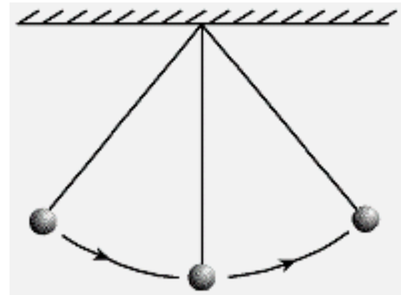
# Oscillations and Waves

IB 12

**Oscillation:** the vibration of an object



**Examples of oscillations:**

a) a mass on a spring (eg. bungee jumping),

b) a pendulum (eg. swing),

c) an object bobbing in water (eg. buoy, boat),

d) a vibrating cantilever (eg. diving board),

e) musical instruments (eg. strings, percussion, brass, woodwinds, vocal chords)

f) an earthquake (eg. P waves and S waves),

**Wave:** a transfer of energy without a transfer of matter

Connection:

| Quantity | Symbol | Units | Definition | Formula |
|---|---|---|---|---|
| Period | | | time taken for one complete oscillation | |
| Frequency | | | number of oscillations per unit time | |
| Angular Frequency (Angular speed) | | | Angular displacement per unit time (measures rotation rate) | |

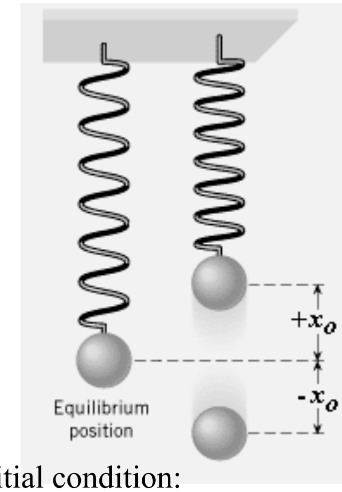
1. A pendulum completes 10 swings in 8.0 seconds.

a) Calculate its period.

b) Calculate its frequency.

c) Calculate its angular frequency.

**Mean Position (Equilibrium Position) –** position of object at rest



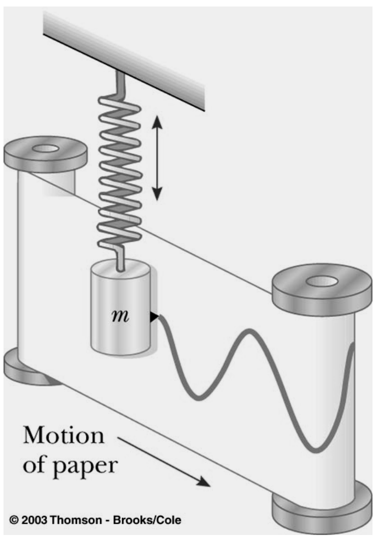

**Displacement** (x, meters) – distance in a particular direction of a particle from its mean position

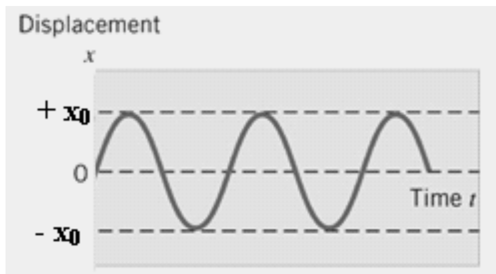
Sign:

**Amplitude** (A or $x_o$, meters) – maximum displacement from the mean position

Sign:

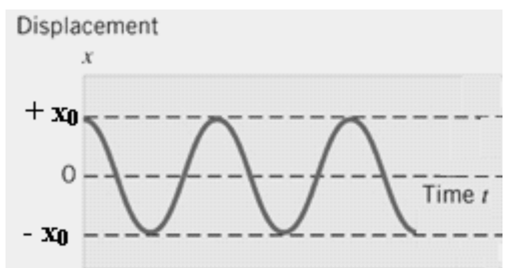
## The Displacement Function




Initial condition:

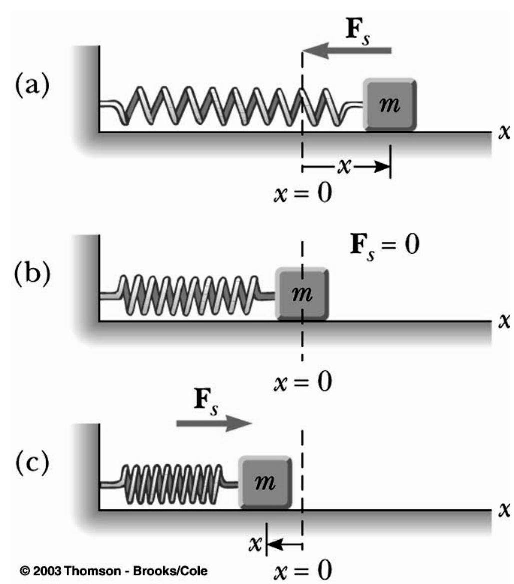
Function:




Initial condition:

Function:

**Restoring Force:** A force that tends to restore the system to its equilibrium position.

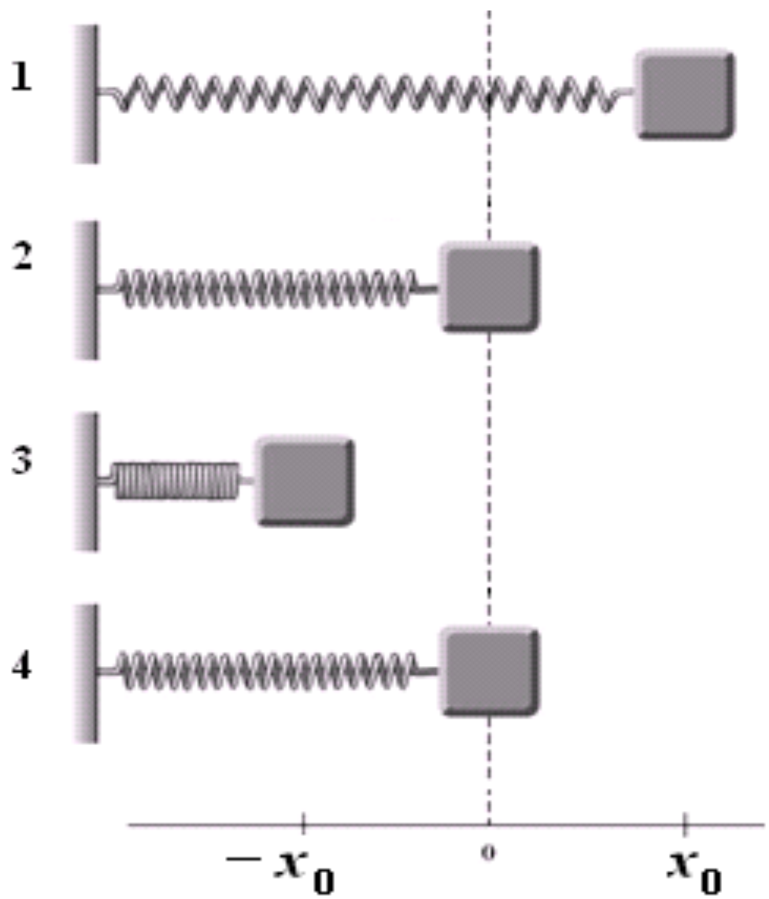



Therefore, the restoring force is . . .

**In general:** **For a spring:**

# Simple Harmonic Motion

A mass oscillates on a horizontal spring without friction. At each position, analyze the restoring force, displacement, velocity and acceleration.

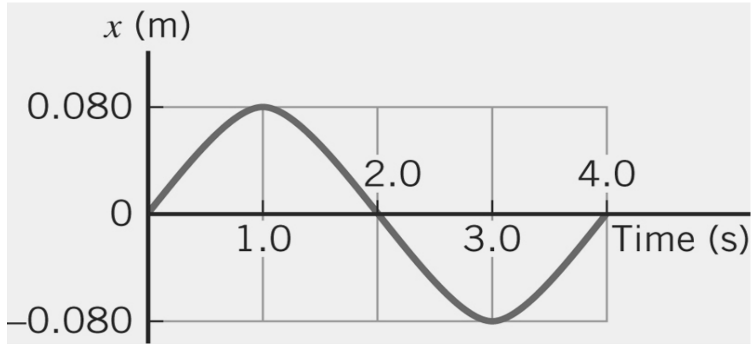



| | Displacement | Velocity | Restoring Force | Acceleration |
|---|---|---|---|---|
| **1** | | | | |
| **2** | | | | |
| **3** | | | | |
| **4** | | | | |

1. When is the velocity of the mass at its maximum value?

2. When is the acceleration of the mass at its maximum value?

4. The graph shows the displacement as a function of time for an oscillating object.




a) What is the amplitude of the motion?

b) Calculate the period, frequency, and angular frequency of the system.

c) At what time(s) is the displacement of the object at a maximum?

d) At what time(s) is the velocity of the object at a maximum?

e) At what time(s) is the acceleration of the object at a maximum?

f) Write a function for the displacement of the object versus time.

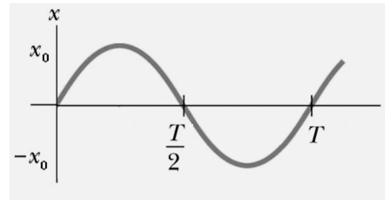
g) Use this function to determine the displacement of the mass at each of the times listed below. Then, check your answer by using the graph.

i) t = 1.0 s?

ii) t = 2.5 s?

# Equations of Motion for Simple Harmonic Motion

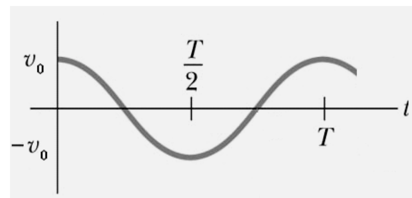
**a) Displacement Function**




**b) Velocity Function**

Maximum displacement:

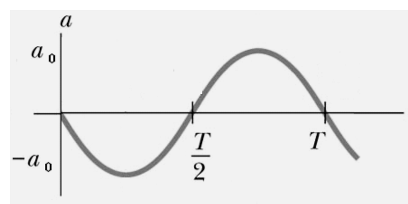
Occurs at t =




Maximum velocity:

Occurs at t =

**c) Acceleration Function**



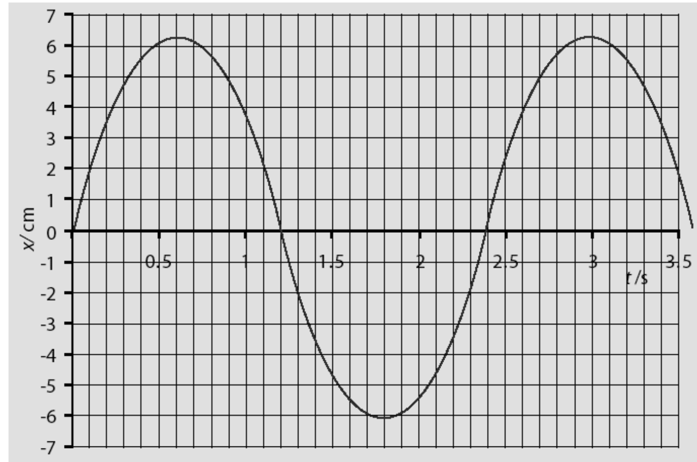

Maximum acceleration:

Occurs at t =

1. What is the phase difference between velocity and displacement?

2. What is the phase difference between acceleration and displacement?

3. The graph shown at right shows the displacement of an object in SHM. Determine the:




a) angular frequency

b) maximum velocity and the times it occurs

c) maximum acceleration and the times it occurs

d) Determine the velocity of the object at 1.3 seconds.

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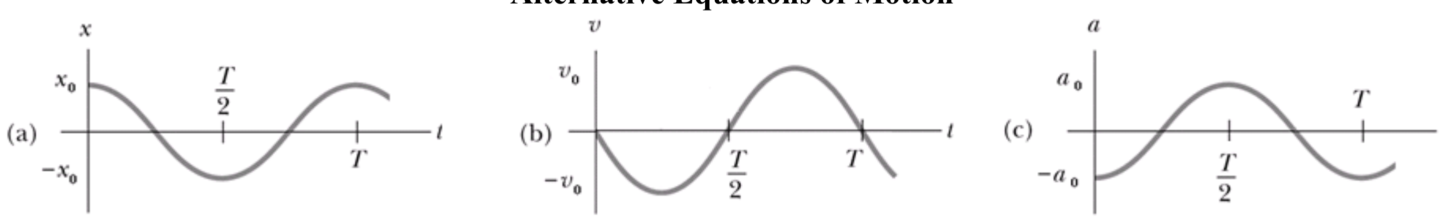
**Alternative Equations of Motion**




When would these equations be used?

**Alternate Velocity Function**

Derivation:

$$v = \omega x_o \cos(\omega t)$$

– – – – – – – – – – –

$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

$$\cos^2(\omega t) = 1 - \sin^2(\omega t)$$

$$\cos(\omega t) = \pm\sqrt{1 - \sin^2(\omega t)}$$

– – – – – – – – – – –

$$v = \omega x_o \left(\pm\sqrt{1 - \sin^2(\omega t)}\right)$$

$$v = \omega\left(\pm\sqrt{x_o^2 - x_o^2 \sin^2(\omega t)}\right)$$

– – – – – – – – – – –

$$x = x_o \sin(\omega t)$$

$$x^2 = x_o^2 \sin^2(\omega t)$$

– – – – – – – – – – –

Formula:

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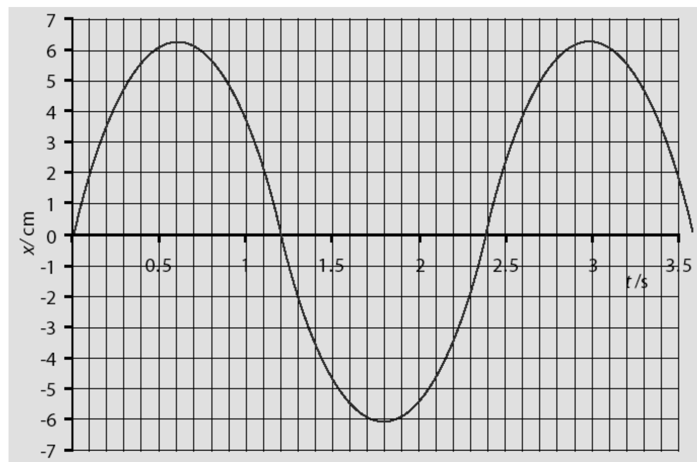
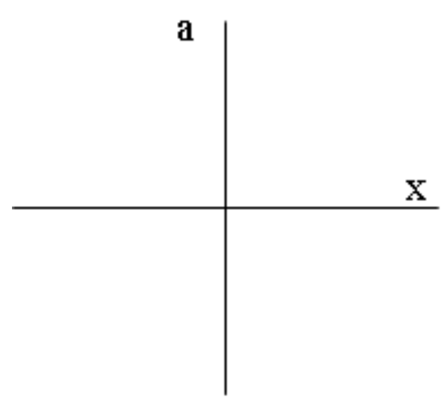
4. (Same graph as question #3) Determine the velocity of the object at 1.3 seconds by this new formula. Compare this to your answer for question #3 part (d) above.



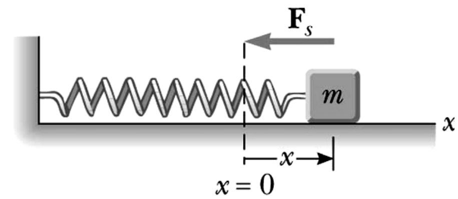

**Simple Harmonic Motion (SHM) –** motion that takes place when the acceleration of an object is proportional to its displacement from its equilibrium position **and** is always directed toward its equilibrium position




**Relationship** **Defining Equation for SHM:**

**Two Simple Harmonic Systems**

## I. Mass on a spring

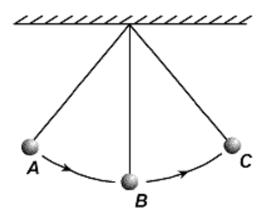



i) Show that this is simple harmonic motion.

ii) Determine the frequency, angular frequency, and period of oscillation.

iii) What factors influence the period of a mass – spring system?

## II. Pendulum




i) What factors influence the period of a pendulum?

ii) Determine the period, frequency, and angular frequency of oscillation.

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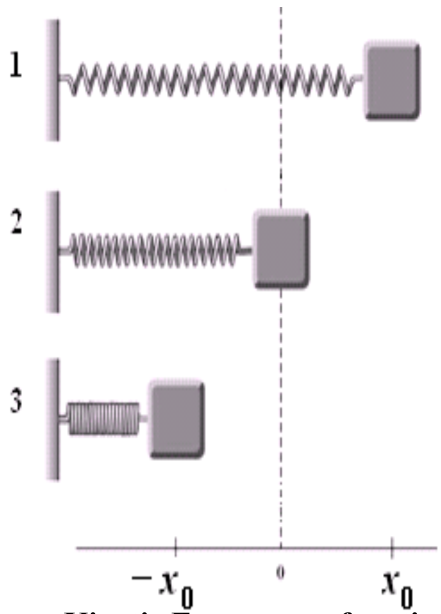
**Isochronous:**

**Natural frequency:**

# Energy and Simple Harmonic Motion

A mass oscillates back and forth on a spring between its extreme positions at 1 and 3.

1. Analyze the energy in the system at each location and derive expressions for each type of energy.




| | Kinetic Energy | Potential Energy | Total Energy |
|---|---|---|---|
| **1** | | | |
| **2** | | | |
| **3** | | | |

**Kinetic Energy as a function of displacement**

**Maximum Kinetic Energy**

**Total Energy**

**Potential Energy as a function of displacement**

**Special Note:** The total energy of a system in simple harmonic motion is proportional to . . .

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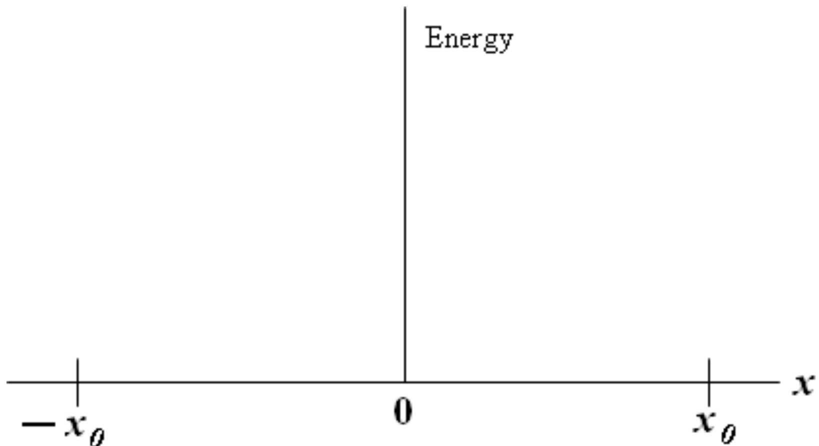
2. Sketch the energies as a function of displacement from equilibrium.




**Energy vs. Displacement**

3. Sketch the energies as a function of time.

4. A mass M is put on a spring, pulled back a distance A from its equilibrium position and set in simple harmonic motion with period T.

a) If the mass is pulled back to 2A and then released, what is the effect on the mass's:

i) period

ii) total energy

b) If the mass is doubled and then released from the original position, what is the effect on the mass's:

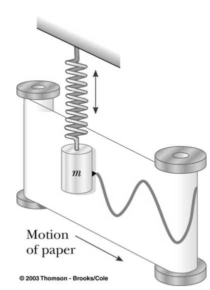
i) period

ii) angular frequency

iii) total energy

## Damping in Oscillations

**Damping**: the result of a force that acts on a system in the opposite direction to the direction of motion of the oscillating particle




When a force acts this way it is known as a . . .

Examples of a dissipative force:

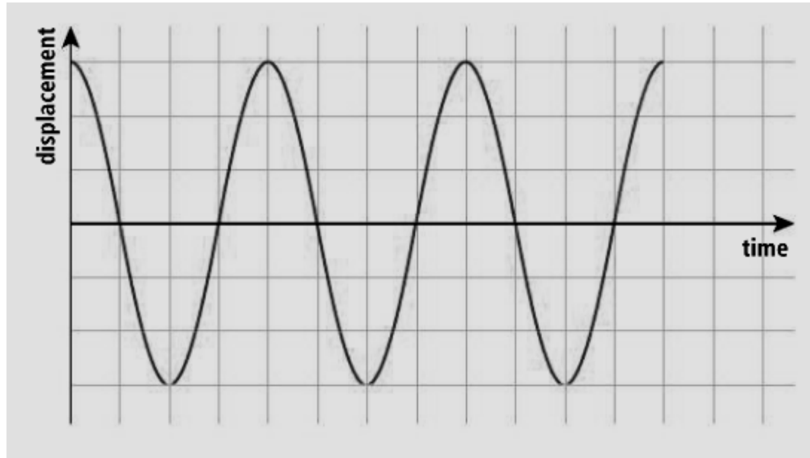
Effects of damping:

a)

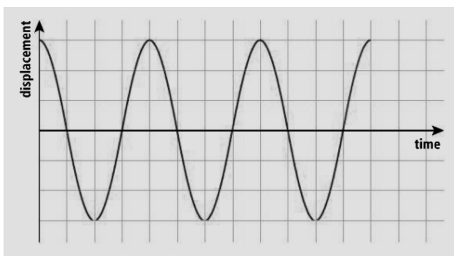
b)

1. The diagram at right shows the oscillation of a system without damping. Sketch the oscillations with damping.

$x = \_\_(x_o \cos(\omega t))$

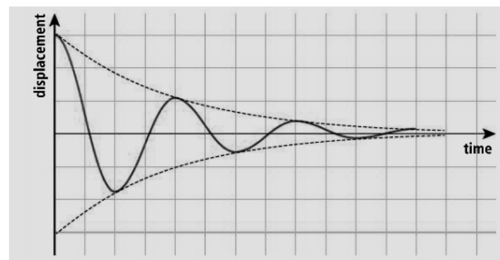



2. Write the equation for each motion.




Without damping

$$x = x_o \cos(\omega t)$$




With light damping

$$x = \_\_\ x_o \cos(\omega t)$$

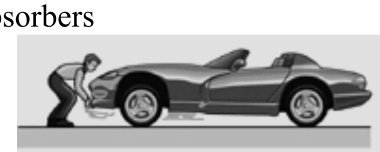
**Degrees of Damping**

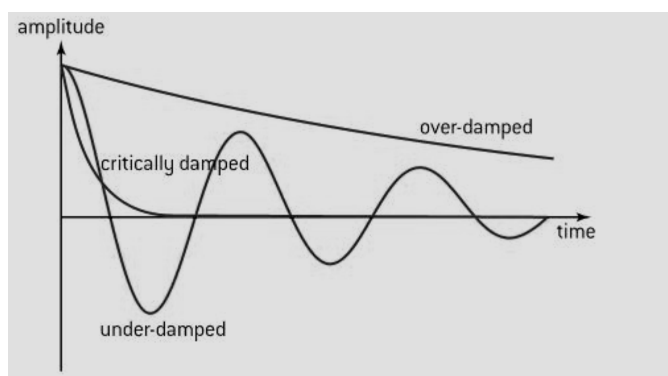
**a) Light damping (under-damping):** small resistive force so only a small percentage of energy is removed each cycle – period is not affected – can take many cycles for oscillations to die out

**b) Critical damping:** the smallest degree of damping that completely eliminates the oscillations – object settles back to rest in the shortest amount of time

**eg.** – electric meters with pointers, automatic door closers

**eg**. – car shock absorbers






Variation of amplitude with time for different degrees of damping.

**c) Heavy damping (over-damping):** large resistive force – can completely prevent any oscillations from taking place – takes a long time for object to return to mean position

**eg.**- oscillations in viscous fluid

# Resonance

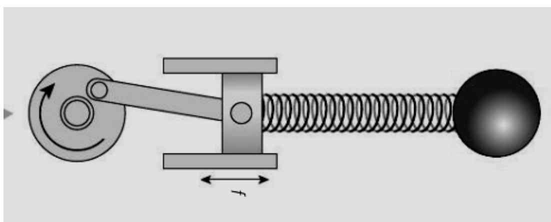


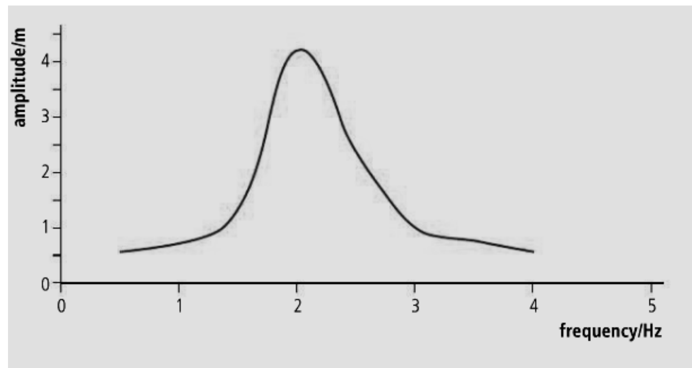
**Natural Frequency of Vibration**: when a system is displaced from equilibrium and allowed to oscillate freely, it will do so at its natural frequency of vibration

**Forced Oscillations –** a system may be forced to oscillate at any given frequency by an outside force that is applied to it (called a driving force) – this driving force supplies a **periodic stimulus** to the system

**Driving Frequency –** the frequency of a periodic stimulus or driving force that is forcing a system to oscillate

1. What happens to a system when it is made to oscillate by a driving force, that is, what is the response of a system to forced oscillations? For example, a mass on a spring is given a periodic stimulus by a rotating piston. A graph of the amplitude of the resulting oscillation as a function of the driving frequency is shown.




What is the significance of the peak on the graph?

**Resonance –** a transfer of energy in which a system is subject to a forced oscillation whose driving frequency matches the natural frequency of the system resulting in a large amplitude of vibration

### Useful Examples of Resonance:

a) **Musical Instruments:** Many musical instruments (including the human voice) produce their sounds by arranging for a column of air or a string to be driven at its natural frequency which causes the amplitude of oscillation (the volume) to increase.

b) **Microwave Generators:** Microwave ovens produce electromagnetic waves in the microwave portion of the EM spectrum. The driving frequency of these microwaves provides energy which is transferred to water molecules in the substance being heated so that their temperature increases.

c) **Quartz Oscillators:** When an AC voltage is applied to a quartz crystal, the crystal vibrates. If the driving frequency of the AC voltage matches the natural frequency of the quartz crystal, the crystal will vibrate with a large enough amplitude to be used to provide accurate clocks for microprocessors and to produce and detect sound waves in a medical test known as ultrasound.

d) **Greenhouse Effect:** The natural frequency of oscillation of the molecules of the greenhouse gases (water vapor, carbon dioxide, methane, nitrous oxide) is in the infrared region of the EM spectrum. Radiation from the Earth's surface can be readily absorbed by the greenhouse gases and then reradiated back down to Earth increasing the surface and air temperature of the Earth.

e) **Lasers:** The optical cavities in lasers set up standing waves for light in order to produce powerful coherent beams.

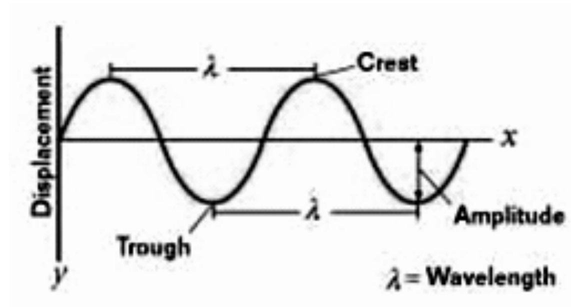
f) **MRI (magnetic resonance imaging) scans:** These use magnetic fields and oscillating radio frequency waves that resonate with protons in hydrogen atoms which emit energy to produce pictures of body tissue.

# Wave Phenomena – Part I



**Pulse** – single oscillation or disturbance

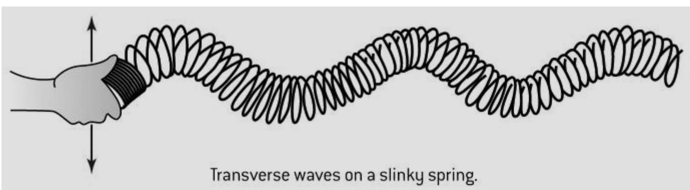
**Traveling wave (progressive wave, continuous wave)** – succession of oscillations (series of periodic pulses)

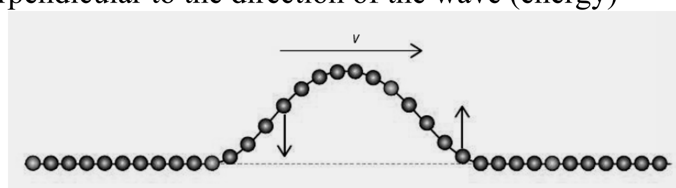



**Period**: (T, seconds)
a) time taken for one complete oscillation
b) time for one complete wave (cycle) to pass a given point

**Wavelength**: (λ, meters)
a) shortest distance along the wave between two points that are in phase
b) the distance traveled by a wave in one period

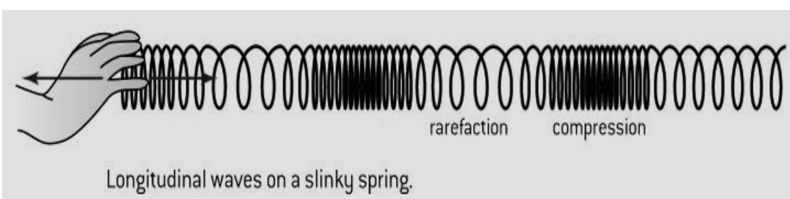
**Transverse Wave** – the particles of the medium move perpendicular to the direction of the wave (energy)

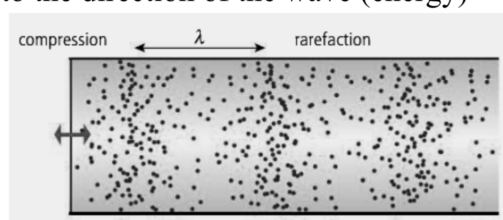



Note the direction of motion of the particles.

**Longitudinal Wave** – the particles of the medium move parallel to the direction of the wave (energy)




A sound wave is a longitudinal wave of varying air pressure.

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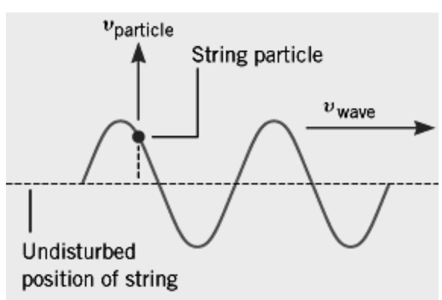
1. Distinguish between the motion of the wave (energy) and the motion of the particles.

**Motion of the Wave (Motion of Energy Transfer)**



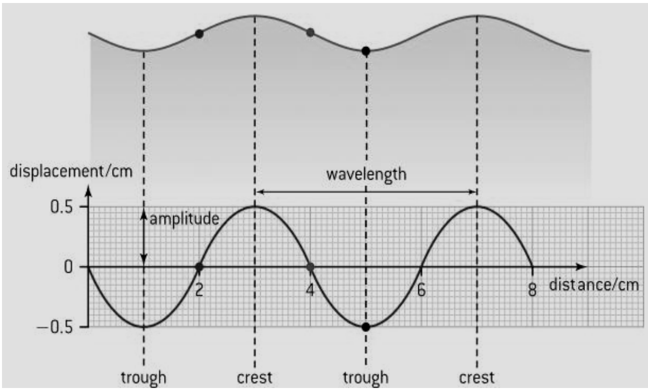

**Motion of a Particle**

Derivation of the Wave Equation

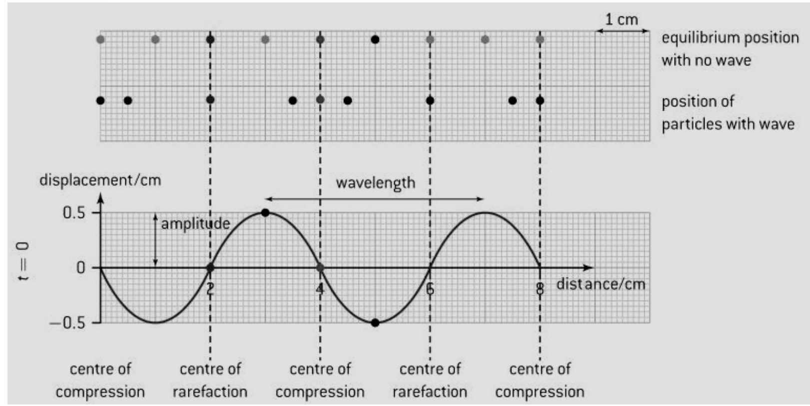
In time t = 1 period:

**Graphs of motion**

a) Displacement versus distance (position): shows where every particle is at any one instant

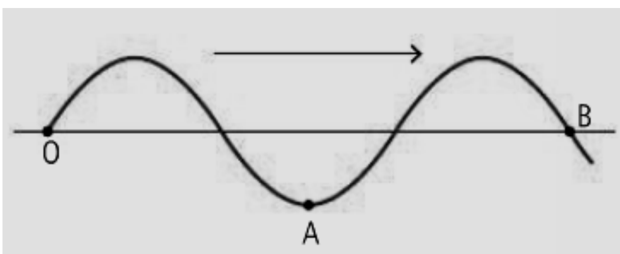



**Transverse Wave**

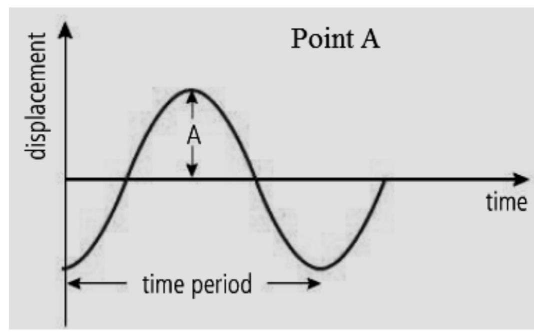


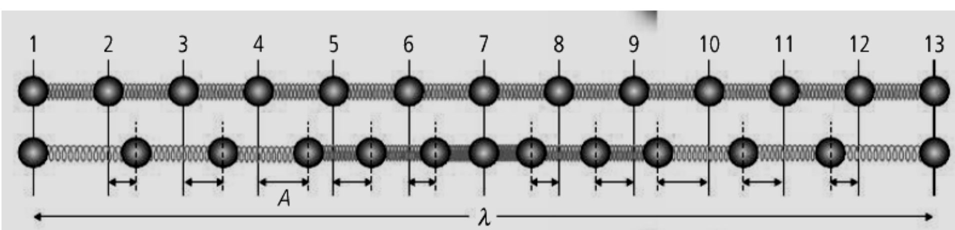

**Longitudinal Wave**

b) Displacement versus time: shows how the displacement of a single particle varies with time

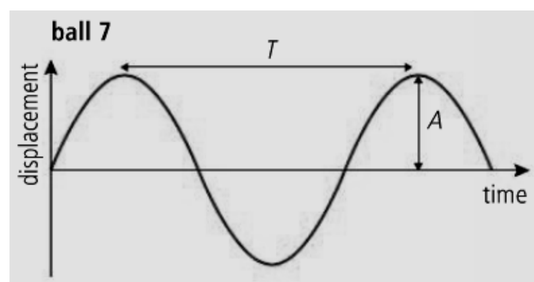



**Transverse Wave**




**Longitudinal Wave**

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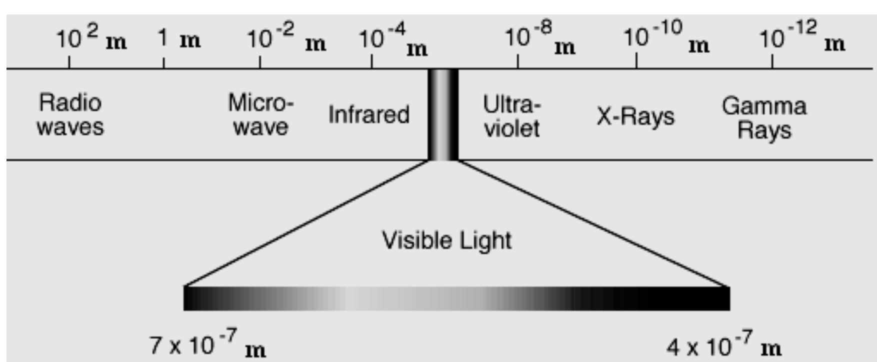
Light:

Control variable:

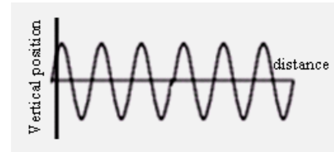
**The Electromagnetic Spectrum**



2. What is the control variable for a wave in a single medium?

Low frequency, high wavelength

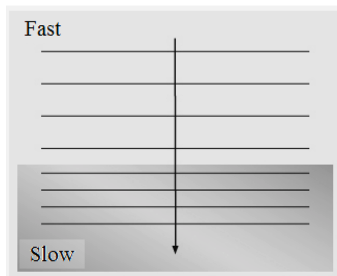

High frequency, low wavelength




Wave speed depends on . . .

3. What is the control variable for a wave as it crosses a boundary between two media?

Fast Medium:




Slow Medium:

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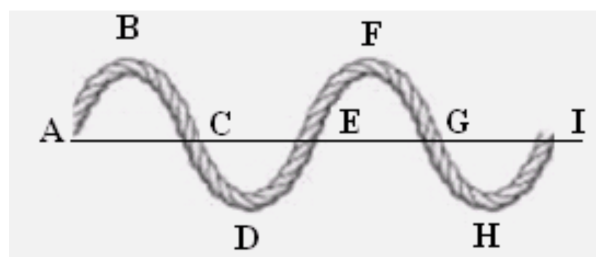
**Phase:** The phase of any particle is its position in its cycle of oscillation.

**Phase Difference –** difference in phase between two points or the particles of two oscillating systems



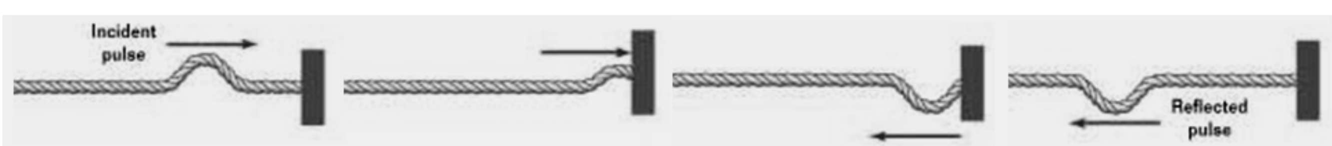

**In phase**: (A,E,I) (B,F) (D,H) (C,G)

The **phase difference** between any points *in phase* is 0.

**Completely out of phase**: (A,C) (B,D) (A,G)

The **phase difference** between any points *completely out of phase* is π or $180^0$.

**Fixed End Reflection (Hard Reflection):** Reflected pulse is . . .out of phase



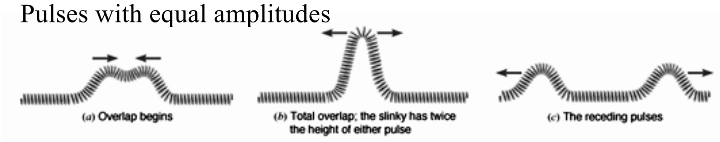

Reason for inversion of pulse: Newton's third law

**Free End Reflection (Soft Reflection):** Reflected pulse is . . . in phase

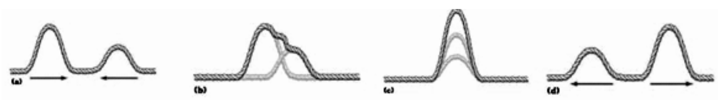
**Principle of Linear Superposition**: When two or more waves meet, the displacement of the resultant wave is the vector sum of the displacements of the component waves.

**Constructive Interference**: superposition of two or more pulses (or waves) in phase

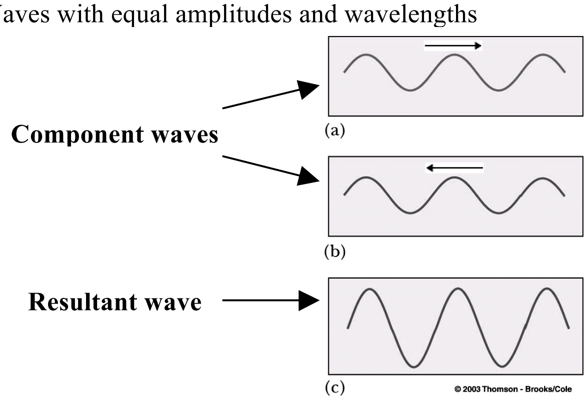
Pulses with equal amplitudes




Pulses with unequal amplitudes

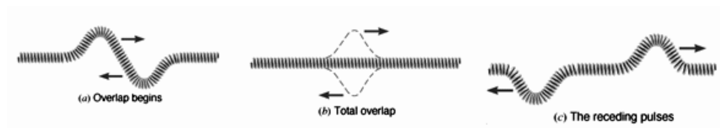



Waves with equal amplitudes and wavelengths



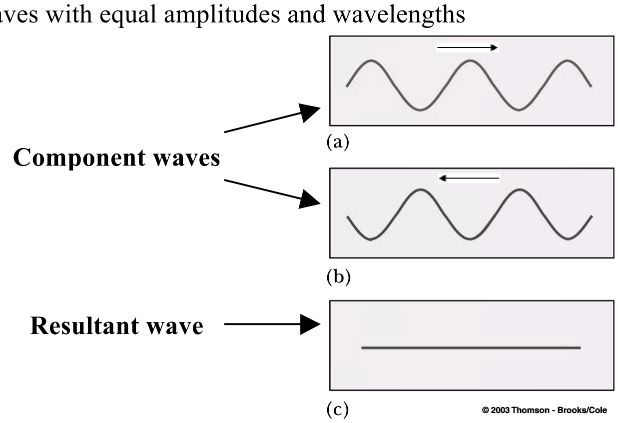

**Destructive Interference**: superposition of two or more pulses (or waves) out of phase

Pulses with equal amplitudes




Pulses with unequal amplitudes


Waves with equal amplitudes and wavelengths




**Standing (stationary) wave -** resultant wave formed when two waves of equal amplitude and frequency traveling in opposite directions in the same medium interfere

How is a standing wave on a string formed?

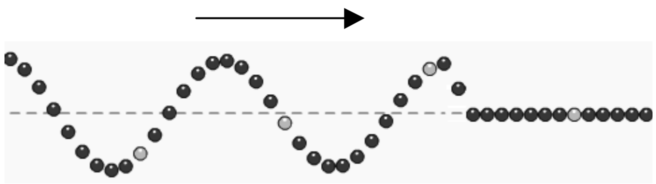
A traveling wave moving in one direction in the string is reflected off the end of the string. This sends a reflected wave traveling in the opposite direction in the string which is nearly identical with the first traveling wave. It has the same frequency, the same wavelength, and almost the same amplitude. These two traveling waves moving in opposite directions in the string are the component waves. These component waves interfere with each other creating the standing wave whose amplitude at any point is the superposition of the components' amplitudes. This standing wave is the resultant wave of the two component traveling waves.

**Node**: location of constant complete destructive interference

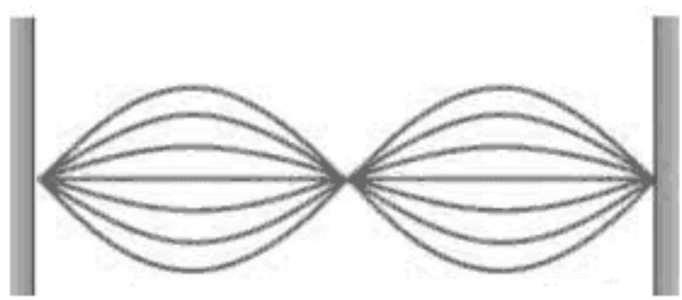
**Anti-Node**: location of maximum constructive interference

http://www.walter-fendt.de/ph6en/standingwavereflection_en.htm

# Comparing Traveling Waves and Standing Waves



**Traveling Wave**



**Standing Wave**

### Comparison of travelling waves and stationary waves

| Property | Travelling wave | Standing wave |
|---|---|---|
| energy transfer | energy is transferred in the direction of propagation | no energy is transferred by the wave although there is interchange of kinetic and potential energy within the standing wave |
| amplitude | all particles have the same amplitude | amplitude varies within a loop – maximum occurs at an antinode and zero at a node |
| phase | within a wavelength the phase is different for each particle | all particles within a "loop" are in phase and are antiphase (180° out of phase) with the particles in adjacent "loops" |
| wave profile (shape) | propagates in the direction of the wave at the speed of the wave | stays in the same position |
| wavelength | the distance between adjacent particles which are in phase | twice the distance between adjacent nodes (or adjacent antinodes) |
| frequency | all particles vibrate with same frequency. | all particles vibrate with same frequency except at nodes (which are stationary) |

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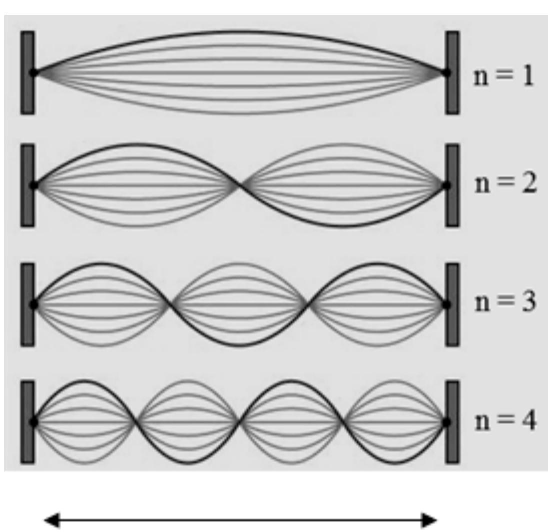
## I. Transverse Standing Wave: string fixed at both ends

**Boundary conditions:**



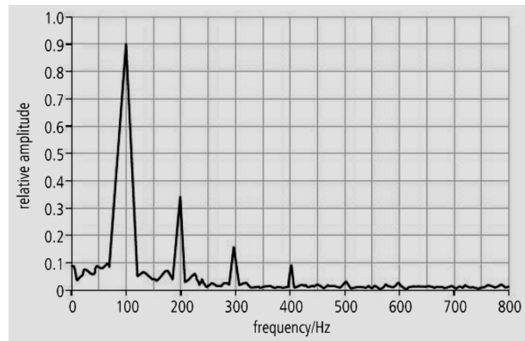

**L**

| Name | Frequency | Wavelength |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

Notice that a stable standing wave will only occur at certain discrete frequencies or certain discrete lengths of the string. It is only at these frequencies or lengths that the wave resonates so they are called *resonant frequencies* or *harmonics* or *resonant modes of vibration* of the string. We say that these resonant frequencies are *quantized*.

**Harmonics:**

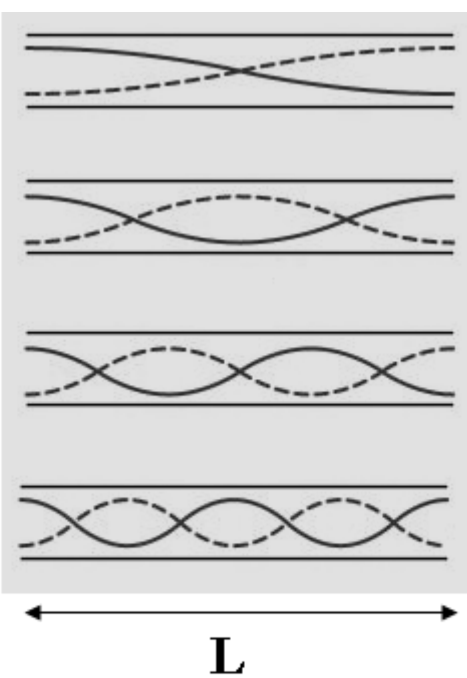
What do you hear when a guitar (or piano, etc.) string is played?




## II. Longitudinal Standing Wave: pipe open at both ends

**How is this standing wave formed?** Vibrations of air at one end produce a traveling (longitudinal) wave that reflects off the open end of the pipe which causes a second traveling wave in the opposite direction. These two component waves interfere to produce a standing wave if they have a frequency which is one of the resonant frequencies of the pipe.

**Boundary conditions:**

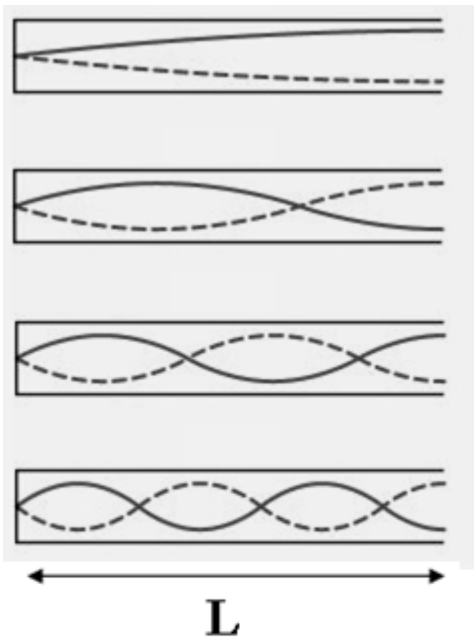



| Name | Frequency | Wavelength |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

**Note**: An anti-node does not represent an up/down motion of air particles but a left/right motion of air particles.

## III. Longitudinal Standing Wave: pipe open at only one end

**Boundary conditions:**




| Name | Frequency | Wavelength |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

**Harmonics:**

---

**Summary:**

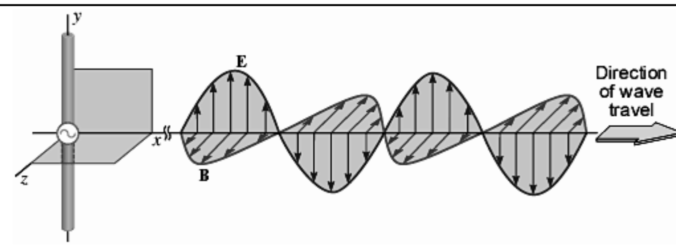
| | Boundary conditions | First Harmonic (fundamental) | Resonant Wavelengths (higher harmonics) | Resonant Frequencies (higher harmonics) |
|---|---|---|---|---|
| **String of length L** | Both ends fixed or both ends free | $L = \frac{1}{2}\lambda_1$<br>$\lambda_1 = 2L$ | $\lambda_n = \frac{2L}{n}$<br>where n = 1,2,3,4... | $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$<br>where n = 1,2,3,4... |
| **Pipe of length L** | Both ends open or both ends closed | | | |
| | | | | |
| **String of length L** | One end fixed, the other end free | $L = \frac{1}{4}\lambda_1$<br>$\lambda_1 = 4L$ | $\lambda_n = \frac{4L}{n}$<br>where n = 1,3,5,7... | $f_n = \frac{v}{\lambda_n} = \frac{nv}{4L}$<br>where n = 1,3,5,7... |
| **Pipe of length L** | One end open, the other end closed | | | |

# Polarization of Light

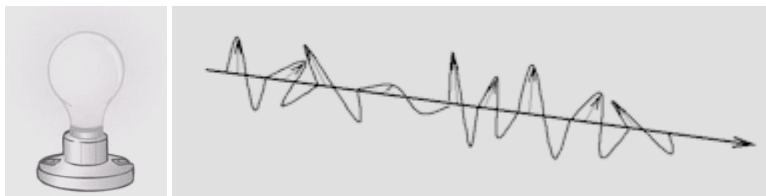


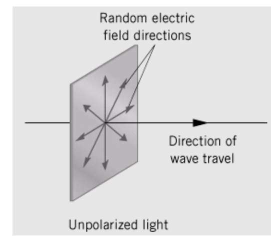
**Production of EM waves:** oscillating electric charge produces varying electric and magnetic fields

**Transverse**: vibration is perpendicular to direction of motion of energy



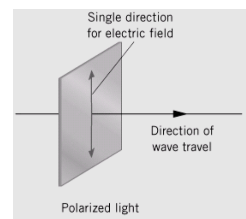

**Unpolarized Light –** light in which the electric field vector vibrates in . . .






Examples:

**Polarized Light –** light in which the electric field vector vibrates in . . .




**Plane of Polarization:** plane in which the electric field vector vibrates

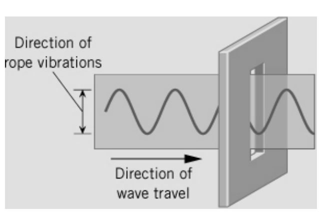
**Methods of Polarization**

a) Polarizing filter

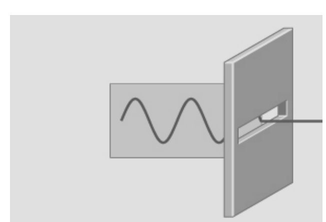
**Polarizer –** material that produces . . .

**Transmission axis –** direction of polarization that . . .

**A simple model of a polarizer using a wave on a rope**




Transmission axis of polarizer is parallel to the plane of polarization of the wave.

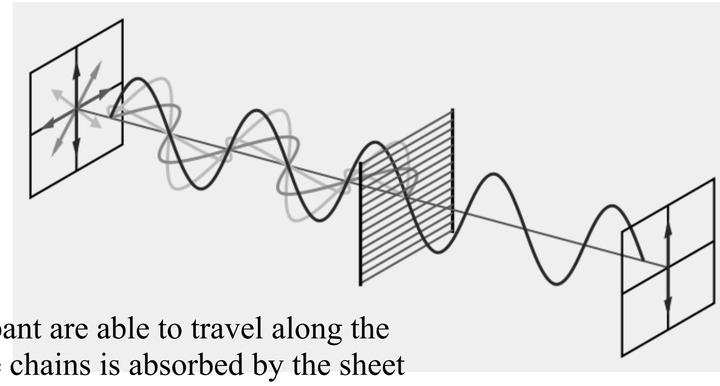


Transmission axis of polarizer is perpendicular to the plane of polarization of the wave.

NOTE:

NOTE:

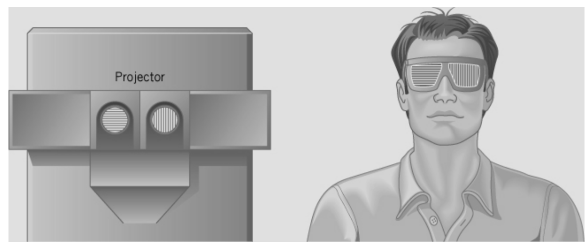
How is a polarizing filter made?



A Polaroid polarizing filter was in its original form an arrangement of many microscopic crystals. Its later form is made from polyvinyl alcohol (PVA) plastic with an iodine doping. Stretching of the sheet during manufacture ensures that the PVA chains are aligned in one particular direction. Electrons from the iodine dopant are able to travel along the chains, ensuring that light polarized parallel to the chains is absorbed by the sheet (resonance); light polarized perpendicularly to the chains is transmitted.

Therefore: The transmission axis is . . .

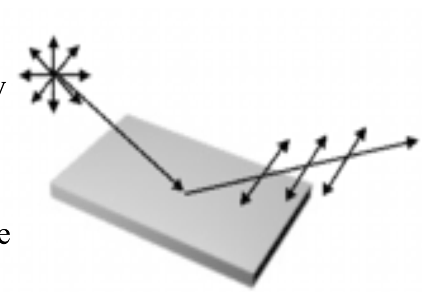
**APPLICATION - IMAX 3-D films -** The movies are recorded on two separate rolls of film, using a camera that provides images from the two different perspectives that correspond to what is observed by human eyes and allow us to see in three dimensions. The camera has two apertures located at roughly the spacing between our eyes. The films are projected using a projector with two lenses. Each lens has its own polarizer and the two polarizers are crossed.




In one type of theater, viewers watch the action on-screen using glasses with corresponding polarizers for the left and right eyes. Because of the crossed polarizers the left eye only sees the image from the left lens of the projector and the right eye sees only the image from the right lens. Since the two images have the approximate perspectives that the left and right eyes would see in reality, the brain combines the images to produce a realistic 3-D effect.

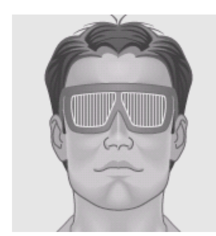
b) Reflection from a non-metallic plane surface

Sunlight is unpolarized. When sunlight is incident on an object, part of the light will be absorbed by the object (or refracted inside the object) while the rest of the light will be reflected by the object. This reflected light is partially to completely polarized, depending on the angle of incidence.



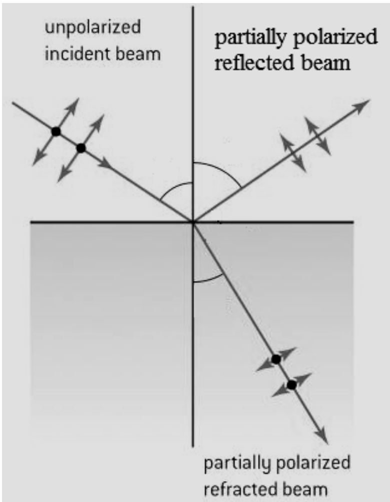
The polarization direction will be in the same direction as the surface of the object, that is, the plane of polarization for the reflected light is parallel to the surface of the object. Sunlight striking the surface of water or a road are examples where reflected light will be polarized. This is often called *glare*.

What is glare?

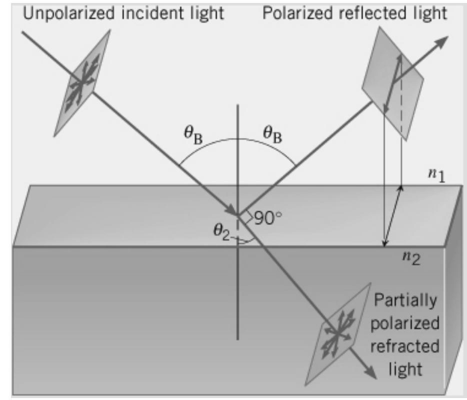


What is the direction of polarization of glare?

How do polarized sunglasses reduce glare?

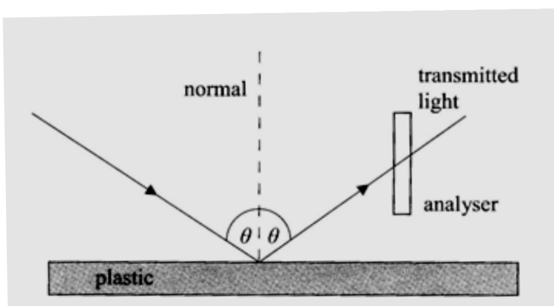


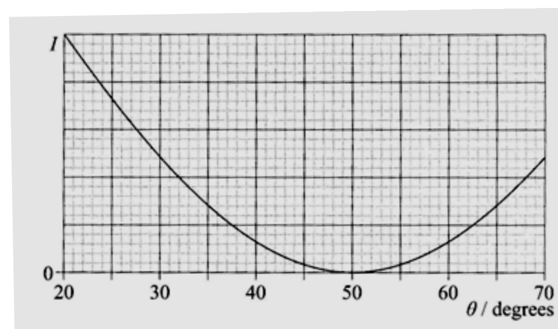

In 1809 the French experimenter Etienne-Louis Malus showed that when unpolarized light reflected off a glass plate it could be polarized to a degree that depended on the angle of incidence (left). In 1812 the Scottish physicist Sir David Brewster showed that when unpolarized light incident on the surface of an optically denser material (such as glass or water), at an angle called the polarizing angle (or Brewster's angle), the reflected ray would be completely polarized. At this angle the reflected ray and the refracted ray are at right angles (right).




**Brewster's Angle ($\theta_B$):**

1. Unpolarized light is incident on the surface of a plastic. The angle of incidence is θ. The reflected light is viewed through a polarizing filter whose transmission axis is vertical. The variation with θ of the intensity *I* of the transmitted light is shown in the graph.



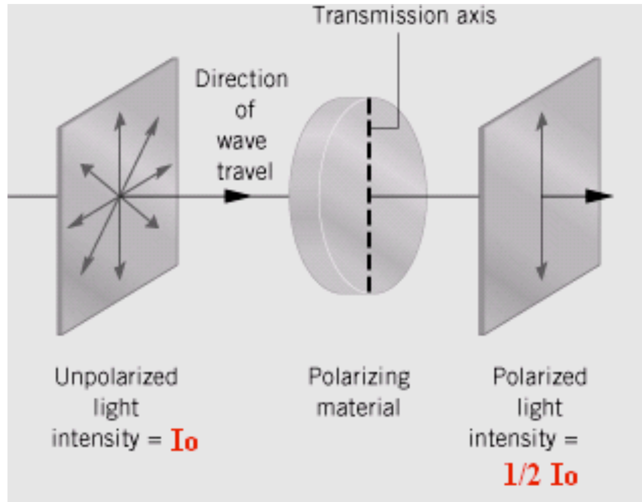




a) Explain why there is an angle of incidence for which the intensity of the reflected light transmitted through the polarizing filter is zero.

b) Determine the index of refraction of the plastic.

## A more sophisticated model of a polarizer using light

$I_0$ =

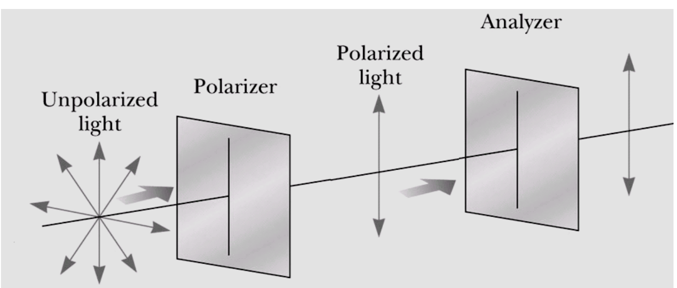



A polarizer allows the parallel component of any wave to pass through and blocks the perpendicular component of any wave.

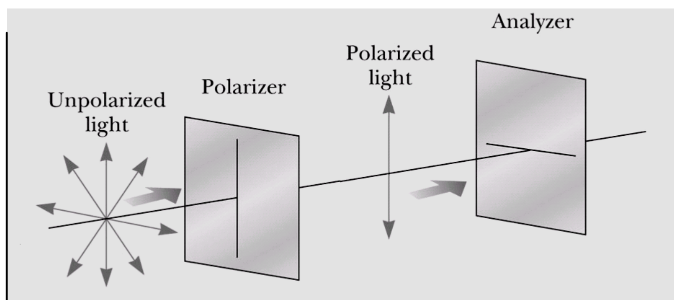
If unpolarized light is incident on a polarizer with intensity $I_0$, what is the intensity of the transmitted polarized light?

Why?

**Analyzer –**



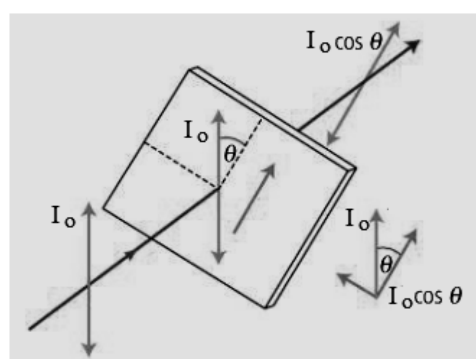

When the transmission axis of the analyzer is parallel to that of the polarizer . . .

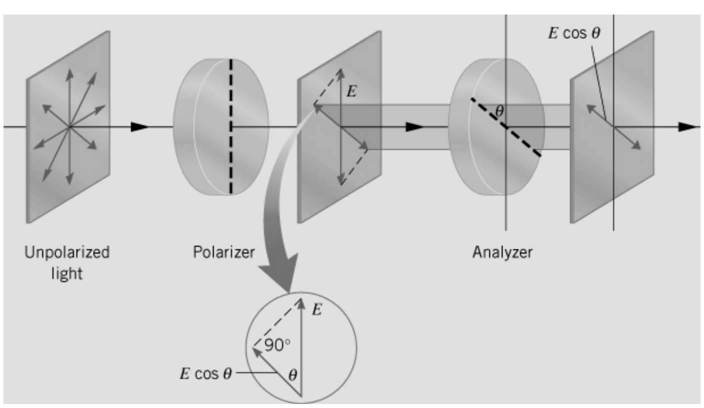



When the transmission axis of the analyzer is perpendicular to that of the polarizer . . .

What happens when the analyzer is neither parallel nor perpendicular to the polarizer?

Only the component of the polarized light . . .



**Malus' Law:**

Where:

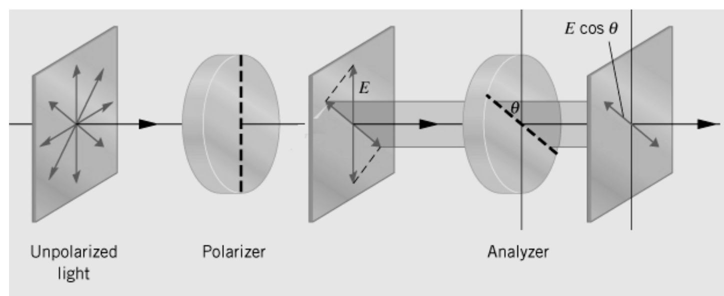
I = transmitted intensity

$I_0$ = intensity of light incident **on the analyzer**

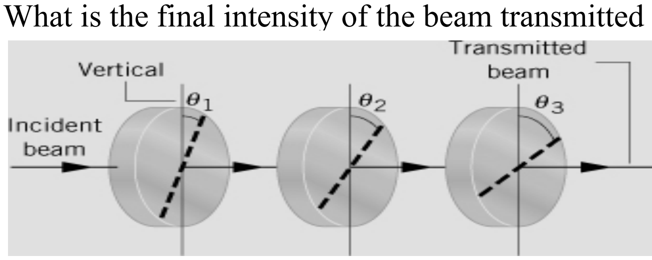
$\theta$ = angle between transmission axis of polarizer and analyzer

2. Sketch the relationship between the intensity of the transmitted light and the angle between the two polarizing filters.


3. Natural, unpolarized light of intensity 6.0 W $m^{-2}$ is incident on two polarizing filters oriented at $60^0$ to each other. Find the intensity of the light transmitted through each of them.

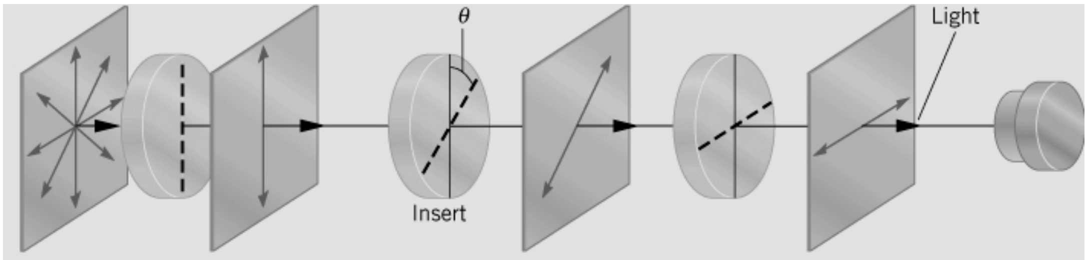



4. A beam of unpolarized light, whose intensity is 1551 $W/m^2$, is incident on the first polarizing filter as shown. The three filters make angles of $\theta_1 = 30^o$, $\theta_2 = 45^o$, and $\theta_3 = 70^o$ with the vertical as shown. What is the final intensity of the beam transmitted through the three filters?



How can light be transmitted through "crossed polarizers?"

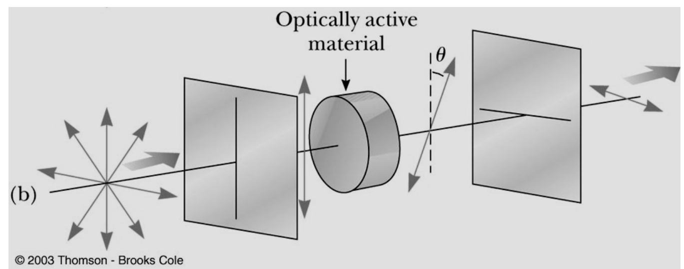
Insert an intermediate polarizer (or an optically active substance) between the original polarizer and the analyzer. Some component of light from the first polarizer will make it through this intermediate polarizer and then some component of this light will make it through the analyzer. The intermediate polarizer "rotates the plane of polarization" at the cost of lost intensity.




**Optically Active Substance –**

a) one that rotates the plane of polarization of the light that passes through it

b) one that changes the plane in which the electric field vector of the light vibrates

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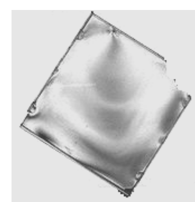
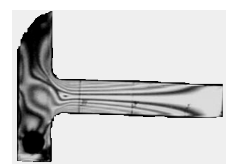
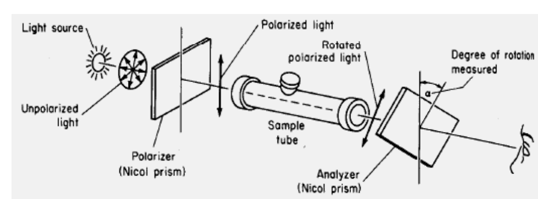
**Applications of polarization**



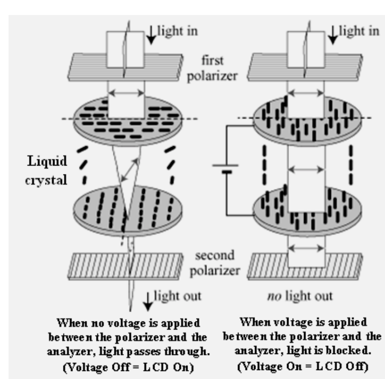


1. **Stress analysis**: Some materials are optically active under stress and allow different colors to pass through at different angles. Engineers can build models out of plastic and subject them to stress. Then when they are placed between a polarizer and an analyzer and viewed, points of probable mechanical failure due to high stress can be determined.

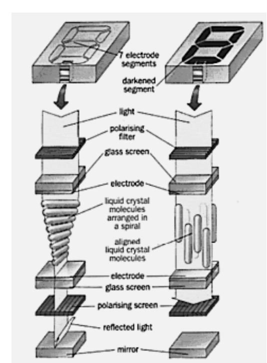
2. **Determining the concentration of solutions**: Sugar solutions, such as glucose, are optically active. The angle by which polarized light is rotated when passing through the solution is related to the concentration of the solution. Therefore, if a container with a sugar solution is placed between a polarizer and an analyzer and the analyzer is rotated until the intensity of the light passing through it is maximum then from measuring the angle of rotation the concentration of the solution can be calculated.




3. **Liquid crystal displays (LCD):** Calculators, watches, computer screens and televisions have displays that are made up of thousands of small dots called pixels. In an LCD, each pixel is made of a tiny liquid crystal. Liquid crystals have a very useful property; normally they rotate the plane of polarization through $90^0$, but when a voltage is applied across them, they don't. So if a liquid crystal is placed between two crossed polarizers the crystal goes dark when the voltage is applied.



Back-lit Display



Front-lit Display