8. A proton is accelerated to a speed of 0.95*c*. Determine its energy, rest energy, and kinetic energy.

$E_0 = 938$ MeV

$E_k = 2064$ MeV

$E =$

$m_0 = 938 \frac{\text{MeV}}{c^2}$

**Particle acceleration**

units for charge: $1.6 \times 10^{-19}\,c = 1\,e.c.$ (or 1 e)

$$E_k = qV = (\gamma - 1)E_0$$

9. An electron is accelerated through a potential difference of 2.0 x $10^6$ V. Calculate its energy, kinetic energy, and speed.

$$E = E_0 + qV$$

$$\gamma = \frac{2.51}{.5} = \frac{E}{E_0} = \frac{M}{m_0}$$

$$= 4.922$$

$$.51\,\text{MeV} + (1e)(2\,\text{MV}) = 2.51\,\text{MeV}$$

$$2\,\text{MeV}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}} \quad \gamma^2 = \frac{1}{1-v^2} \quad 1 - v^2 = \frac{1}{\gamma^2} \quad v^2 = 1 - \frac{1}{\gamma^2} \quad v = \sqrt{1 - 1/\gamma^2}$$

$$= .979c$$

G. Relativistic Momentum and Energy

| | | |
|---|---|---|
| Newtonian momentum and kinetic energy | $p = \cancel{mv}$ $\quad E_k = \cancel{\frac{1}{2}mv^2}$ $\quad E_k = \cancel{\frac{p^2}{2m}}$ | |
| Relativistic momentum and kinetic energy | $p = \gamma m_0 v$ $\quad E_k = (\gamma - 1)E_0 = (\gamma - 1)m_0c^2 = qV$ | |
| units for Newtonian momentum | $(kg\,m/s)$ | units for relativistic momentum $[\text{MeV}/c]$ |
| **Relativistic total energy** | $E = \gamma E_0 = \gamma m_0 c^2 = mc^2$ | $E^2 = p^2c^2 + m_0^2c^4$ <br> $E^2 = p^2 + m_0^2$ |

1. a) A proton is at rest. Determine its:

i) mass

$938\, MeV/c^2$ $\qquad m_0 = 938\, \frac{MeV}{c^2}$

ii) energy

$938\, MeV$

$\gamma = 1$

iii) momentum

$0$

b) The proton is accelerated through a potential difference V until it reaches a speed of 0.900c. Determine:

$\gamma = 2.29$ $\qquad v = 1c$ $\qquad \gamma = \frac{1}{\sqrt{1-.9^2}}$

i) its kinetic energy

$1210\, MeV$

~~ii)~~ the potential difference

$qV = 1210\, MeV$

$(1e)\, 1210\, MV$

iii) its mass

$m = \gamma m_0$

$= 2150\, MeV/c^2$

iv) its energy

$E = E_0 + E_K$

$= \gamma E_0$

$= 2150\, MeV$

v) its momentum

$p = \gamma m_0 v$

$= 2.29 \cdot 938\, MeV/c^2 \cdot .9c = 1930\, \frac{MeV}{c}$

$E^2 = p^2 + m_0^2 \rightarrow p = \sqrt{E^2 - m_0^2}$

$= \sqrt{(2150\, MeV)^2 - (938\, MeV)^2}$

2. A proton is accelerated through a potential difference of $3.0 \times 10^9$ V.

a) Calculate the energy of the proton after its acceleration.

$V = 3000\, MV$

$E_k = 3000\, MeV$

$E = 3938\, MeV$

b) Calculate the final momentum of the proton.

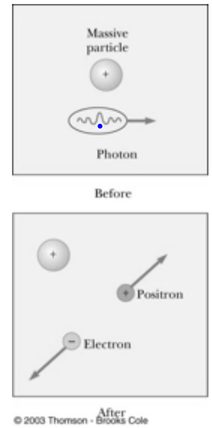
$p = \sqrt{(3938\, MeV)^2 - (938\, MeV)^2}$

$= 3820\, \frac{MeV}{c}$

$p = \underbrace{\gamma m_0}_{m} v$

3. "Pair production" is a process by which antimatter pairs of particles are produced from energy. This can happen when a high energy gamma ray photon is in the vicinity of a heavy nucleus. For example, if a gamma photon is near a lead atom, the reaction pictured at right might occur, where the photon creates an electron-positron pair. If the energy of the photon is 3.20 MeV calculate the following quantities. (Neglect the recoil of the lead atom and assume the energy is shared equally between the particles.)

$$\gamma \rightarrow e^- + e^+$$




a) The energy and kinetic energy of each particle.

$e^-$ $E_0 = .51\,\text{MeV}$ $E = 1.6\,\text{MeV}$ $E_k = 1.09\,\text{MeV}$

$e^+$ $E_0 = .51\,\text{MeV}$ $E = 1.6\,\text{MeV}$ $E_k = 1.09\,\text{MeV}$

b) The speed of each particle.

$\gamma = \frac{1.6}{.51} = 3.137$

$.95c$

c) The mass of each particle.

$1.6\,\text{MeV}/c^2$

$\gamma \cdot .51$

d) The momentum of each particle.

$p = \sqrt{E^2 - m_0^2}$

$= m \cdot \_\_ c$

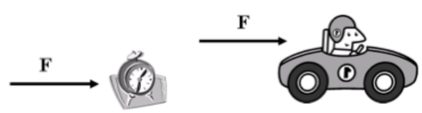
tot, $\gamma m_0$

$\frac{1.52\,\text{MeV}}{c}$

**General Theory of Relativity:** a more general theory of relativity that takes into account non-inertial (accelerating) reference frames and relates them to the effects of gravity

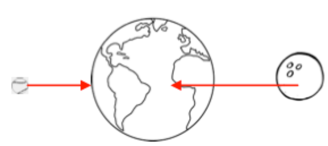
1. **Inertial Mass** – A property of an object that determines how much it resists accelerating.

   Different masses have different accelerations when the same net force acts on them.




2. **Gravitational Mass** – the property of an object that determines how much gravitational force it feels when near another object.

   Different masses have different gravitational forces acting on them them.



Observation: All experiments to measure each type of mass for an object have shown that, within the experimental uncertainty,

an object's gravitational mass is numerically equal to its inertial mass