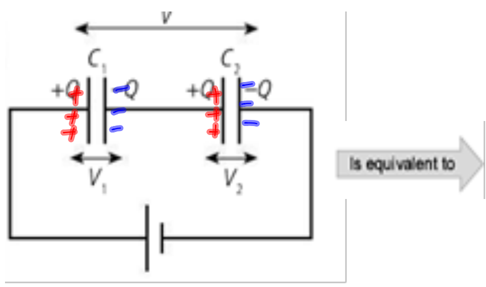
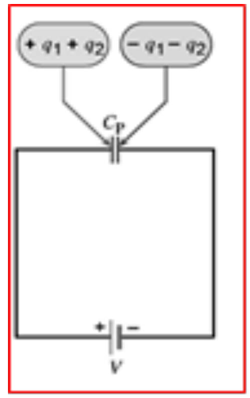
Series Capacitors





Control variable: charge

Conservation of . . . energy

Derivations:

$$V_T = V_1 + V_2$$

$$\frac{q_T}{C_T} = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

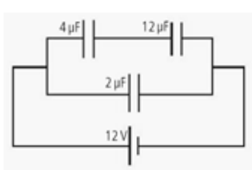
$$\text{but } q_T = q_1 = q_2$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{C_1}{C_2} = \frac{\frac{q_1}{V_1}}{\frac{q_2}{V_2}} = \frac{\frac{1}{V_1}}{\frac{1}{V_2}} = \frac{V_2}{V_1}$$

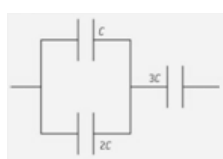
1. Calculate the equivalent capacitance of each network below.

a)



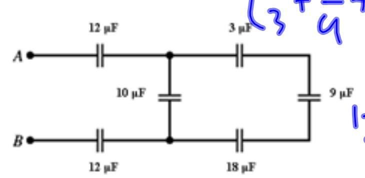
5 uF

b)



1.5

c)



$$\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{18}\right)^{-1} 4$$

$+10$

12

$$\left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right)^{-1}$$

$= 4 \mu F$

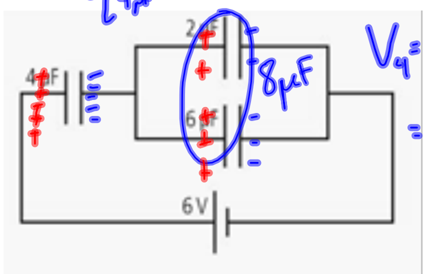
2. In each circuit, calculate the potential difference across the 4μF capacitor.

$C = q/V$

a)

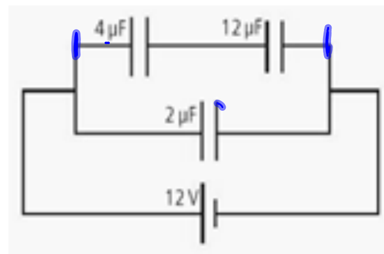
$C_T = 2.7\mu F$

$q_{4\mu F} = q_T = C_T V_T = 2.7\mu F \cdot 6V = 16\mu C$



$V_4 = q/C = \frac{16\mu C}{4\mu F} = 4V$

b)

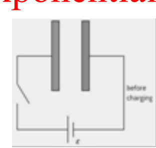


Highest capacitance = lowest voltage = inverse ratio

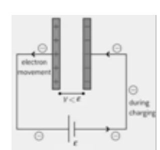
Charging a Capacitor

A capacitor which is initially uncharged is connected to a cell in order to be charged, as shown.

a) Charge on capacitor increases at a decreasing rate (exponential)

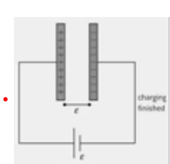


b) Potential difference across capacitor increases at a decreasing rate.

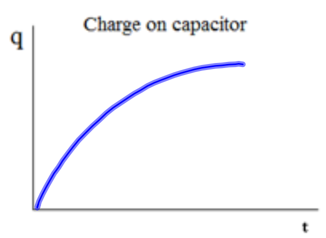


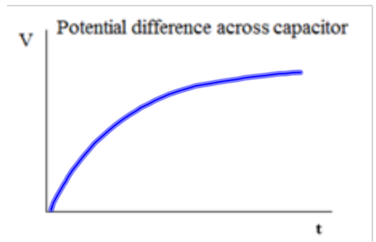
c) Current in circuit deceases at a decreasing rate.

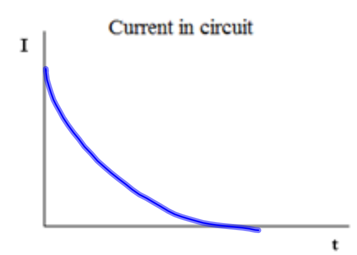
d) After a "long time" the circuit reaches a **steady state**.



1. Sketch the following graphs for the charging circuit shown.





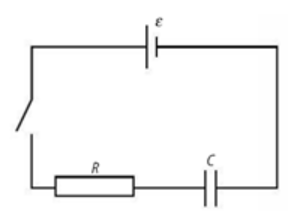


Type of relationships: exponential (growth or decay)

R-C circuit: circuit with capacitor and resistor used to control charging rate of capacitor

Effect of adding resistor: slow down the rate of charging

Common use: timing circuit



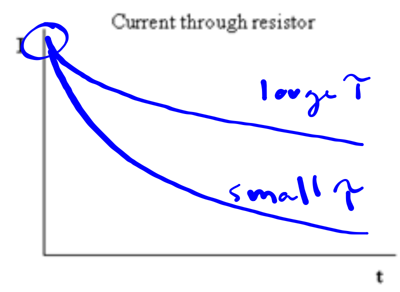
2. What determines the amount of time it takes the capacitor to charge? Both resistance and capacitance

Time constant: $\tau = RC$ $\left[\frac{V}{A}\cdot\frac{C}{V}\right] = \left[\frac{C}{C/s}\right] = [s]$

Units: seconds

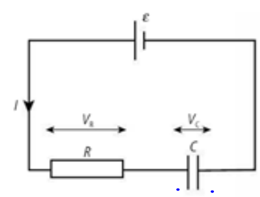
3. On the axes at right, sketch a graph of the charging current for a circuit with a large time constant and one with a small time constant. Label each.

$I_0 = \frac{\varepsilon}{R}$



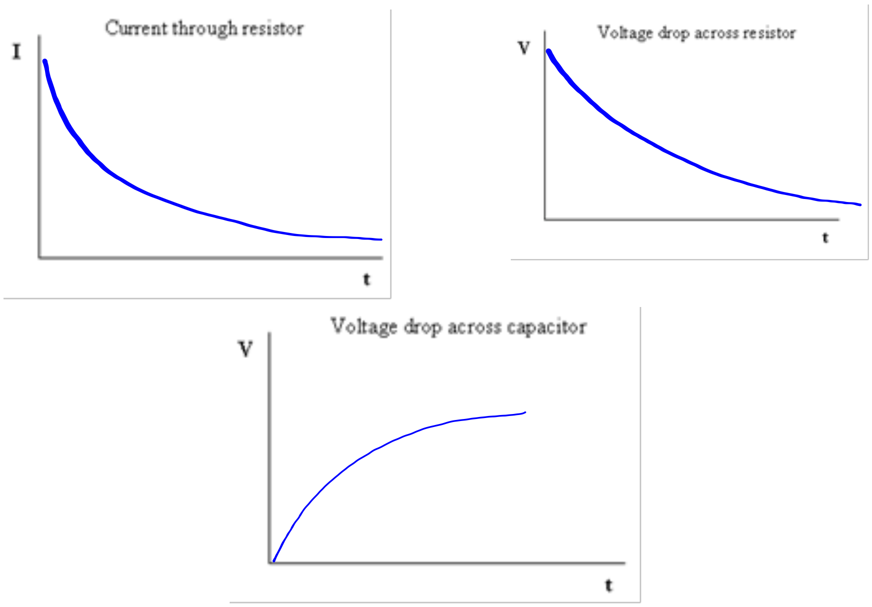
Maximum current: $I_o = \varepsilon/R$

4. What are the steady state values?



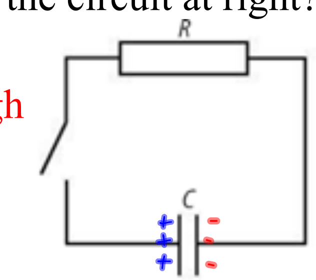
Steady State: $I = 0$ $V_c = \varepsilon$ $V_R = 0$

5. Sketch the following graphs for the R-C charging circuit above.



Discharging a Capacitor

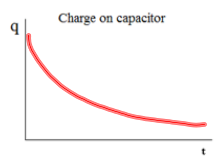
6. What happens when the switch is closed in the circuit at right?

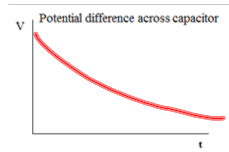


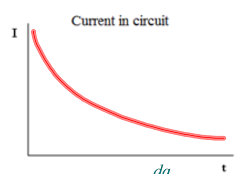
Capacitor discharges as electrons move through the resistor from negative plate to positive plate until capacitor is neutral

Note: no cell needed

7. Sketch the following graphs for the R-C discharging circuit above.







Derivation:

$$V_R + V_C = 0$$
$$V_C = -V_R$$
$$\frac{q}{C} = -IR$$
$$q = -RCI$$

$$q = -RC\frac{dq}{dt}$$
$$q = q_0 e^{-\frac{t}{RC}}$$
$$q = q_0 e^{-\frac{t}{\tau}}$$

Time constant:

$\tau = RC$

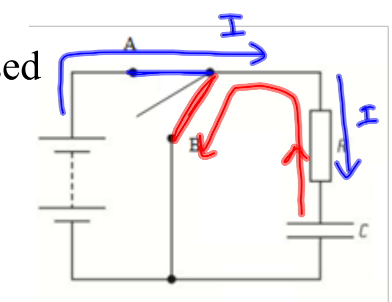
time to discharge to 37% of initial value = e^{-1}
Or to charge to 63% of max value

Math Models:

$$q = q_0 e^{-\frac{t}{\tau}}$$

$$q = q_0 e^{-\frac{t}{\tau}} \qquad V = V_0 e^{-\frac{t}{\tau}}$$
$$CV = CV_0 e^{-\frac{t}{\tau}} \qquad IR = I_0 R e^{-\frac{t}{\tau}}$$
$$V = V_0 e^{-\frac{t}{\tau}} \qquad I = I_0 e^{-\frac{t}{\tau}}$$

8. A circuit is shown at right that can be used to both charge and discharge a 220 μF capacitor through a 330 kΩ resistor.



a) In which position should the switch be to:

i) charge the capacitor? A

ii) discharge the capacitor? B

b) Calculate the time constant for this circuit. $\tau = RC = 330 \times 10^3 \Omega \times 220 \times 10^{-6} F$

72.6s

c) If the capacitor has been charged to 30 V, what will be the potential difference across it after discharging for 20 seconds?

$V = V_0 e^{-t/\tau}$

$30V e^{-20s/72.6s} = 23V$

d) How long will it take the capacitor to discharge to 10 V?

e) Calculate the amount of charge that has passed through the circuit in the time it takes to discharge to 10 V.

$q_0 = C \cdot V_0 = 220\mu F \cdot 30V = 6.6mC$

$q_{10} = C \cdot V_{10} = 220\mu F \cdot 10V = 2.2mC$

$\boxed{4.4mC}$

$q = C \cdot V e^{-t/\tau}$

$V = V_0 e^{-t/\tau}$

$\frac{V}{V_0} = e^{-t/\tau}$

$\frac{V_0}{V} = e^{t/\tau}$

$\ln\left(\frac{30V}{10V}\right) = t/72.6s$ $t = 80s$

f) How much time will it take the capacitor to lose 90% of its charge?

Losing 90% means 10% left

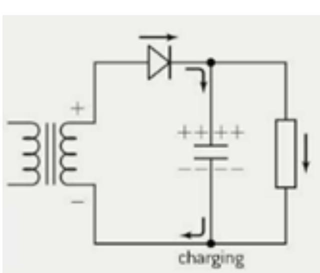
9. The discharge current of a 470 μF capacitor through an unknown resistance falls from 87 mA to 12 mA in two minutes. Determine the unknown resistance.

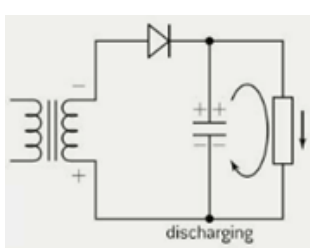
Effect of adding a capacitor on diode bridge rectification circuits

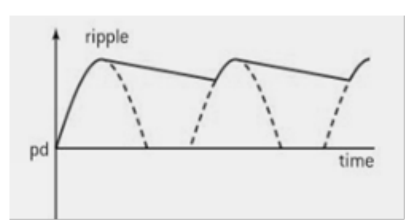
Effect: smoothes the pulsing of the output voltage and current

– makes output more steady

a) Half-wave rectification







b) Full-wave rectification