Evidence to Support Special Relativity

1. **Michelson-Morley experiment**

   Evidence of: speed of light in a vacuum is the same for all observers

2. **Hafele-Keating experiment**

In 1971, experimenters J.C. Hafele and R.E. Keating from the U.S. Naval Observatory undertook an experiment to test time dilation. They made two flights around the world aboard commercial airliners, once east and once west, with each circuit taking about three days. They carried with them four cesium beam atomic clocks, accurate to within $\pm 10^{-9}$ s. The researchers expected that the relative motion of the clocks would produce a measurable time dilation effect ("moving clocks run slow").
In a frame of reference at rest with respect to the center of the earth,

a) the clock aboard the plane moving eastward, in the direction of the earth's rotation, is moving faster than a clock that remains on the ground, therefore it should

b) while the clock aboard the plane moving westward, against the earth's rotation, is moving slower than a clock that remains on the ground, therefore it should

When they returned, they compared their clocks with a ground based clock at the Observatory in Washington, D.C. The time intervals measured by the clocks that had traveled on the aircraft differed from those time intervals measured by the ground based clocks and provided confirmation of the time dilation effects of relativity.

Note that in this experiment, both time dilation due to motion or kinematics (special relativity) and time dilation due to gravity (general relativity) are significant and had to be taken into account.

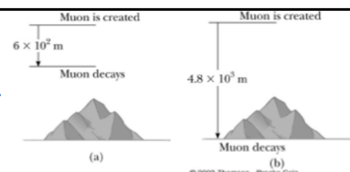
Evidence of: time dilation

Experimental Results:

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

3. **Muon Decay experiments**

A muon is an elementary particle that, like an electron, is classified as a lepton. It is unstable and decays into other particles with an average lifetime of about 2.2 microseconds. Muons are created naturally by collisions of incoming cosmic radiation from outer space with particles in the earth's upper atmosphere. Hence, this type of muon is known as a "cosmic ray muon."



The question is: Why do so many muons make it to the bottom of the mountain?
The answer lies in special relativity.

From the frame of reference of Earth, time runs slowly for the muon so it has time to reach the ground before decaying.

Evidence of: time dilation

From the muon's frame of reference, the height of the atmosphere contracts so the muon has very little distance to travel. It can easily cross this distance within its 2.2 μs lifetime.

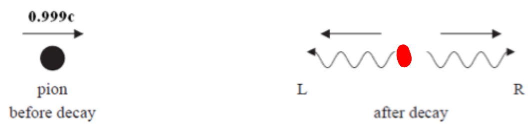
Evidence of: length contraction

1) A muon traveling at 0.99c has a gamma factor of 7.1.

   a) What is the lifetime of the muon, as measured from Earth?

4. Pion Decay experiments

A pion (short for pi meson) is an unstable subatomic particle (denoted π^+, π^-, or π^0). Experiments at CERN in 1964 showed that fast moving pions decayed into two high energy gamma-ray photons moving in opposite directions. Measurements of the speed of the photons showed that they were still moving at 3.00×10^8 m/s no matter how fast the original pion was moving.



Evidence of: speed of light in a vacuum is the same for all observers

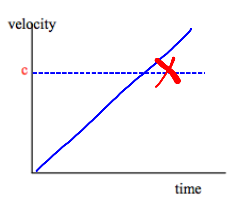
Newton mechanics: A constant force produces a constant acceleration.

Implication: Therefore, any speed can be attained if the force is applied for enough time

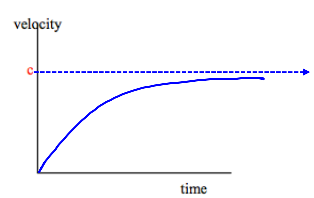
Relativistic mechanics: As the object's speed approaches the speed of light, the acceleration decreases even if the force is constant.

Implication: Its mass is increasing

Newtonian mechanics



Relativistic mechanics



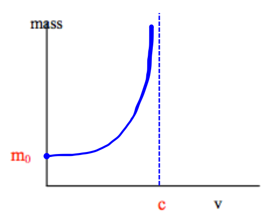
Rest mass (m_0): the mass of an object as measured in a frame of reference where the object is at rest

NOTE: rest mass is an invariant quantity

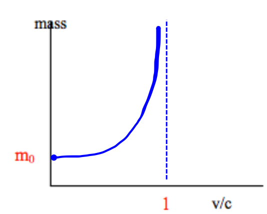
Mass (m): "relativistic mass" – mass of moving object – resistance to acceleration – inertial mass

Relationship: $m = \gamma m_0$

Mass versus actual speed



Mass versus relative speed



Consequence: No object can ever attain the speed of light in a vacuum or go faster

Explanation: The greater the speed of an object, the greater its mass. As mass increases, so does force needed to accelerate it. Mass becomes infinite as speed approaches c. Infinite amount of force or energy would be needed.

Rest energy (E_0): energy equivalent of the mass of an object at rest

Relationship: $E_0 = m_0 c^2$

1. What is the energy equivalent of a 0.20 kg golf ball at rest?

$E_0 = (.2\,kg)(3 \times 10^8\,m/s)^2 = 1.8 \times 10^{16}\,J$

2. What is the rest energy of an electron?

$m_0 = 9.11 \times 10^{-31}\,kg$

$9.11 \times 10^{-31}\,kg \cdot (3 \times 10^8\,m/s)^2$

$= 8.2 \times 10^{-14}\,J$

Alternate units for energy

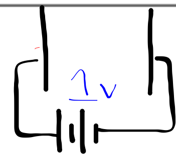
1 eV = energy gained by one electron accelerated through a potential difference of 1 volt

1 eV = 1.6×10^{-19} J

1 MeV = 1.6×10^{-13} J

3. What is the rest energy of an electron?

$$8.2 \times 10^{-14}\,\text{J} \times \frac{1\,\text{MeV}}{1.6 \times 10^{-13}\,\text{J}} = 0.51\,\text{MeV}$$



$$[V] = \left[\frac{J}{C}\right]$$

Alternate units for mass

4. What is the rest mass of an object whose energy equivalent is 1 MeV?

$$E_0 = m_0 c^2$$

$$m_0 = \frac{1\,\text{MeV}}{c^2}$$

5. What is the rest mass of an electron?

$$E_0 = m_0 c^2$$

E_0: .51 MeV; m_0: .51 MeV/c^2

Quantity	Units	Units
Energy	J	MeV
Mass	kg	MeV/c^2
Momentum	kg m/s	MeV/c

Formula representing equivalence of mass and energy: $E = \gamma E_0 = \gamma m_0 c^2 = mc^2$

For an object at rest: $\gamma = 1 \quad E = E_0$

For an object in motion: $\gamma > 1 \quad E > E_0$

6. What is the energy equivalent of an electron accelerated to a speed of 0.90*c*?

$\gamma = \frac{1}{\sqrt{1-.9^2}} = 2.29$

$E = \gamma E_0$

$= 2.29 \cdot .51\,MeV = 1.17\,MeV$

Total energy of a moving object =

$E_T = E_k + E_0$

$E = E_0 + E_k$

Derivation:

$\gamma E_0 = E_0 + E_k$

$E_k = \gamma E_0 - E_0$

$= (\gamma - 1) E_0$

Relativistic kinetic energy formula: $E_k = (\gamma - 1) m_0 c^2$ ~~$\frac{1}{2} mv^2$~~

7. What is the kinetic energy of an electron accelerated to a speed of 0.90*c*? The rest mass of an electron is 0.51 MeV c^{-2}.

$E_k = (\gamma - 1) E_0$

$= 1.29 \cdot .51\,MeV$

$= .663\,MeV$

$\gamma = 2.29$

$.663\,MeV$

$+ .51\,MeV$

8. A proton is accelerated to a speed of 0.95*c*. Determine its energy, rest energy, and kinetic energy.

$E_0 = 938\ \text{MeV}$ $m_0 = 938\ \frac{\text{MeV}}{c^2}$

$E_k = 2064\ \text{MeV}$

$E =$

Particle acceleration

units for charge:

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9. An electron is accelerated through a potential difference of 2.0 x 10^6 V. Calculate its energy, kinetic energy, and speed.

G. Relativistic Momentum and Energy

Newtonian momentum
and kinetic energy

Relativistic momentum
and kinetic energy

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units for Newtonian momentum	units for relativistic momentum

Relativistic total energy