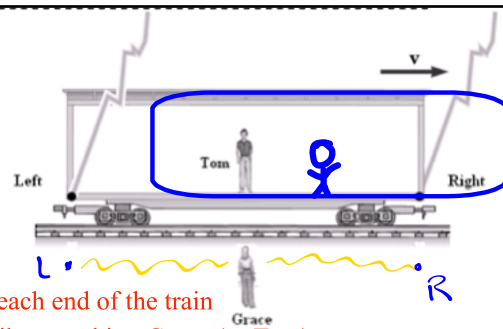
Simultaneity and the Relativity of Time

Event: something happening at a particular time and at a particular point in space

1. Two events occurring at different points in space and which are simultaneous for one observer cannot be simultaneous for another observer in a different frame of reference.
2. Two events occurring at the same point in space and which are simultaneous for one observer are simultaneous for all observers.

Einstein's Train *Gedanken* (Thought Experiment)

1. A train is traveling to the right with constant speed v with respect to the ground. Tom is in the midpoint of the train car. At the moment Tom passes Grace, two bolts of lightning strike the ends of the car, as seen by Grace. What does each observer notice and why?



Events occurring at different locations: lightning striking each end of the train

Events occurring at the same location: light from the strikes reaching Grace (or Tom)

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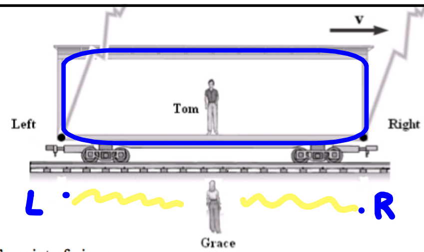


From Grace's point of view:

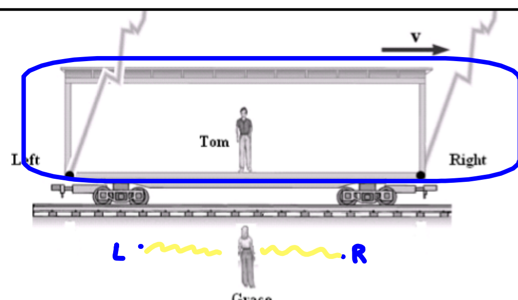
The strikes are simultaneous to her but not to Tom

Grace's observation about herself:

The strikes are simultaneous since the light from each strike has to travel the same distance to her at the same speed (c) and hence will take the same amount of time to get to her.

Grace's observation about Tom:

The strikes are not simultaneous since Tom is moving towards the light traveling from the Right strike and traveling away from the light traveling from the Left strike. Since the speed of light in a vacuum is the same for all observers in inertial reference frames, the light from the Right strike will reach him first. Hence, he will report that the lightning struck the Right end first and then the Left end.



From Tom's point of view:

The strikes are not simultaneous to him but are to Grace

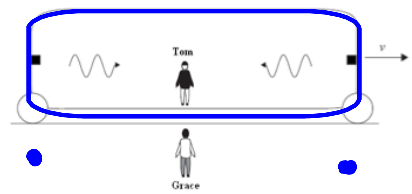
Tom's observation about himself:

His car is stationary. The lightning will strike the Right end first and then the Left end.

Tom's observation about Grace:

Grace will see the strikes as simultaneous since she is moving to the left, away from the light traveling to her from the Right end and toward the light traveling to her from the Left end. Since the speed of light in a vacuum is the same for all observers in inertial reference frames, and the light has a longer distance to travel from the Right end, it will reach Grace at the same time as the light from the Left end.

2. Grace is at rest with respect to the ground. Tom is in a carriage that is moving with speed v relative to Grace in the direction shown. Two flashes of light are emitted from the back and the front of the carriage. According to Tom's clock they arrive at Tom's position simultaneously. Explain why the arrival of the light pulses at Tom will also be simultaneous to Grace.



The arrival of the light flashes occurs at the same location in space and since they are simultaneous for one observer (Tom) they are simultaneous for all observers (Grace). This means that Grace reports they reach Tom simultaneously (but not that they were emitted simultaneously since the flashes were emitted at two different locations.)

Tom would report that the flashes were emitted from each end of the carriage simultaneously and since they travel the same distance to him at the same speed would arrive at the same time.

Grace would report that the flash from the left end occurred first. Since Tom is moving away from that flash which has a longer distance to travel and towards the flash from the right end which has s shorter distance to travel at the same speed, the flashes arrive at him simultaneously.

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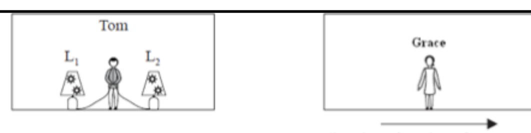
3. Tom and Grace are two observers each in a separate reference frame. The reference frames are moving relative to each other in the same straight line with constant velocity.



direction of motion of Grace's reference frame relative to Tom's reference frame

Two lamps L_1 and L_2 are operated by the same switch. Tom is at the mid-point between the lamps as measured in his frame of reference. The lamps and the switch are at rest relative to Tom.

Tom switches on the lamps and to him they light simultaneously. Explain why the lamps will not light simultaneously, according to Grace.

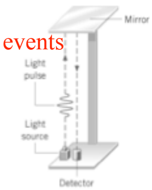
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Electrical signals (electric fields, electricity) travel at the speed of light. According to Grace, Tom is moving to the left. Hence the signal will reach lamp L_2, first since it is moving towards the signal, and then lamp L_1, since it is moving away from the signal. So lamp L_2 will light first, according to Grace.

But as the light flash emitted by lamp L_2 travels towards Tom, Tom is moving away from it and towards the light flash emitted from lamp L_1. Grace will thus predict that Tom will report that the lamps lit simultaneously since L_2 lit first but had a longer distance to travel (at the same speed) than the flash from L_1 which was emitted later but had a shorter distance to travel.

B. Time Dilation

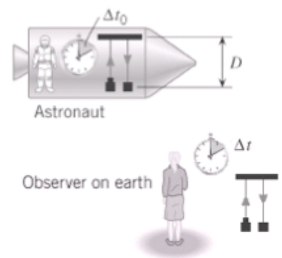
Light Clock:
a beam of light reflected between two parallel mirrors used to measure the time interval between two events



Beginning Event: the light pulse is emitted from the source

Ending Event: the light pulse is detected at the detector

Each observer uses a light clock to measure the time, as seen from their frame of reference, between the pulse being emitted and detected. When the space ship is at rest with respect to the observer on Earth, the two clocks measure the same amount of time.

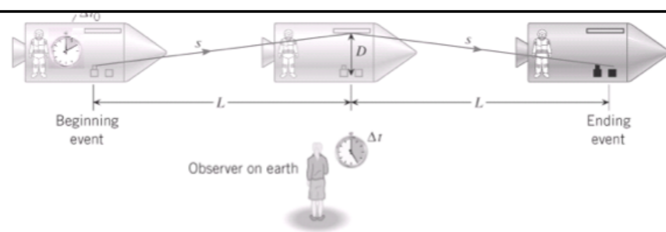


Δt_0 = time measured in the astronaut's frame of reference

Δt = time measured in the earth observer's frame of reference

If the two frames of reference are at rest with respect to one another, then $t = t_0$

If the spaceship moves to the right with a speed v, the observer on Earth sees the light pulse travel a greater distance between the two events. Since each observer measures the same speed for the light pulse, if it traveled a greater distance then it must have taken a longer time. The observer on Earth thus measures a greater time interval between the two events than the astronaut does.



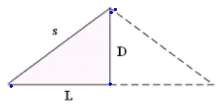
If the astronaut's frame of reference is moving with respect to the observer on Earth, then $t \geq t_0$

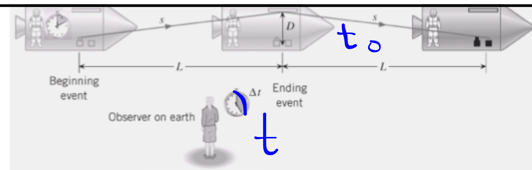
Time dilation: stretching of time – moving clock runs more slowly than stationary clocks

NOTE: situation is symmetric – astronaut sees Earth observer's clock run more slowly since ship could be at rest and the earth observer moving in the opposite direction

Proper time interval (Δt_0): the time between events as measured in a frame where the events take place at the same point in space (in moving frame)

NOTE: The proper time is the shortest possible time that any observer could correctly record for the time between events.





Derivation of time dilation formula:

Observer sees ship move: $L = vt$

Observer sees light pulse move: $S = ct$

Astronaut sees light pulse move: $D = ct_0$

$$L^2 + D^2 = S^2$$
$$(vt)^2 + (ct_0)^2 = (ct)^2$$
$$\left(\frac{v}{c}t\right)^2 + t_0^2 = t^2$$
$$t_0^2 = t^2 - \left(\frac{v}{c}t\right)^2$$

$$t_0^2 = t^2\left(1 - \frac{v^2}{c^2}\right)$$
$$t_0 = t\sqrt{1 - \frac{v^2}{c^2}}$$
$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Lorentz gamma factor

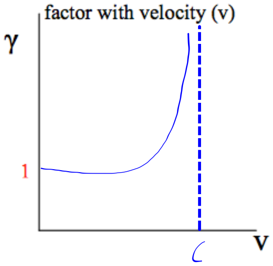
$$\frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

Note: $\gamma \geq 1$

$$t = \gamma t_0$$

Lorentz factor	For an object at rest:	At low (non-relativistic) velocities:	At high (relativistic) velocities:
$\frac{1}{\sqrt{1-(\frac{v}{c})^2}}$	$\gamma = 1$	$\gamma \approx 1$	$v \to c$ $\gamma \to \infty$

Variation of Lorentz gamma factor with velocity (v)

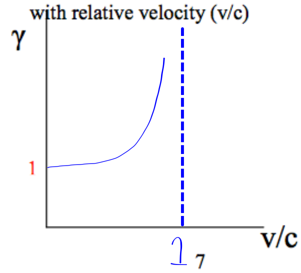


Relative velocity:

$v = .5c$

$$\gamma = \frac{1}{\sqrt{1 - .5^2}} = 1.2$$

Variation of Lorentz gamma factor with relative velocity (v/c)



Example: A certain particle created in an experiment has a lifetime of 2.2μs when measured in a reference frame in which the particle is at rest.

a) Describe a reference frame in which the particle could be considered at rest.

a lab, it's own frame

b) What is the "proper lifetime?"

$t_0 = 2.2\,\mu s$

c) In another experiment, the particle is accelerated in a "particle accelerator" to a speed of 2.7×10^8 m/s. This is the speed of the particle as measured relative to a stationary frame of reference. Give an example of such a frame of reference.

d) Calculate the Lorentz factor for this particle.

$$\frac{1}{\sqrt{1-\left(\frac{2.7\times10^8\,m/s}{3\times10^8\,m/s}\right)^2}} = 2.29$$

e) Calculate the lifetime of the particle as measured in the stationary reference frame.

$t = \gamma t_0 = 2.29 \times 2.2\,\mu s$

$\sim 5\,\mu s$

f) What would be its lifetime if it traveled at 0.98c?

5.03 $\quad$ 11 μs

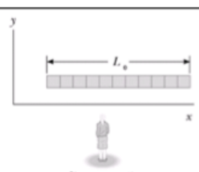
$$c = \frac{d}{t} = \frac{L}{t_0} = \frac{L_0}{t}$$

C. Length Contraction

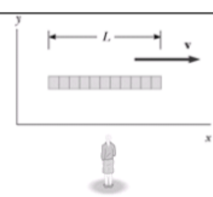
Because of Special Relativity, observers moving at a constant velocity relative to each other measure different time intevals between two evetnts. Bt if *speed = distance/time* and the speed is the same for each observer, then the two observers must measure different distances or lengths as well. This effect is known as **length contraction.**

length contraction: according to a stationary observer, moving objects contract (shrink) in the direction of motion (but not in perpendicular directions)

For example, a ruler at rest appears to have a length of L_0. This is known as its **proper length.**



For a stationary observer on Earth, a moving ruler would appear to be shorter but just as thick. It only shrinks in the horizontal direction.



proper length: the length of an object recorded in a frame of reference where the object is at rest

NOTE: This is the greatest possible length for the object

$$L = \frac{L_0}{\gamma} \leftarrow t = t_0 \times \gamma$$