d) What is the area of a circle whose radius is measured to be 6.2 cm ± 0.1 cm?

Area:

Determining uncertainty:

Maximum area:

Minimum area:

Evaluating Results

There are many ways to evaluate the accuracy of your results. One common method is to compare your results to a previously established value, called the "accepted value" or "literature value."

1. How does a value get to become an "accepted value?"

Many scientists measure value and agree on most probable result

2. Where would you look to find an "accepted value?" (Hint: Why do you think it's also called the "literature value?")

Technical handbooks or scientific journals or official websites

A simple method of comparing your results to the accepted value is known as "percent error."

$$\frac{|\text{lab - accepted}|}{\text{accepted}} \times 100\%$$

3. A student takes measurements and determines the density of a liquid to be 0.78 g/ml. The accepted value for this liquid's density is 0.82 g/ml. Calculate her percent error and make a conclusion.

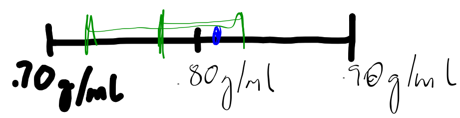
$$\frac{|.78\ g/mL - .82\ g/mL|}{.82\ g/mL} = 4.9\%$$

Her results agree with accepted value to within 5%

A more sophisticated method of evaluating your results is to determine if the literature value falls within your results' uncertainty range.

4. A student takes measurements and determines the density of a liquid to be 0.78 g/ml ± 0.05 g/ml. The accepted value for this liquid's density is 0.82 g/ml. Make a conclusion about her results.

Her results agree with accepted value, as the literature value falls within experimental uncertainty range



Data Processing Practice

1. Five people measure the mass of an object. The results are 0.56 g, 0.58 g, 0.58 g, 0.55 g, 0.59 g.

How would you report the measured value for the object's mass?

0.57 g ± 0.02 g

2. Adella Kutessen measured 8 floor tiles to be 2.67 m ±0.03 m long. What is the length of one floor tile?

0.334 m ± 0.004 m

3. The first part of a trip took 25 ± 3 s, and the second part of the trip took 17 ± 2s. How long did the whole trip take?

42 s ± 5 s

4. The sides of a rectangle are measured to be 4.4 ± 0.2 cm and 8.5 ± 0.3 cm. Find the area of the rectangle.

4.6 cm × 8.8 cm = 40.48 cm²

4.2 cm × 8.2 cm = 44.44 cm²   37 cm² ± 3 cm²

$\frac{.3}{8.5} \sim 3.5\%$

$\frac{.2}{4.4} \sim 4.5\%$

± 8%

± ½ range

5. A car traveled 600 m ± 10 m in 32 ± 3 s. What was the speed of the car?

19 m/s ± 2 m/s

6. The radius of a circle is measured to be 2.4 cm ± 0.1 cm. What is the area of the circle?

$\pi(2.5\,cm)^2 = 19.6\,cm^2$

$\pi(2.3\,cm)^2 = 16.6\,cm^2$

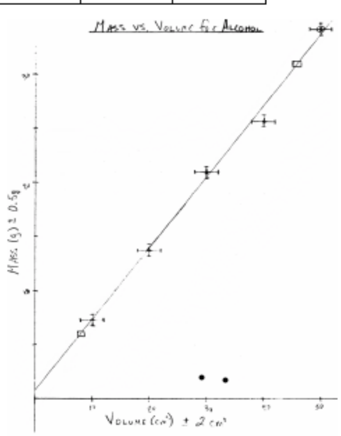
18 cm² ± 2 cm²

$\frac{.1}{2.4} \sim 4\%$

Conventions for Graphing Data

Volume (cm^3) ± 2 cm^3	10.	20.	30.	40.	50.
Mass (g) ± 0.5 g	7.2	12.8	21.0	25.7	35.2

1. Use a clean sheet of graph paper, a ruler, and a sharp pencil and plan your graph to take up most of the page.
2. Label each axis with a variable name, symbol, and units, for example **Volume (V) (cm^3)**. Usually, the independent variable is graphed on the x-axis, though there are often times and reasons for graphing it on the y-axis, such as, if it better matches your math model this way.
3. Choose an appropriate scale for each axis. Usually you should begin the graph at (0, 0). (On a **very rare** occasion, you may need a "break" in the graph where you skip to higher values. Avoid doing this if at all possible.) Space out the units appropriately and evenly. The value of the spacing does not have to be the same on each axis.
4. Title your graph: *Dependent vs. Independent* For example: *Mass vs. Volume for a Sample of Alcohol*
5. Plot your points carefully – make the dot large enough to see.
6. Should you put a data point at (0,0)? If it was not one of the measured data points, you will have to make a judgment as to whether or not it should be included as part of your graphed data.
7. Include ***error bars*** drawn to scale for each data point in at least one direction (x or y). Choose the most significant error bars (proportionally largest) to draw.
8. Do not play "connect-the-dots" with your data points. Look at the general shape made by your data points and decide what relationship it looks like. If it looks linear, draw in with a ruler a ***best-fit line*** (regression line) that fits within all or most of your error bars. Try to have as many points above the best-fit line as below it.



9. Consider any *outlier*. This is a point where the best-fit line doesn't go through the error bars. Maybe you made some mistake when taking this data point. You will have to explain why it's an outlier in your lab evaluation. If you have too many outliers, maybe the shape isn't really linear.
10. If it looks like some other relationship (quadratic, inverse, etc.), draw in that ***best-fit curve*** smoothly by hand. (Sometimes even a curve is called a best-fit "line.")
11. If it's a straight line, calculate the ***slope*** *(gradient)* of the line. Use two points **on** the line - these may or may not be data points. Put a box around each point used, state their actual coordinates (don't count boxes) and show your calculations, including formula and substitutions. Express the result as a decimal to the appropriate number of significant digits and include units. For example:

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{31.0g - 6.0g}{46.0cm^3 - 8.0cm^3} = 0.66\frac{g}{cm^3}$$

December 9, 2019

12. Write the ***experimental relationship*** for the line you've drawn by filling in the specific symbols for your data and the slope and y-intercept into the general equation for a line.

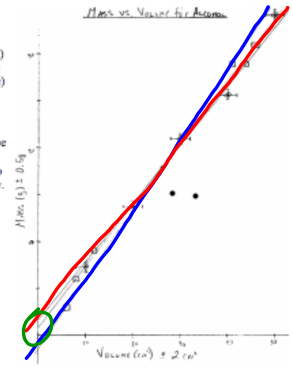
General Equation: $y = mx + b$ → slope $= 0.66 \frac{g}{cm^3}$, y-intercept $= 0.8$ g → *Experimental Relationship:* $M = (0.66\ g/cm^3)V + 0.8\ g$

13. Consider the physical significance of the slope and/or the y-intercept. Does it have a meaning or a special name? Compare your equation to a known ***mathematical model*** in order to draw conclusions.

Mathematical Model: *D= M/V* *or* *M = D x V*

Conclusion:
By comparing the experimental equation to the mathematical model, the slope represents the density of the liquid so the alcohol's density is 0.66 g/cm³.

14. Draw the line of maximum slope that fits your error bars and the line of minimum slope that fits your error bars. These are called your *max/min lines.* All three lines (best-fit, max, min) should cross at or near the midpoint of your data. The quickest and easiest way to do this is to connect the top(left) of the first error bar to the bottom(right) of the last error bar (for a minimum line) and the bottom(right) of the first error bar to the top(left) of the last error bar (for a maximum line) unless the first or last points are clear outliers – use your judgment.



15. Determine the slopes of your max and min lines. You do not need to show these calculations. Calculate therange of your slopes (max slope – min slope) and use ½ the range as the uncertainty for the slope of your best-fit line. Round the uncertainty to one sig fig and be sure to match the decimal place between the best-fit slope and its uncertainty. (You may need to round the best-fit slope to do this.)

Maximum slope: $0.74\ g/cm^3$

Best-fit slope: $0.66\ g/cm^3$

Minimum slope: $0.63\ g/cm^3$

Range: $0.74\ g/cm^3 - 0.63\ g/cm^3 = 0.11\ g/cm^3$

Uncertainty: $\frac{1}{2}(0.11\ g/cm^3) = 0.055\ g/cm^3 = 0.06\ g/cm^3$

Slope with uncertainty: **$0.66\ g/cm^3 \pm 0.06\ g/cm^3$**

16. If there is a physical significance for your slope and if there is a known literature value for this quantity, then decide if your results "agree with" the literature value by considering the uncertainty range.

Conclusion:
The literature value for the density of ethyl alcohol is 0.71 g/cm³. Our results agree with the literature value since 0.71 g/cm³ lies within the uncertainty range of 0.66 g/cm³ ± 0.06 g/cm³.

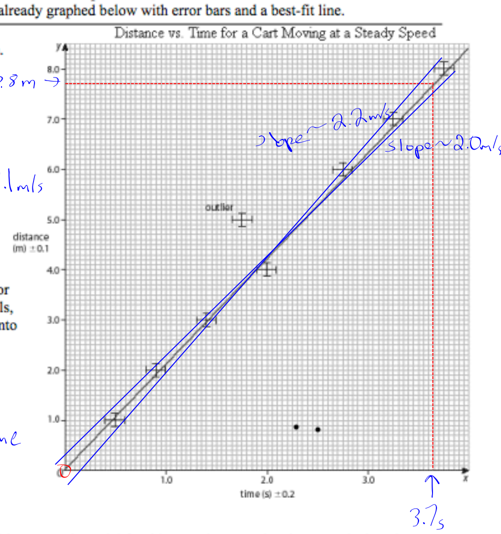
17. Does the mathematical model predict that the y-intercept should be zero? If so, see if your results agree with this by inspecting the graph to see if (0, 0) falls within the max and min lines you drew. If (0, 0) does **not** fall within the uncertainty range, you probably have a systematic error in your experiment that you will now have to account for.

Practice – Analyzing Data Graphically

An experiment was done to determine the relationship between the distance a cart moved and the time it took to do this. The data is already graphed below with error bars and a best-fit line.

1. Calculate the slope of the best-fit line. Show your work, including equation and substitution with units.

$Slope = \frac{\Delta Y}{\Delta X} = \frac{7.8m - 0}{3.7s - 0} = 2.1 m/s$



2. Write the experimental relationship for this data. (Substitute specific symbols, the slope and y-intercept with units into the general equation for a line.)

Speed = dist / time
(Y) (X)

dist = speed · time

D = 2.1m/s · t

3. Compare your experimental relationship to a math model for this experiment and make a conclusion about the meaning of the slope of the best-fit line.

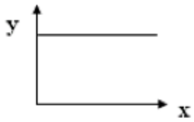
using D=v*t, the slope represents a speed of 2.1 m/s

4. Use a ruler and sharp pencil to draw in the max/min lines. Calculate the slopes of these lines and find the range of slopes. Finally, write the value for the slope with its uncertainty. (Remember, slope uncertainty = ½ range.)

+/- ½ range = ± ½ (2.2m/s − 2.0m/s)

$\boxed{2.1 m/s \pm .1 m/s}$

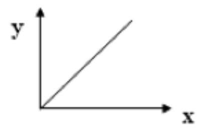
Graphical Representations of Relationships Between Data Sets

1. 

Name:
constant

General equation:
$y = c$

---

2. 

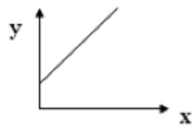
Name:
direct, proportional

Constant of proportionality c

Proportion: proportional
$y \propto x$

General equation:
$y = c \cdot x$

---

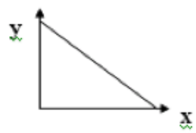
3. 

Name:
linear

NOT direct

General equation:
$y = mx + b$

---

4. 

Name:
linear (w/neg slope)

NOT inverse!

$y = mx + b$
General equation:

---

5. 

Name:
inverse

Proportion: $y \propto \frac{1}{x}$

General equation:
$y = c\,x^{-1}$

---

6. 

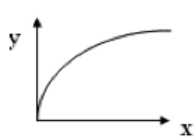
*

Name:
square

Proportion: $y \propto x^2$

General equation:
$y = \underline{c}\,x^2$

---

7. 

Name:
square root

Proportion: $y \propto \sqrt{x}$

General equation: $y = c\,x^{1/2}$

---

8. 

Name:
inverse square,

inverse square root?

Proportion: $y \propto x^{-2}$
$y \propto x^{-1/2}$

General equation:

Graph Straightening

Linearizing (straightening) a graph:

Transforming a non-linear graph into a linear one by an appropriate transformation of the variables and a re-plotting of the data points.

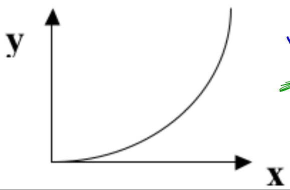
Purpose:

To find the constant of proportionality and write the experimental equation so the relationship can be compared to a mathematical moc

For each relationship shown below, give the name and the general equation for it.
Then, show the transformed variables that should be graphed in order to straighten the graph.

1.

Original Graph



Name and General Equation

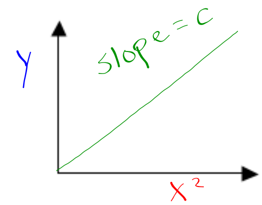
$y = c x^2$

square

Transformation of variables

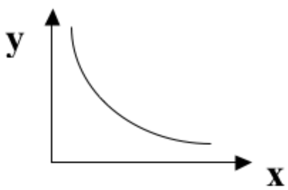
x	y	x^2

Straightened Graph



2.

Original Graph



Name and General Equation

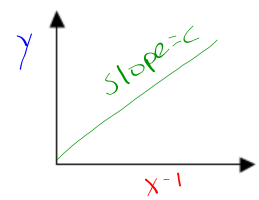
inverse

$y = c x^{-1}$

Transformation of variables

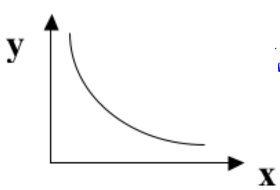
x	y	x^{-1}

Straightened Graph



3.

Original Graph



Name and General Equation

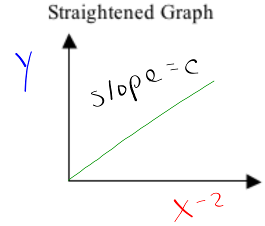
inverse square

$y = c x^{-2}$

Transformation of variables

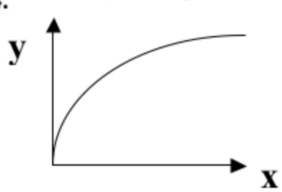
x	y	x^{-2}

Straightened Graph



4.

Original Graph



Name and General Equation

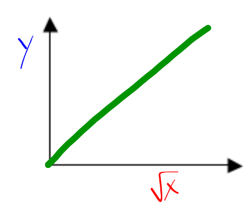
$y = c x^{1/2}$

square root

Transformation of variables

x	y	$\sqrt{x}$

Straightened Graph



How does straightening the graph help in writing the experimental equation for a non-linear relationship?

The slope shows the constant of proportionality