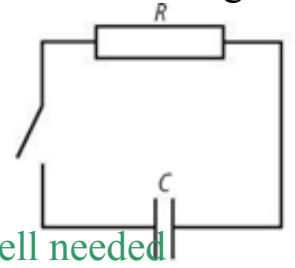
5. Sketch the following graphs for the R-C charging circuit above.



Discharging a Capacitor

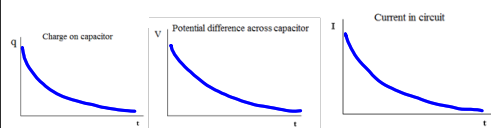
6. What happens when the switch is closed in the circuit at right?

Capacitor discharges as electrons move through the resistor from negative plate to positive plate until capacitor is neutral



Note: no cell needed

7. Sketch the following graphs for the R-C discharging circuit above.



$$I = \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

$$C = q/V \quad I = V/R$$

Derivation: $V_R + V_C = 0 \quad q = -RC \frac{dq}{dt}$

$$V_C = -V_R \quad \frac{q}{-RC} = \frac{dq}{dt}$$

$$\frac{dq}{dt} = -1/R \quad q = q_0 e^{-t/RC}$$

Time constant:

$$\tau = RC$$

time to discharge to 37% of initial value = e^{-1}

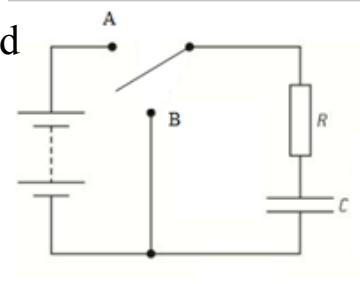
Math Models:

$$q = q_0 e^{-\frac{t}{\tau}} \quad V = V_0 e^{-\frac{t}{\tau}}$$

$$q = q_0 e^{-\frac{t}{\tau}} \quad CV = CV_0 e^{-\frac{t}{\tau}} \quad IR = I_0 R e^{-\frac{t}{\tau}}$$

$$V = V_0 e^{-\frac{t}{\tau}} \quad I = I_0 e^{-\frac{t}{\tau}}$$

8. A circuit is shown at right that can be used to both charge and discharge a $220\ \mu\text{F}$ capacitor through a $330\ \text{k}\Omega$ resistor.



a) In which position should the switch be to:

i) charge the capacitor? ii) discharge the capacitor?

b) Calculate the time constant for this circuit. 72.6s

c) If the capacitor has been charged to $30\ \text{V}$, what will be the potential difference across it after discharging for $20\ \text{seconds}$?

$$V = V_0 e^{-t/\tau}$$

\uparrow 30v \nwarrow 20s

d) How long will it take the capacitor to discharge to $10\ \text{V}$?

$$V = V_0 e^{-t/\tau}$$

$$10 = 30 e^{-t/\tau}$$

$$\ln\left(\frac{1}{3}\right) = -t/\tau$$

$$t = 79.8\text{s}$$

e) Calculate the amount of charge that has passed through the circuit in the time it takes to discharge to 10 V.

$$C = q/V$$

$$q_0 = CV_0 = 220 \mu\text{F} \cdot 30\text{V} = 6.6 \text{mC}$$

$$q_{(79.8\text{s})} = q_0 e^{-t/\tau} = 2.2 \text{mC}$$

$$= CV_{10}$$

$$\Delta q = q_0 - q = \underline{4.4 \text{mC}}$$

f) How much time will it take the capacitor to lose 90% of its charge? **167s**

Losing 90% means 10% left

9. The discharge current of a 470 μF capacitor through an unknown resistance falls from 87 mA to 12 mA in two minutes. Determine the unknown resistance.

$$I = I_0 e^{-t/RC}$$

$$\uparrow$$

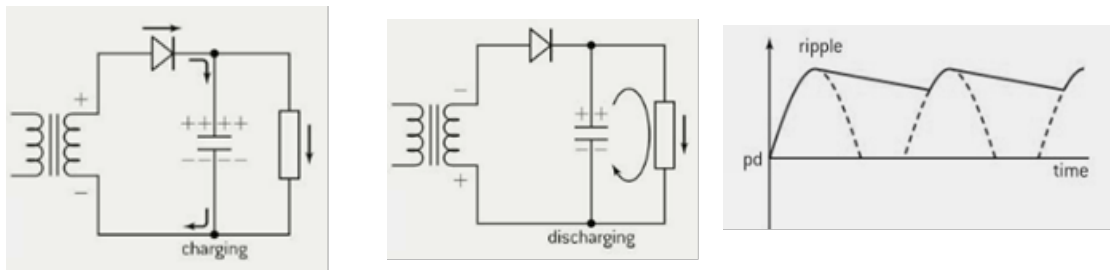
$$1.3 \times 10^5 \Omega$$

Effect of adding a capacitor on diode bridge rectification circuits

Effect: **smoothes the pulsing of the output voltage and current**

– makes output more steady

a) Half-wave rectification



b) Full-wave rectification

