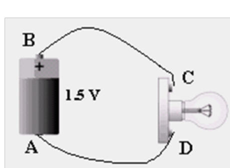
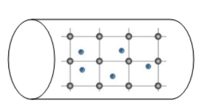
Electric Circuits

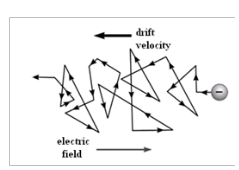
Quantity	Symbol	Units	Alternate Units	Formula
Charge	Q,q	coulomb (C)		
Electric Potential	V	volt (V)	1 V = 1 J/C	$V = E_P/q$ $V = W/q$
Work, energy	W, E_P	joule (J)		$W = qV$ $E_P = qV$
Current	I	ampere (A)	1 A = 1 C/s	$V = IR$ $I = \Delta q/\Delta t$
Drift speed	v	meters/sec (m/s)		$I = nAvq$
Charge density	n			(number of charge carriers per unit volume (m^3))
Resistance	R	ohm (Ω)	1 Ω = 1 V/A	$R = V/I$ $R = \rho L/A$
Resistivity	ρ	ohm·meter (Ω·m)		$R = \rho L/A$
Power	P	watt (W)	1 W = 1 J/s = 1 VA	$P = VI = I^2 R = V^2/R$
Energy, work	E, W, Q	joule (J)		$E = W = Q = Pt = VIt = I^2 Rt = V^2 t/R$

1. How does a battery cause a light bulb to light up?

Chemical processes inside a battery cause one terminal of the battery to be at a high electric potential (+) and the other to be at a low electric potential (-). The negative terminal is usually taken to be the base level for the electric potential (0 volts). This difference in electric potential sets up an electric field in both the wire and the bulb's filament causing free electrons everywhere in the circuit to start moving at once towards the positive terminal (electron current). This can be alternatively described as positive charge carriers moving towards the negative terminal (conventional current). As the charge carriers move, they collide with the stationary positive lattice ions making up the wire and filament thus transferring kinetic energy. The resulting increase in kinetic energy of the lattice ions in the filament is exhibited as thermal energy, that is, the filament gets hot enough to glow in the visible portion of the EM spectrum.







2. A copper wire of diameter 0.65 mm carries a current of 0.25 A. There are 8.5 x 10^{28} charge carriers in each cubic meter of copper.
Calculate the drift speed of the charge carriers.

3. What is the difference between a source of emf and a potential difference?

Electromotive force (EMF): conversion from some other form of energy into electrical energy voltage rise, potential increase

Potential difference (PD): conversion from electrical energy into some other form of energy voltage drop, potential decrease

Device					pd or emf?
Cell	*converts energy from*	chemical	*into*	electrical	emf
Resistor		electrical		internal	pd
Microphone		sound		electrical	emf
Loudspeaker		electrical		sound	pd
Lamp		electrical		light (and internal)	pd
Photovoltaic cell		light		electrical	emf
Dynamo		kinetic		electrical	emf
Electric motor		electrical		kinetic	pd

4. What is the difference between a cell and a battery?

Cell: a container in which a chemical reaction occurs to convert chemical potential energy to electrical energy – a source of EMF

Battery: two or more cells connected together

5. What is the difference between a primary cell and a secondary cell?

Primary cell: non-rechargeable **Secondary cell**: rechargeable

6. A cell-phone battery is marked as "90 mA h 12 V 1.08 Wh".

a) What quantity is being measured as 90 mAh?

(Charge) Capacity: a quantity used to measure the ability of a cell to release charge

A battery whose capacity is 90 mA h means that before it "dies" and needs recharging you can run it:

at 90 mA for ______ hour or

at 45 mA for ______ hours or

at 9 mA for ______ hours, etc.

b) Determine how much energy is stored in the battery

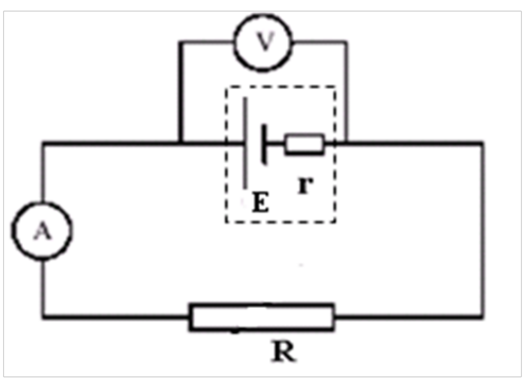
7. A cell has a capacity of 1400 mA h. Calculate the number of hours for which it can supply 1.8 mA.

8. How does a real cell differ from an ideal cell?

Ideal cell: no internal resistance, voltage across terminals (terminal pd) remains constant over time

Real cell: has small internal resistance that increases over time as chemicals are used up, votage across terminals (terminal pd)

Schematic of circuit with real cell



r = internal resistance

R = total external resistance

ε, E = emf

V = terminal potential difference Vterm

Internal Resistance of Batteries

9. What is the difference between emf and terminal potential difference?

Electromotive force (emf):

total energy per unit charge supplied around a circuit by the battery

- Energy that is used by both the exteriaor circuit and by the interior chemical processes of the cell
- Remains constant as battery discharges

Terminal Voltage (V_{term}):

potential difference across the terminals of the battery

- Energy that is available for use by the exterior circuit
- Decreases as battery discharges

In an ideal cell..emf=Vterm

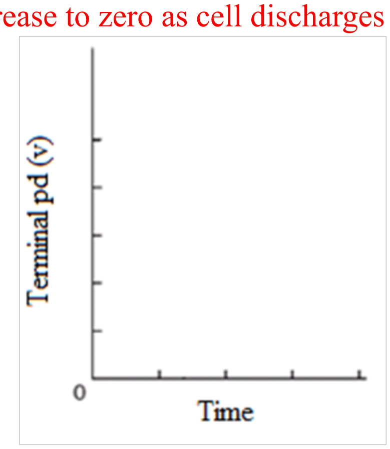
in a real cell... emf>Vterm

10. What are the discharge characteristics of a cell, that is, how does the terminal potential difference vary with time?

a) loses its initial value quickly

b) has a stable and constant value for most of its lifetime

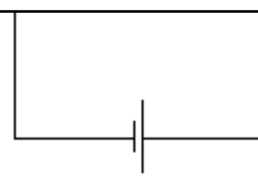
c) rapid decrease to zero as cell discharges completely



11. In which direction should current flow in order to recharge a secondary cell? Why?

Backwards through the cell – from positive to negative to reverse the chemical reaction within the cell

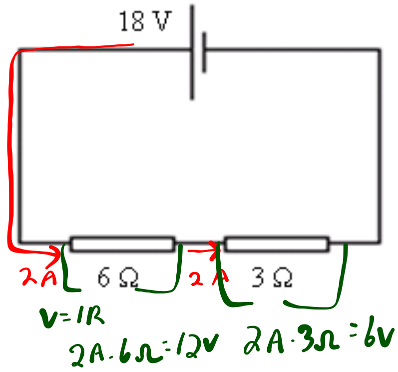
Series Circuits



Current	same everywhere	$I_T = I_1 = I_2 = \dots$
Voltage	split, in proportion to resistance	$V_T = V_1 + V_2 + \dots$
Resistance	total adds up	$R_T = R_1 + R_2 + \dots$
Power	total add up	$P_T = P_1 + P_2 + \dots$

Ratios: $\frac{R_1}{R_2} = \frac{V_1}{V_2} = \frac{P_1}{P_2}$ Control: I

1. Determine the current through and the voltage drop across each resistor in the circuit below.



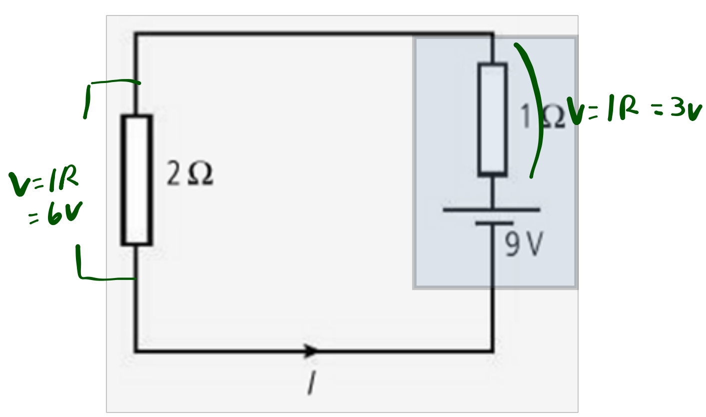
$R_T = 6\Omega + 3\Omega = 9\Omega$

$I_T = V_T / R_T = 18\,V / 9\Omega = 2A$

2. A battery with an emf of 9 V and an internal resistance of 1 ohm is connected to a 2-ohm resistor as shown. How much current is in the circuit and what is the terminal potential difference?

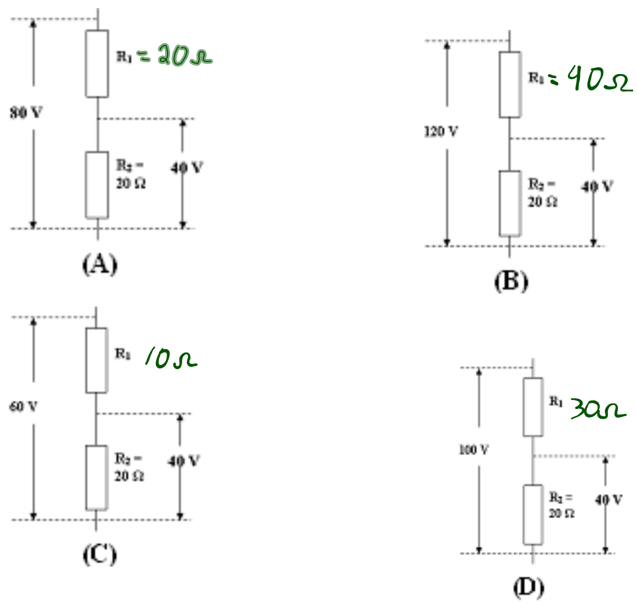
$R_T = 3\Omega$

$I_T = 3A$

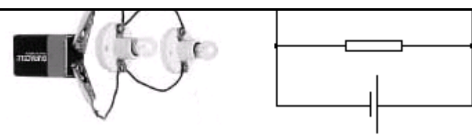


Potential Divider: Resistors in series act as a "potential (voltage) divider." They split the potential of the source between them.

3. A 20Ω device requires 40 V to operate properly but no 40 V source is available. In each case below, determine the value of added resistor R_1 that will reduce the voltage of the source to the necessary 40V for device R_2.



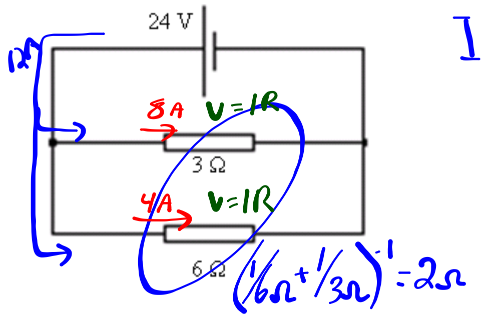
Parallel Circuits



Current	split in *inverse* proportion to resistance	$I_T = I_1 + I_2 + \ldots$
Voltage	same for all resistors	$V_T = V_1 = V_2 = \ldots$
Resistance	total adds down	$1/R_T = 1/R_1 + 1/R_2 + \ldots$
Power	total adds up	$P_T = P_1 + P_2 + \ldots$

Ratios: $\frac{R_1}{R_2} = \frac{I_2}{I_1} = \frac{P_2}{P_1}$ Control: V

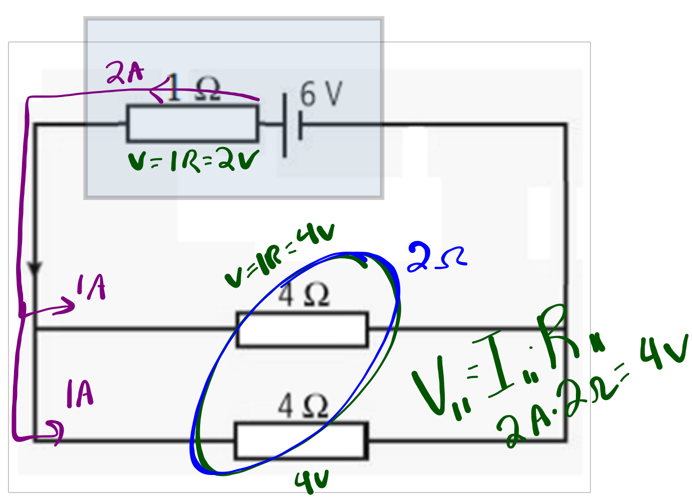
4. Determine the current through and the voltage drop across each resistor in the circuit below.



$I_T = 12A$

5. A cell with an emf of 6 volts and an internal resistance of 1 ohm is connected as shown. Determine the total current in the circuit and the terminal potential difference.

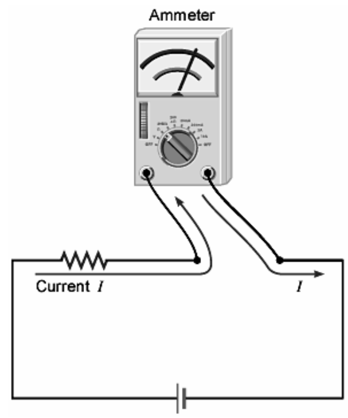
$R_T = 3\Omega$
$I_T = 2A$



Ammeter: measures current

Placement: in series

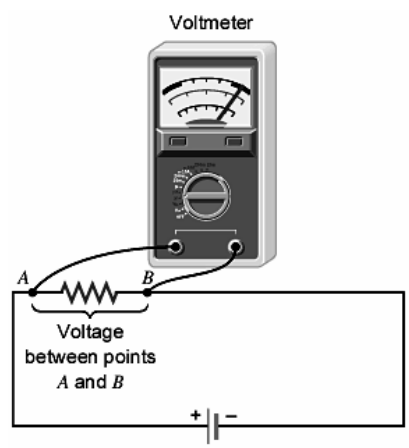
Ideal ammeter: resistance = zero



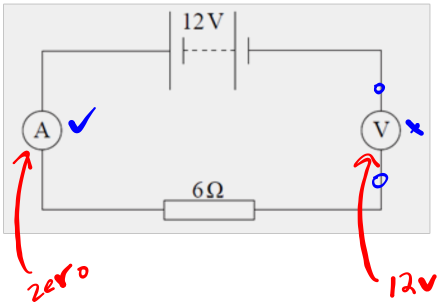
Voltmeter: measures potential difference

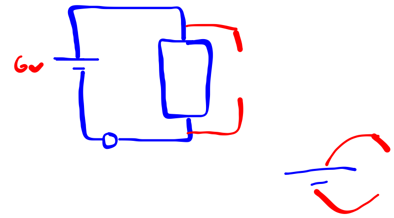
Placement: in parallel

Ideal voltmeter: resistance is infinite



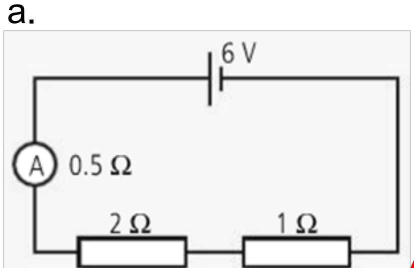
6. Which meter is improperly placed? What is the reading on each meter?





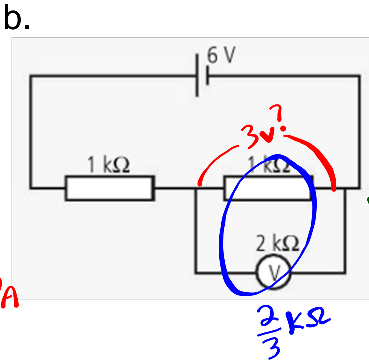
7. What is the reading on each of these non-ideal meters?

a.



2A? $I = V/R = \frac{6V}{3.5\Omega} = 1.7A$

b.



$I_T = \frac{V}{R} = \frac{6v}{1.67k\Omega}$

$= 3.6mA$

$V_{\parallel} = I_{\parallel} R_{\parallel}$

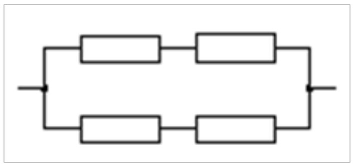
$3.6mA \cdot .67k\Omega$

$= 2.4v$

Combination Series-Parallel Circuits

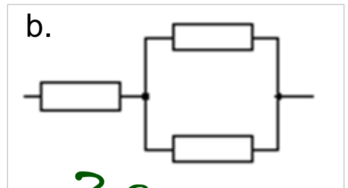
1. Determine the equivalent resistance of each circuit segment shown below. Each resistor is an identical 2-ohm resistor.

a.



2Ω

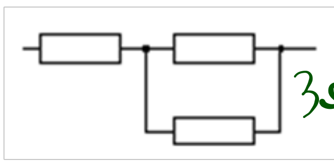
b.



3Ω

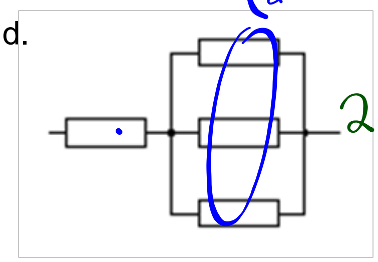
$\left(\frac{1}{2\Omega}+\frac{1}{2\Omega}+\frac{1}{2\Omega}\right)^{-1} = \frac{2}{3}\Omega$

c.



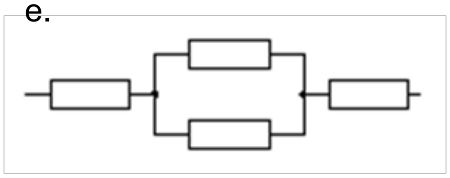
3Ω

d.

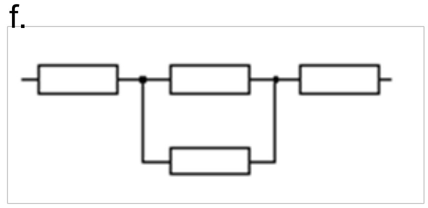


2.67Ω

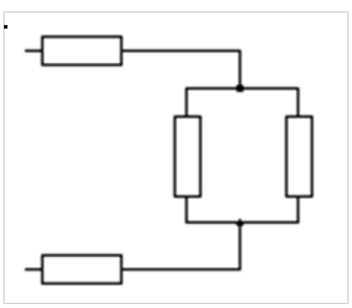
e.



f.

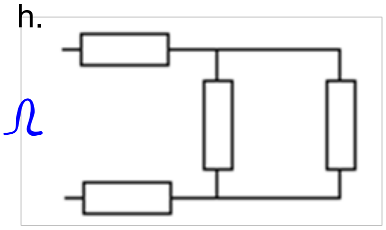


g.



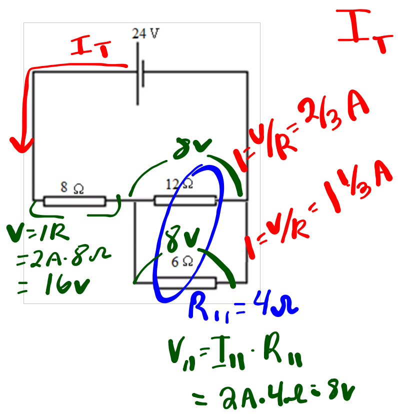
5Ω

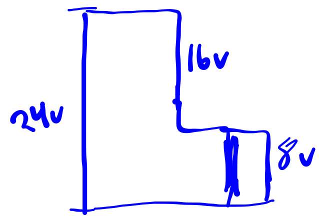
h.



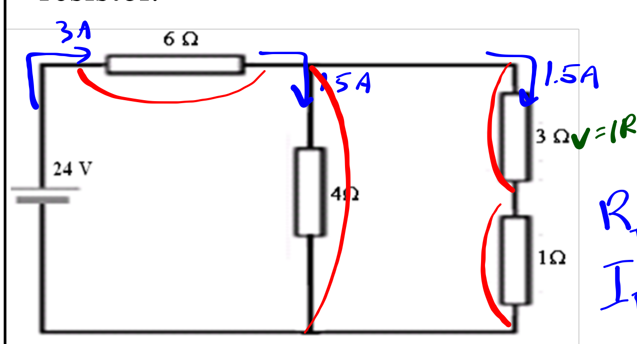
2. Determine the current through and the voltage drop across each resistor.

$I_T = V/R = \frac{24V}{12\Omega} = 2A$





3. Determine the current through and the voltage drop across each resistor.



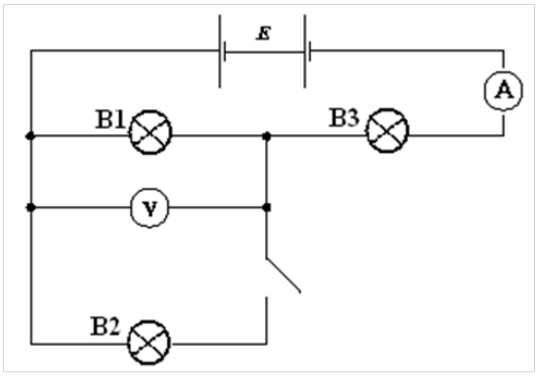
$R_T = 8\Omega$

$I_T = 3A$

	I	V
1	1.5A	1.5v
3	1.5A	4.5v
4	1.5A	6v
6	3A	18v

4. A battery with emf E is connected in a circuit with three identical light bulbs, as shown.

a) Determine the reading on the voltmeter when the switch is open and when it is closed.

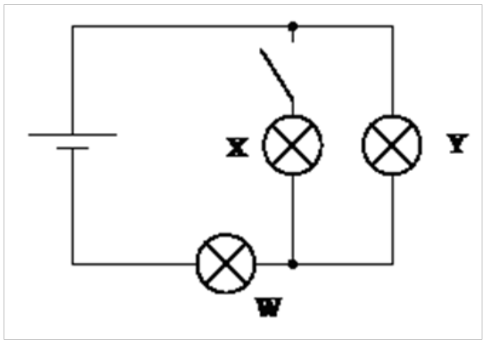


b) State what effect closing the switch has on the total resistance and total current as well as the current through each bulb and the brightness of each bulb.

When the switch closes . . .

	Current	Voltage	Power
B1			
B2			
B3			

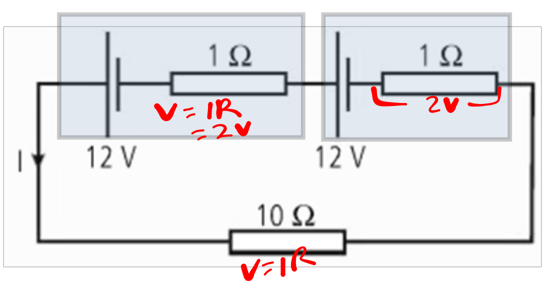
5. Three identical filament lamps (assume constant resistance) are connected to a source of emf as shown.
Predict what will happen when the switch is closed.

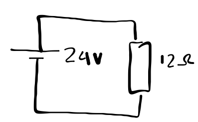


Circuits with Multiple Emfs

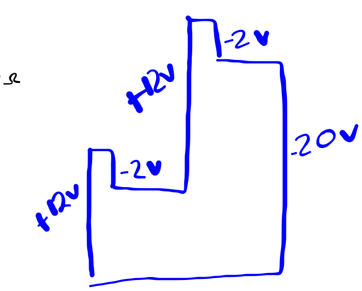
I. Emfs in series in the same direction: add the voltages

1. Determine the current in the following circuit.



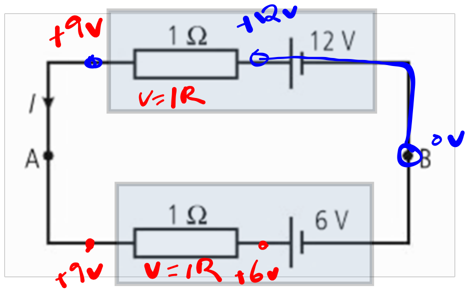


2A



II. Emfs in series in opposite directions: subtract the voltages

2. Determine the current in the following circuit.

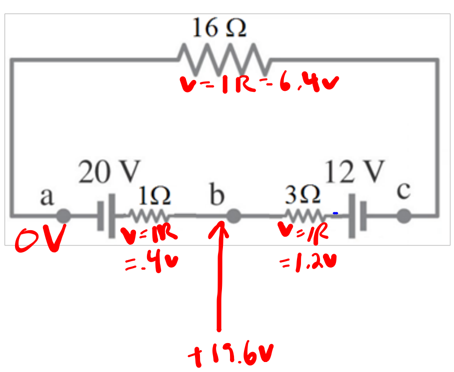


$$I = \frac{V}{R} = \frac{(+12v - 6v)}{2\,\Omega} = 3A$$

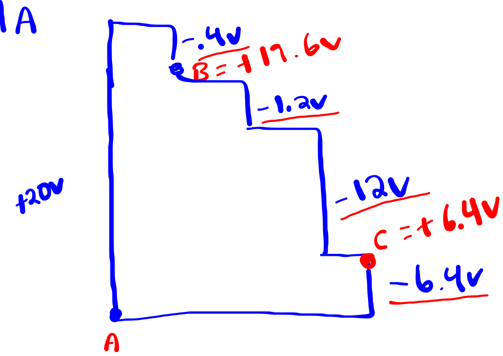
Application of series emfs in opposite directions: recharging a secondary cell

Direction of current flow: backwards through the secondary cell

3. Determine the current in the following circuit. Which is the primary cell and which is the secondary cell?

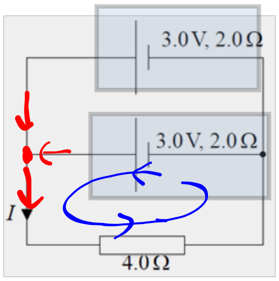


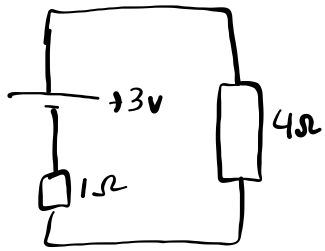
$I = 4A$



III. EMFs in parallel: same EMF but add internal resistances

4. Determine the current in the following circuit. Each identical cell has an emf of 3.0 V and an internal resistance of 2.0 ohms.





$I = \frac{V}{R} = \frac{3V}{5\Omega} = .6A$

Kirchoff's Circuit Laws

Loop Rule: Around any closed loop in a circuit, the sum of the EMFs equals the sum of the potential differences. (total voltage rises = total voltage drops)

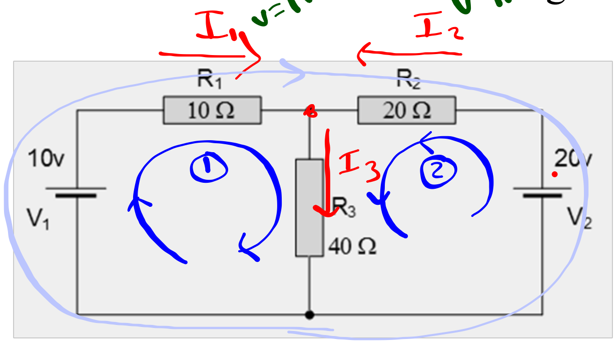
Conservation of . . .energy **Formula:** $\mathbf{\Sigma V = 0}$

Junction Rule: At any junction in a circuit, the sum of the currents entering the junction equals the sum of the currents leaving the junction. (total current in = total current out)

Conservation of . . .electric charge **Formula:** $\mathbf{\Sigma I = 0}$

1. Determine the current through resistor R_3.

Loop 3
$+10v - (I_1 \cdot 10\Omega)(I_2 \cdot 20\Omega) - 20v = 0$



Loop 1

$+10v - I_1 \cdot 10\Omega - I_3 \cdot 40\Omega = 0$

Loop 2

$+20v - I_2 \cdot 20\Omega - I_3 \cdot 40\Omega = 0$

Junction

$I_1 + I_2 = I_3$

$$
\begin{array}{cccc}
I_1 & I_2 & I_3 & \\
\end{array}
$$

$$
\begin{bmatrix}
10 & 0 & 40 & 10 \\
0 & 20 & 40 & 20 \\
1 & 1 & -1 & 0
\end{bmatrix}
$$

$1 - I_1 - 4(I_1 + I_2) = 0$

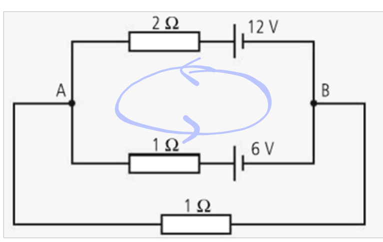
$1 - I_2 - 2(I_1 + I_2) = 0$

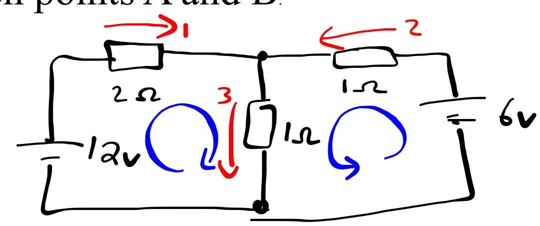
$I_1 = -.143A$

$I_2 = .429A$

$I_3 = .286A$

2. Determine the currents in the following circuit as well as the potential difference between points A and B.





Redraw to look like circuit in Example #1

$12 - 2I_1 - I_3 = 0$

$6 - I_2 - I_3 = 0$

$I_1 + I_2 = I_3$

$I_1 = 3.6A$

$I_2 = 1.2A$

$I_3 = 4.8A$

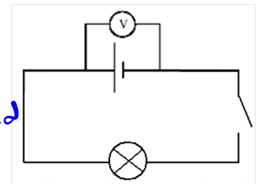
Internal Resistance of Cells

Electromotive force (emf): total energy per unit charge supplied around a circuit by the cell

Terminal Voltage (V_{term}): potential difference across the terminals of the cell

1. Compare the terminal voltage of the cell when the switch is open to the terminal voltage when it is closed.

$V_{term_{open}} > V_{closed}$



2. Compare the emf of the cell to its terminal voltage when the switch is:

a) open $\varepsilon = V_{Term}$

b) closed $\varepsilon > V_{Term}$

3. Explain these observations.

Internal resistance (r): the resistance supplied by the materials within the cell

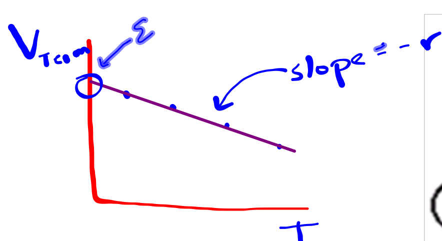
think of a cell as . . . a perfect emf and a small resistor.

Mathematical model:

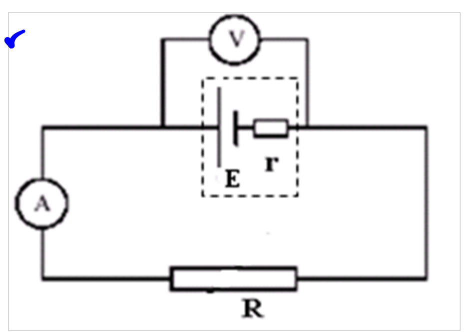
$\varepsilon = I\,R_T$

$\varepsilon = I\,(R + r)$

$\varepsilon = \underbrace{IR}_{V_{term}} + Ir$



$V_{Term} = -rI + \varepsilon$



4. Use the math model to make some inferences about the behavior of a circuit containing a cell with internal resistance.

open circ. voltage

a) Emf = V_{term} . . . when no current is flowing (when R is infinite or open circuit) or if it is an ideal cell (r = 0)

b) When R>>r . . . emf = IR + Ir   emf ≈ IR   emf ≈ Vterm

c) When R = 0 . . . emf = Ir   I = emf/r $I = I_{max}$

short circuit current

5. A resistor is connected to a 12 V source and a switch. With the switch open, a voltmeter reads the potential difference across the battery as 12 V yet with the switch closed, the voltmeter reads only 9.6 V and an ammeter reads 0.40 A for the current through the resistor. Calculate the internal resistance of the source and the maximum possible current.

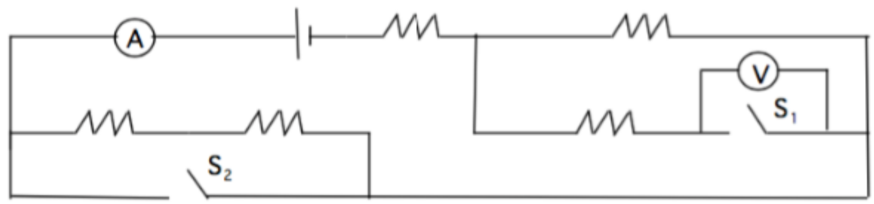
$\mathcal{E} = IR_T = IR + Ir$

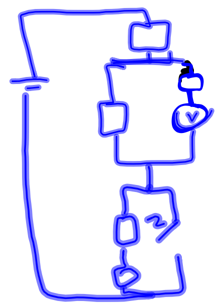
$\underset{12v}{\mathcal{E}} = \underset{9.6v}{V_{Term}} + \underset{.4A}{I}r$

$r = \frac{2.4v}{.4A} = 6\Omega$

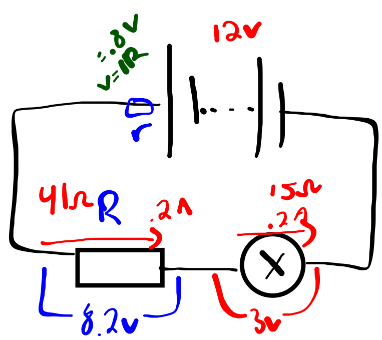
$\underline{I_{max}}$ $R = 0$

$I = \frac{\mathcal{E}}{r} = \frac{12v}{6\Omega} = 2A$





6. A resistor R and a filament lamp L are connected in series with a battery. The battery has an emf of 12 V and internal resistance 4.0 Ω. The potential difference across the filament of the lamp is 3.0 V and the current in the filament is 0.20 A. Determine the resistance R.



Resistance: ratio of potential difference applied across a piece of material to the current through the material

$R = V/I$

$V = IR$

Ohm's Law: for a conductor at constant temperature, the current flowing through it is proportional to the potential difference across it over a wide range of potential differences

Relationship: $V \propto I$

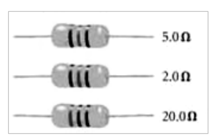
Georg Simon Ohm
(1787 – 1854)

Ohmic and Non-Ohmic Devices

Ohmic Device: a device that obeys Ohm's law for a wide range of potential differences

Meaning: a device with constant resistance

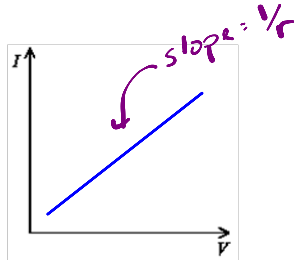
Example: resistor

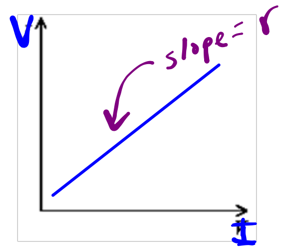


1. On the axes, sketch the **I-V characteristics** for a resistor.

a) R = V/ I at any point

b) resistance is related to slope





Non-Ohmic Device: a device that does not obey Ohm's law

Meaning: resistance is not constant

Example: filament lamp

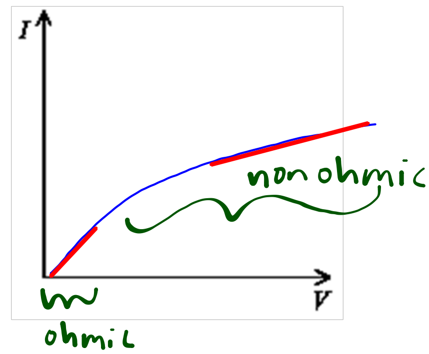
2. On the axes, sketch the **I-V characteristics** for a filament lamp.

a) as current increases, wire fillament heats up, resistance increases

b) R = V/I at any point

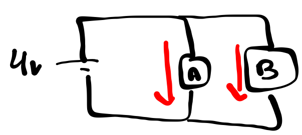
c) I increases, but slower than V. So ratio V/I increases (I increases, but at decreasing rate)

d) Resistance is NOT related to the slope (except in initial linear, proportional region)



3. The I-V characteristics of two devices, A and B, are shown in the graph

a) The two devices are connected in parallel to a 4.0 V source.
i) What is the total circuit current?

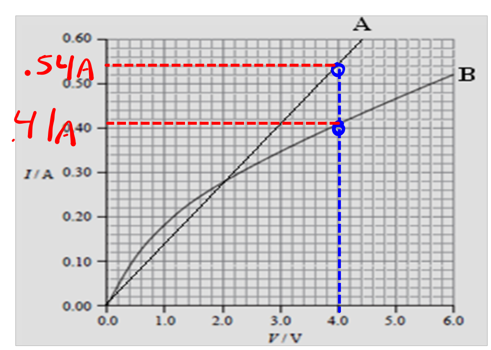


$I_T = .95A$

ii) What is the resistance of each device?

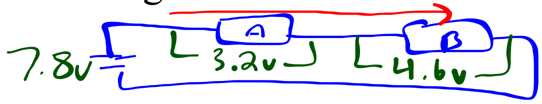
B: $R = V/I = 4V/.41A = 9.8\Omega$

A: $= 4V/.54A = 7.4\Omega$



b) The two devices are connected in series with a different source. The new circuit has 0.44 A of current.

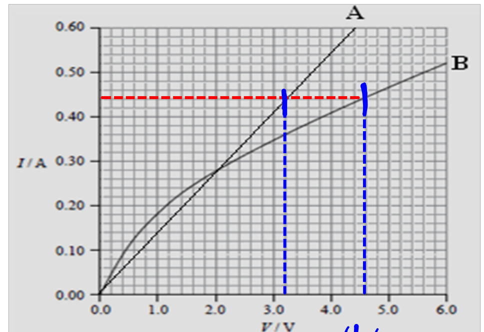
i) What is the total voltage of the source .44A



ii) What is the resistance of each device?

$$\frac{V}{I} = \frac{3.2v}{.44A} = 7.4\Omega$$

$$= \frac{4.6V}{.44A} = 10.5\Omega$$



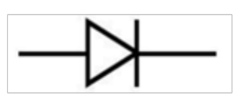
32v 4.6v

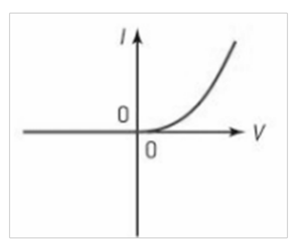
4. What are some other non-Ohmic devices?

i) **Diode:** a semiconductor device that only allows current to flow in one direction (after an initial pd is applied)

Note: forward bias needed and V_i

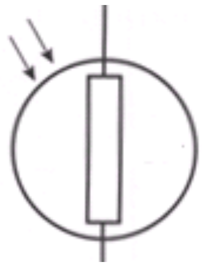
Schematic Symbol





ii) **Light-Dependent Resistor (LDR)** or **Light Sensor**: A photo-conductive cell made of semiconducting material. When light strikes it, charge carriers are released. As more light strikes it, more charge carriers are released and thus its resistance goes down.

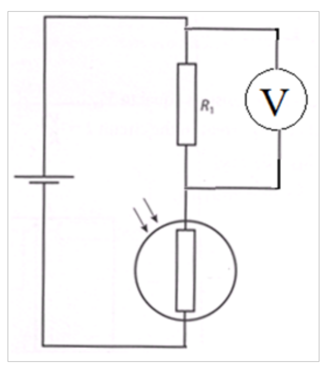
Schematic Symbol



Relationship: as intensity of light increases, resistance decreases

What happens to the reading on the voltmeter as the amount of light on the LDR increases?

increases



Practical Application: night lights, security lights

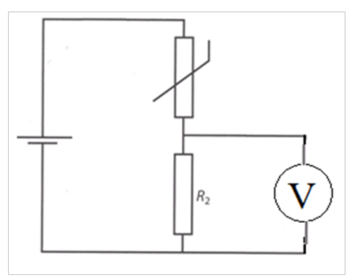
iii) **Negative Temperature Coefficient (NTC) Thermistor** or **Temperature Sensor**:
A sensor made of semiconducting material, also known as a *thermistor* (thermal resistor). As the thermistor gets hotter, more charge carriers are released and thus its resistance goes down

Schematic Symbol

Relationship:

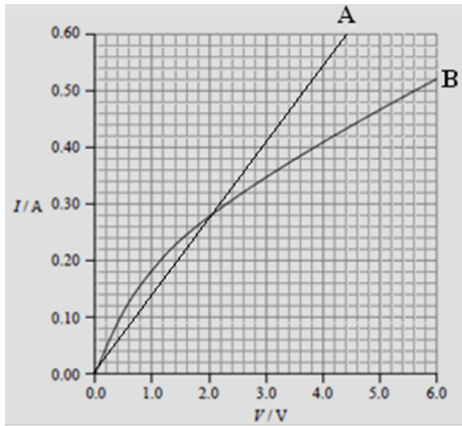
What happens to the reading on the voltmeter as the thermistor gets hotter?

increases



Practical Application: heat sensors for fire alarms

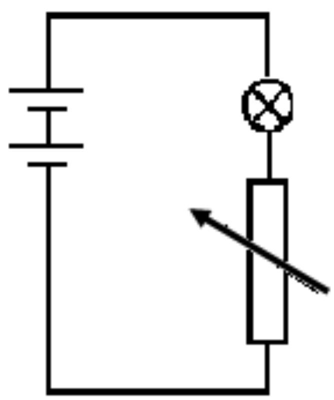
Measure I-V Characteristics



<u>I-V Characteristics</u>:

relationship between current and voltage for a device

1. Comment on the appropriateness of the following circuit to determine the I-V characteristics of the filament lamp.



When R = 0:
The lamp gets maximum current and pd but not 12 V due to internal resistance of battery and variable resistor.

When R = Maximum (10 Ω):
Pd is divided between lamp and variable resistor so current and pd for lamp drop to minimum but never to zero.

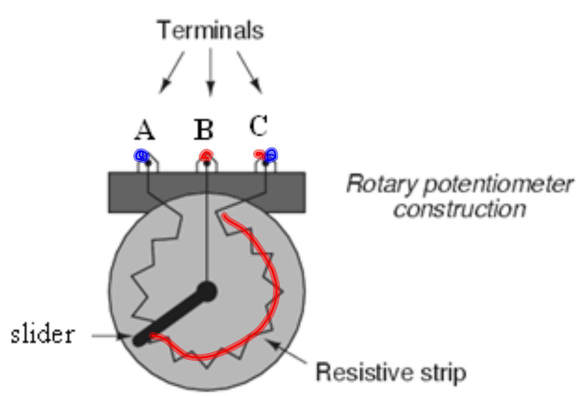
Conclusion: Full range of I-V characteristics is not covered

Potentiometer:

a type of variable resistor with three contact points

common use

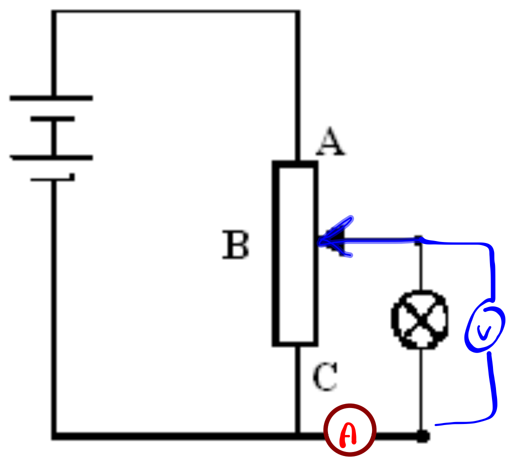
as a potential divider to measure the I-V characteristics of a device



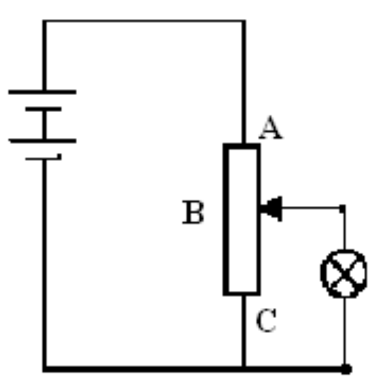
As the slider is rotated counterclockwise, what happens to the resistance between points:

A and B? B and C? A and C?

3. The schematic shows how a potentiometer can be used as a potential divider to measure the I-V characteristics of a filament lamp. It is placed in parallel with the lamp and the slider (center contact point) effectively splits the potentiometer into two separate resistors AB and BC. By moving the slider, the ratio of the voltage drops across the resistors AB and BC is varied.

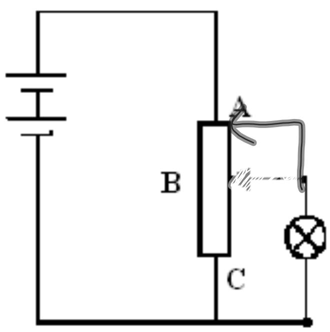


Redraw the schematic with an ammeter and a voltmeter correctly placed to measure the I-V characteristics of the filament lamp.



Comment on the circuit characteristics as the slider is moved from A to B to C.

Slider at A



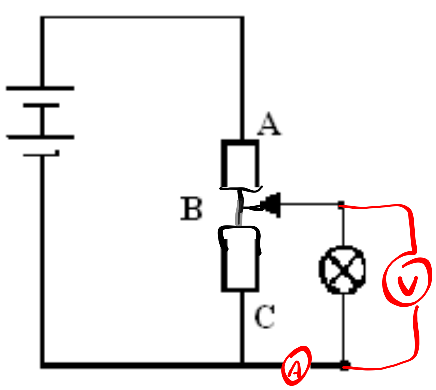
Lamp has max V
$V_l \approx V_t$ (except for internal resistance of batteries and wires)

Max current for lamp

Max brightness

Lowest R_t

Slider at B



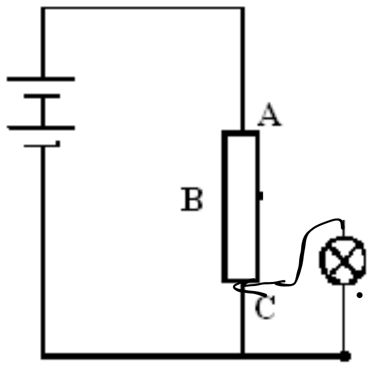
Potential divider

$V_L < V_t$

Dimmer

I_L decreases

Slider at C



Short circuit around lamp

$I_L \approx 0$ (except for internal resistance of wires)

Lamp goes out

$V_L \approx 0$