Spacetime Diagrams and World Lines

One implicit assumption in our traditional motion diagrams is that time is the same for all frames of reference. We can visualize relativistic motion, but we need a more sophisticated diagram.

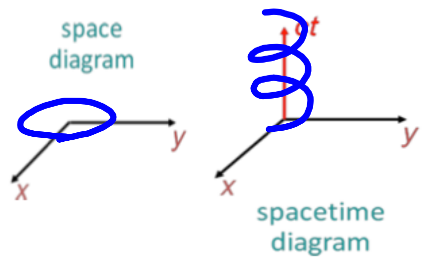
Spacetime diagrams are a very clear and illustrative way to show graphically how different observers in relative motion to each other have measurements that differ from each other.

1. As an example of the difference between a spacetime diagram and a traditional space diagram, contrast the plots of a particle in uniform circular motion in the x-y plane.

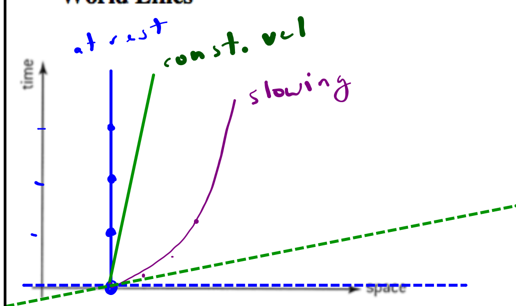
a. In the traditional diagram note that the particle keeps repeating its coordinates

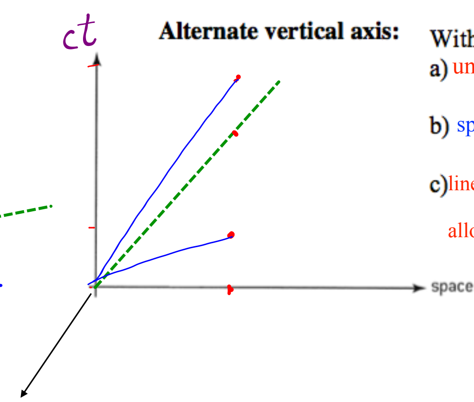
b. In the spacetime diagram the particle **never** repeats its coordinates.

c. It *will* repeat spatial coordinates as regularly as the in the traditional diagram, but *never* repeat its time coordinate.



World Lines





Alternate vertical axis:

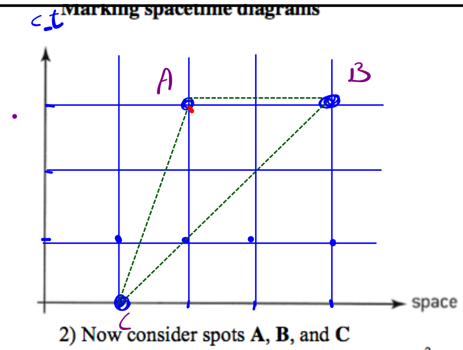
With a ct axis:

a) units are 'dist v dist'

b) speed of light = slope of 1

c) line w/slope of 1 = edge of 'light cone' – nothing allowed below

Marking spacetime diagrams



Consider yourself with a radar gun and a friend w/ a mirror (similar to a light clock)

- Your friend moves down the x axis and you fire the radar gun, which measures the time it takes the light to travel to and from your friend's mirror($\Delta t = 2D / c$, where D=desired unit)
- For ex. you want D=1m, each meter is $\Delta t = 2(1) / 3\times10^8$ seconds in elapsed time.

$(\Delta s)^2 = (\Delta x)^2 - (c\Delta t)^2$

2) Now consider spots **A**, **B**, and **C**

a) the square of the spacetime interval $(\Delta s)^2$ between each pair of pts.

AC: $(\Delta s)^2 = 1^2 - 3^2 = -8 \leftarrow$ time-like

AB: $(\Delta s)^2 = 2^2 - 0^2 = 4 \leftarrow$ space-like

BC: $(\Delta s)^2 = 3^2 - 3^2 = 0 \leftarrow$ light-like (null)

b) Which paths are "legal" paths for a particle to follow?

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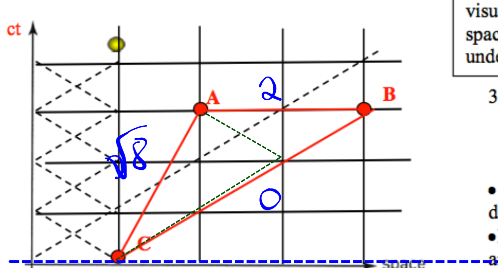
Marking spacetime diagrams



$|\Delta s^2|^{1/2}$

- Minkowski's view of space is non-Euclidean. In Euclidean space, which can also be four-dimensional, the distance formula would be

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 + (c\Delta t)^2.$$

- It is precisely the **subtraction** of the $(c\Delta t)^2$ term in

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2,$$

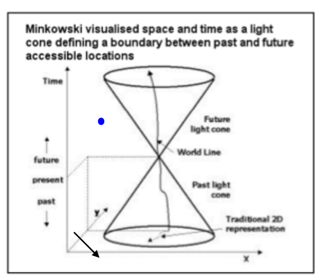
required by Einstein's 2nd postulate, that makes Minkowski spacetime non-Euclidean.

We cal this spacetime interval invariant

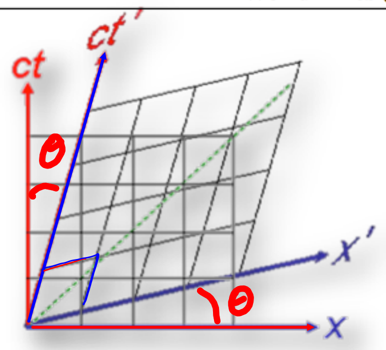
Nature of science: Visualization of models- The visualization of the description of events in terms of spacetime diagrams is an enormous advance in understanding the concept of spacetime.

3) Find the shortest distance between A and B.

- If we go directly from A to B the spacetime distance is 2.83 m.
- However, if we go from from A to C (2.00 m) and then from C to B (0.00 m) we cut off 0.83 m!
- Euclidean and Minkowski spacetime are very different!



World Lines (x) with another IRF (x')



Your Turn

Note:

a) if we set 'c=1', tan(θ)=v/c

b) for two IFR, we can align light lines, and 'constant time' lines for moving IFR are parallel to x' axis

c) the spacetime interval between any two points is invariant for all IFR

$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2$$

d) observers in S' still measure a right-angle geometry in their IRF

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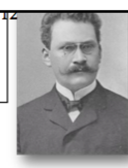
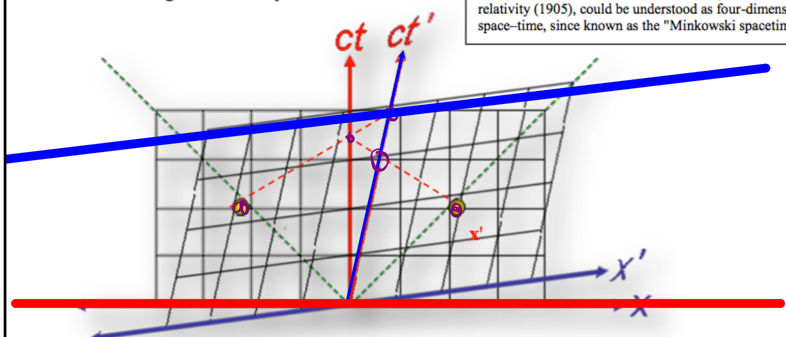
Revisiting Simultaneity

Hermann Minkowski (1864 – 1909) was a German-Jewish mathematician and professor. In 1907 he showed that his former student Albert Einstein's special theory of relativity (1905), could be understood as four-dimensional space–time, since known as the "Minkowski spacetime".





-An observer in IRF S is in the exact center of a train car. At the same instant, lights at each end of the car are turned on.

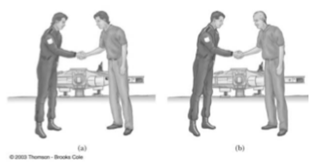
-Above, the world line of the same lights in a spacetime diagram for an observer in S', another IRF moving to the right relative to S. The two observers are directly opposite to each other at the instant the lights are turned on.

1) For the moving observer, which photon will be observed 1st?

2) Find the velocity of S' relative to S, in terms of *c*.

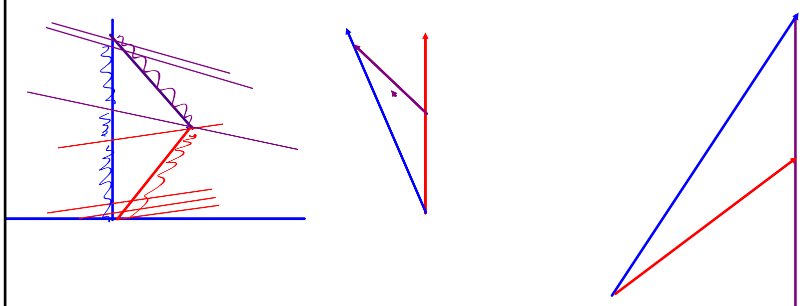
According to special relativity, there is no preferred inertial reference frame so the time dilation effect is the same for all observers. Since each observer sees the other as moving past at a constant speed, each observer measures the other's clock as running slowly – the effect is symmetric. But what about this?

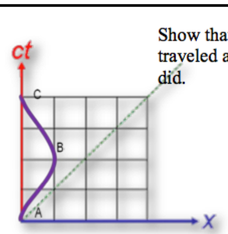
Two twins, Ein and Stein, grow up. Ein becomes and astronaut and Stein becomes a physics teacher. One day, Ein says goodbye to his brother and leaves on a space voyage to a distant star. Some time later, when he returns home, he meets his brother again. However, by now his brother is 30 years older than he is. You might think that this is because of relative motion. The clock in the space ship runs more slowly than the clock on the Earth, so Ein has aged less. But what about the symmetry of the time dilation effect? According to astronaut Ein, his ship was at rest while brother Stein and the Earth moved in the other direction. Since Stein's clock is now the moving one, shouldn't his clock run more slowly and Ein return to Earth as the older brother? Whose view of the situation is correct? In fact, shouldn't the brothers still be the same age since there is no preferred inertial frame of reference?



Explanation:

situation is **not** symmetric since formulas for special relativity are only symmetrical when the two observers are in constant velocity relative motion
- brother on space ship was not in *the same* inertial frame of reference for the entire trip
- he accelerated (to change FOR)
- brother on ground was not subject to forces or acceleration, did not change FOR, so his view of the situation is correct.





Show that in spacetime geometry, Einstein traveled a shorter distance than his twin on Earth did.

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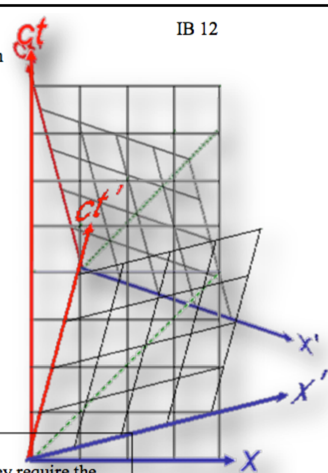

Above, the Twin Paradox scenario from the perspective of the twin on Earth (S). **Note:**

• The accelerations (curves) of Einstein's spacetime trajectory in S at the beginning, the turn-around, & the ending of his rocket trip.

• At no point is the slope of the tangent of Einstein's trajectory less than 1 (his speed is always $v < c$)

Theory of knowledge:
• Can paradoxes be solved by reason alone, or do they require the utilization of other ways of knowing?