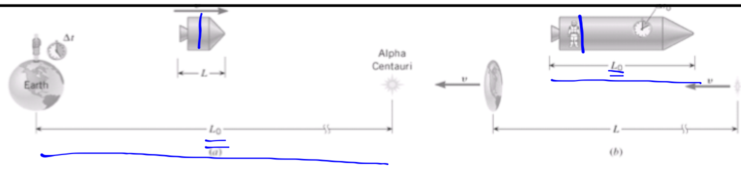



EXAMPLE: An astronaut is set to go on a journey to Alpha Centauri, a nearby star in our galaxy that the astronaut measures from her observatory to be 4.07 x $10^{16}$ m away. The astronaut boards the ship at rest on Earth before take-off and uses a meter stick to measure the length of the ship as 82 m and the diameter as 21 m. After take-off, an observer on Earth notices the space ship traveling past him at a speed of $v$= 0.950$c$ in route to Alpha Centauri.

a) How long does the trip to Alpha Centauri take as measured by:

i) the Earth bound observer?

$$t = \frac{d}{v} = \frac{4.07\times10^{16}\,m}{.95\times3\times10^{8}\,m/s} = 1.43\times10^{8}\,s \sim 4.5\,yr$$

$$\gamma = \sqrt{1-.95^2} = 3.2$$

ii) the moving astronaut?

$$V = \frac{d}{t}$$

$$t_0 = \frac{t}{\gamma} = \frac{4.5\,yr}{3.2} \sim 1.4\,yr$$

$$V = \frac{L}{t_0}$$

b) What is the distance between Earth and the star as measured by:

i) the Earth bound observer?

$$L_0 = 4.07\times10^{16}\,m$$

ii) the moving astronaut?

$$L = \frac{L_0}{\gamma} = \frac{4.07\times10^{16}\,m}{3.2} \sim 1.27\times10^{16}\,m$$

c) While the ship is on its journey, what is the length of the ship as measured by:

i) the Earth bound observer?

$$L = \frac{L_0}{\gamma} = \frac{82m}{3.2} \sim 26m$$

ii) the moving astronaut?

$$L_0 = 82m$$

d) While the ship is on its journey, what is the diameter of the ship as measured by:

i) the Earth bound observer?

21m

ii) the moving astronaut?

IB 12

**Light-year (ly):** Distance can also be measured in light-years which is the distance light will travel in one year.

$$1\,L_y = 9.46\times10^{15}\,m = c \times yr$$

e) If the distance to Alpha Centauri is 4.3 ly, how long will it take the spaceship:

i) as measured by the astronomer on Earth?

$$V = \frac{d}{t} = \frac{4.3(c\cdot yr)}{t}$$

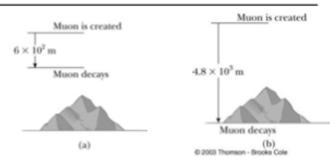
$$t = \frac{4.3(\cancel{c}\cdot yr)}{.95\cancel{c}} = 4.5\,yr$$

ii) as measured by the astronaut in the ship?

$$t_0 = \frac{t}{\gamma}$$

## Cosmic Ray Muon Experiment

**Muon:** unstable elementary particle




Experiment:

1) can be produced by collisions of cosmic radiation with atoms in upper atmosphere
2) due to unstable nature should only survive for a short time before decaying – shouldn't reach surface of earth
3) measurements of number of muons at top of mountain approximately same as at bottom of mountain

Question: Why do so many muons reach the ground before decaying?

$v = \frac{L_o}{t}$

From Earth frame of reference: time runs slowly for muon so it has time to reach ground before decaying

From muon's frame of reference: height of atmosphere contracts so has very little distance to travel

$v = \frac{L}{t_o}$

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EXAMPLE: A muon having a lifetime of 2.2 μs as measured in its own frame of reference is created in the upper atmosphere and travels toward Earth at a speed of 0.99c.

1. How far can a muon travel before it decays, as measured in its own frame of reference?

$d = v \cdot t_o$ 653m $7.089 = \gamma$

2. What is the lifetime of the muon, as measured from the Earth?

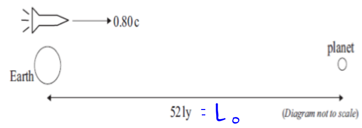
$t = \gamma t_o$ 15.6μs

3. How far will the muon travel through the atmosphere, as measured from the Earth?

$d_o = vt$ or 4630m

$L \cdot \gamma = L_o$

EXAMPLE: A spacecraft leaves Earth at a speed of 0.80 c as measured by an observer on Earth. It heads towards, and continues beyond, a distant planet. The planet is 52 light years away from Earth as measured by an observer on Earth. When the spacecraft leaves Earth, Amanda, one of the astronauts in the spacecraft, is 20 years old.




a) Calculate the time taken for the journey to the planet as measured by an observer on Earth.

$\gamma = \frac{1}{\sqrt{1-.8^2}} = 1.67$

$v = \frac{d}{t}$

$t = \frac{52\,\cancel{c}\cdot yr}{.8\,\cancel{c}} = 65\,yr$

b) Calculate the distance between Earth and the planet, as measured by Amanda.

$L = \frac{L_0}{\gamma} = 31\,ly$

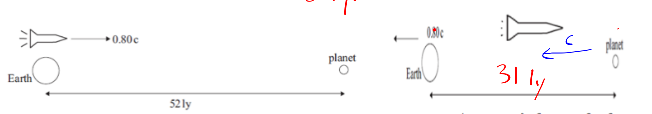
$t = \gamma t_0$ (65yr, 1.67)

c) Calculate Amanda's age as the spacecraft goes past the planet, according to:

i) an observer on Earth.

59yr

ii) Amanda. $t = \frac{31\,c\,yr}{.8c} = 39\,yr$




***Earth-based observer's frame of reference*** ***Astronaut's frame of reference***

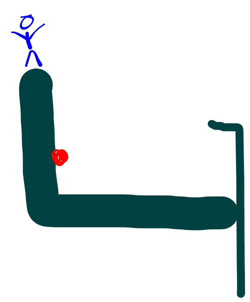
d) As the spacecraft goes past the planet, Amanda sends a radio signal to Earth. Calculate, as measured by the spacecraft observers, the time it takes for the signal to arrive at Earth.

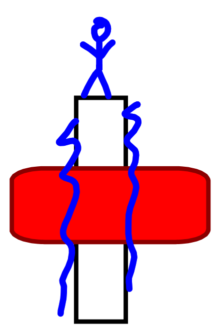
Total time: T

Total dist: CT

$CT = 31\,ly + .8cT$

$.2C = 31\,ly$

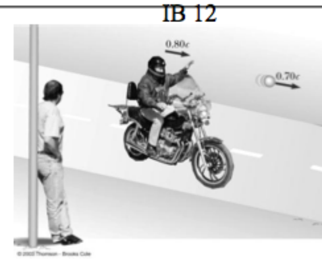




## Lorentz Transformations and Relativistic Formulas for Addition of Velocities

IB 12

1. A motorcyclist drives past a stationary observer at a speed of $0.80c$ and throws a ball forward at $0.70c$, as shown. The stationary observer is 30,000. m from a stoplight. After 3 seconds:
   **-how far is the motorcyclist from the stoplight?**
   **-how fast is the ball moving relative to the stationary observer?**




$\mathbf{x'}$ = position of an object as measured in moving frame
$\mathbf{x}$ = position of an object as measured in stationary frame
$\mathbf{u_x'}$ = velocity of object in x-direction as measured in moving frame
$\mathbf{u_x}$ = velocity of object in x-direction as measured in stationary frame
$\mathbf{v}$ = velocity of frame 2 in x-direction as measured in stationary frame
$\mathbf{c}$ = speed of light in a vacuum

Galilean transformation:

$x' = x - vt$

$30{,}000 - .8 \cdot 3\times10^{8} \cdot 3s \sim 9\times10^{8} m$

$\underset{.7c}{u'} = u - \underset{.8c}{v}$ $\quad u = 1.5c$

Relativistic transformation:


$x' = \gamma(x - vt)$ $\quad v = .8c$

$\gamma = 1.67$

$u = \dfrac{u' + v}{1 + \frac{u'v}{c^2}} = \dfrac{.7c + .8c}{1 + .7\times.8} = .96c$

**Relativistic transformation formula:**

$x' = \gamma(x - vt)$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$u' = \dfrac{u - v}{1 - uv/c^2}$

NOTE: For non-relativistic speeds (where v<<c), the answers provided by the two formulas are equivalent.

2. Suppose the motorcyclist in the above example shines a flashlight ahead of him. How fast does the stationary observer see the light beam travel?

$u =$

$u' = c$

$V = .8c$

3. Two bicyclists approach each other at a speed of 0.60*c*. What is their relative velocity of approach? What is the velocity of approach as measured by someone at rest with respect to the ground? $1.2c$

$- \longleftrightarrow +$

$.6c = V$ $\longrightarrow$

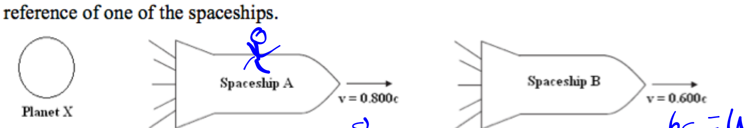
$\longleftarrow .6c$

moving obs.

$u = -.6c$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-.6c - .6c}{1 - (-.6)(.6)} = -.8c$$

4. Two spaceships are moving with the speeds indicated, as measured by an observer on Planet X. Calculate the relative velocity of approach, as measured in the frame of reference of one of the spaceships.




$V = .8c$ $\quad .6c = u$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{.6c - .8c}{1 - .6 \cdot .8} = -.385c$$

$.385c$

5. Two spaceships are moving with the speeds indicated, as measured by an observer on Planet X. Calculate the relative velocity of approach, as measured in the frame of reference of one of the spaceships.


$.946c$