Relativity

Frame of Reference:

Galilean Transformations and Relative Velocities

Galilean Transformation – transformation of measurements made in one frame of reference to another frame of reference without taking into account the theory of relativity

Galilean transformation of positions and velocities

Stationary frame:

Moving frame:

$x' =$

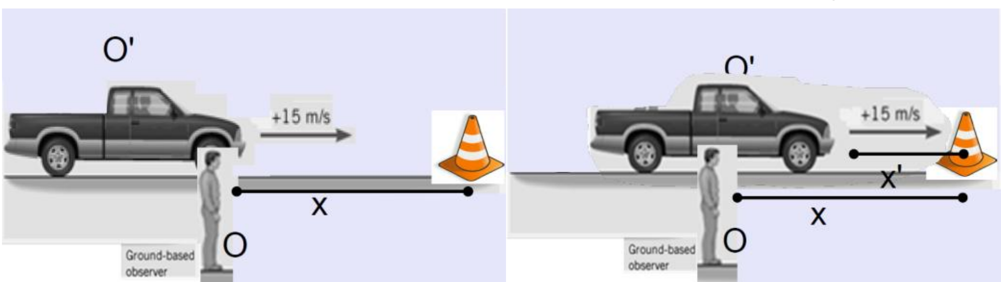
$x =$

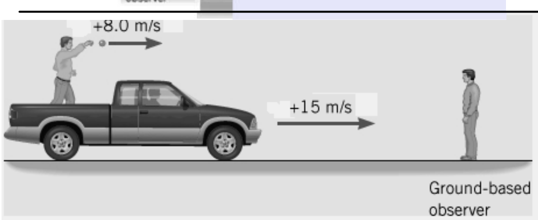
$u_x' =$

$u_x =$

$v =$

1) A truck passes a person on the ground at t = 0, and they are a distance 'X=60. m' from a road cone. Some time *'t=2s'* later, where is the cone, as measured by the truck?

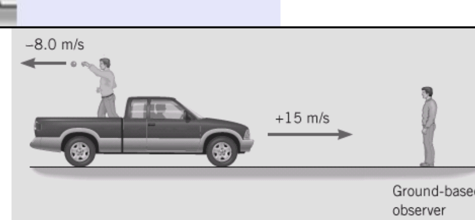




1. What is the velocity of the ball:

a) as measured by the person in the truck?

b) as measured by the ground-based observer?



2. What is the velocity of the ball:

a) as measured by the person in the truck?

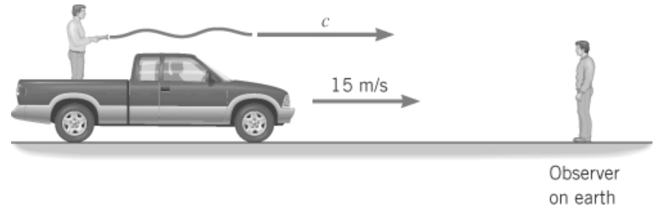
b) as measured by the ground-based observer?

Galilean principle of relativity:

1.

2.

What is the problem with the Galilean principle of relativity?



- different observers will measure different values for the speed of light
- but according to the laws of electromagnetism (Maxwell's equations), the speed of light in a vacuum is a fixed value

How can this contradiction be resolved? Is the speed of light variable or is it fixed? Two possibilities exist:

1. The Galilean transformation laws are incomplete or incorrect. This means that the formulas for adding and subtracting relative velocities will need to be revised so that the speed of light is the same for all observers.
2. The laws of electromagnetism are not the same in all inertial reference frames. This means that there must exist a preferred reference frame in which the speed of light is a constant value but in other reference frames the speed of light can vary according to the Galilean transformations.

Possible solution: Find an absolute frame of reference in which light travels at its predicted constant speed and then all other reference frames can be compared to this absolute frame using the Galilean transformations.

Luminiferous ether:

Absolute Frame:

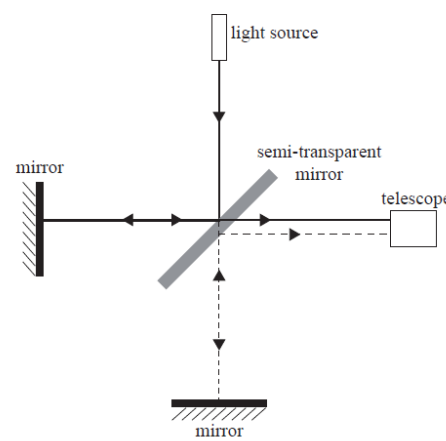
Michelson-Morley experiment – (1887) by Albert Michelson and Edward Morley at what is now Case Western Reserve University in Cleveland, Ohio.

Purpose:

- To measure the speed of the Earth in its orbit relative to the ether
- To detect the ether
- To find an absolute frame of reference

Experiment:

A beam of light is split by a half-silvered (semi-transparent) mirror into two beams which then reflect off mirrors and recombine at the detector (telescope) where an observer looks at the interference pattern these two beams make. The interference pattern should be caused by the path difference between the two beams due to the difference in speed of light relative to the supposed ether. If the ether existed, then when the apparatus is turned 90^{o} the interference pattern would change or shift because the rotation would exchange the beam directions relative to the Earth's motion through the ether. The amount of the shift would allow the speed of the Earth relative to the ether to be determined.



Essential features of the Michelson-Morley apparatus

The Michelson-Morley experiment is perhaps the most famous "null-result" experiment in all of physics. More recent experiments have confirmed the absence of the ether to sensitivities of 10^{-17}.

Results:

No shift was detected so there was no change in the optical path lengths of two beams (no change in path difference). Therefore time taken by the light to travel these distances is independent of the path taken.

Implications:

1)

2)

3)

4)

Special Relativity

Special Theory of Relativity (1905): Einstein's attempt to resolve the paradox about the speed of light and the laws of electromagnetism

Two Postulates of Special Relativity:

I.

Consequence:

II.

Consequence:

Inertial Frame of Reference:

- a frame of reference in which Newton's law of inertia is valid (an object with no unbalanced forces will remain at rest or move at a constant velocity)
-

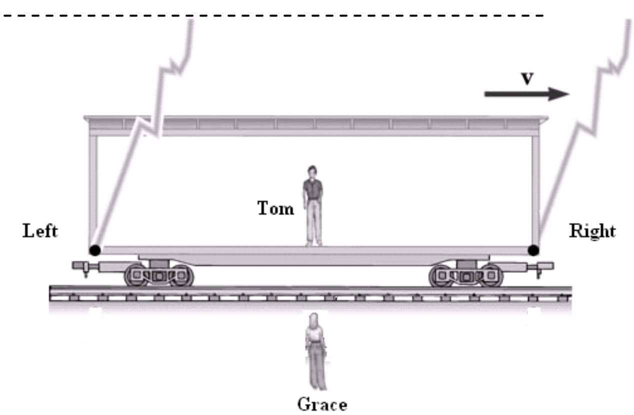
Simultaneity and the Relativity of Time

Event: something happening at a particular time and at a particular point in space

1. Two events occurring at different points in space and which are simultaneous for one observer cannot be simultaneous for another observer in a different frame of reference.
2. Two events occurring at the same point in space and which are simultaneous for one observer are simultaneous for all observers.

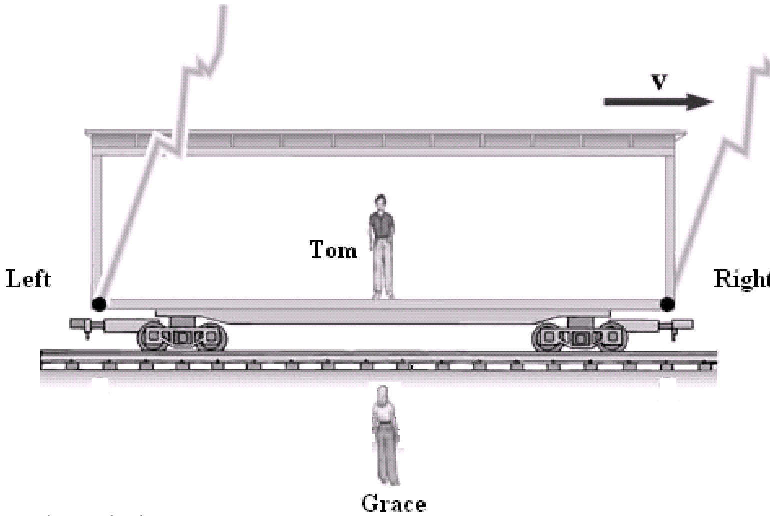
Einstein's Train *Gedanken* (Thought Experiment)

1. A train is traveling to the right with constant speed v with respect to the ground. Tom is in the midpoint of the train car. At the moment Tom passes Grace, two bolts of lightning strike the ends of the car, as seen by Grace. What does each observer notice and why?



Events occurring at different locations:

Events occurring at the same location:



From Grace’s point of view:

Grace’s observation about herself:

Grace’s observation about Tom:

The strikes are not simultaneous since Tom is moving towards the light traveling from the Right strike and traveling away from the light traveling from the Left strike. Since the speed of light in a vacuum is the same for all observers in inertial reference frames, the light from the Right strike will reach him first. Hence, he will report that the lightning struck the Right end first and then the Left end.

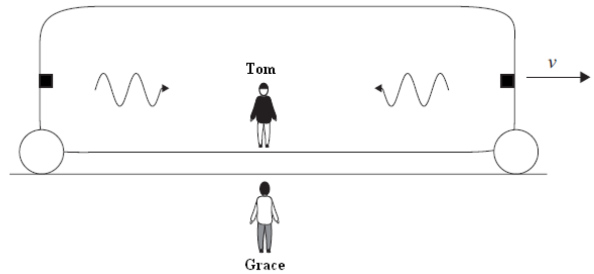
From Tom’s point of view:

Tom’s observation about himself:

Tom’s observation about Grace:

Grace will see the strikes as simultaneous since she is moving to the left, away from the light traveling to her from the Right end and toward the light traveling to her from the Left end. Since the speed of light in a vacuum is the same for all observers in inertial reference frames, and the light has a longer distance to travel from the Right end, it will reach Grace at the same time as the light from the Left end.

2. Grace is at rest with respect to the ground. Tom is in a carriage that is moving with speed *v* relative to Grace in the direction shown. Two flashes of light are emitted from the back and the front of the carriage. According to Tom's clock they arrive at Tom's position simultaneously. Explain why the arrival of the light pulses at Tom will also be simultaneous to Grace.



The arrival of the light flashes occurs at the same location in space and since they are simultaneous for one observer (Tom) they are simultaneous for all observers (Grace). This means that Grace reports they reach Tom simultaneously (but not that they were emitted simultaneously since the flashes were emitted at two different locations.)

Tom would report that the flashes were emitted from each end of the carriage simultaneously and since they travel the same distance to him at the same speed would arrive at the same time.

Grace would report that the flash from the left end occurred first. Since Tom is moving away from that flash which has a longer distance to travel and towards the flash from the right end which has s shorter distance to travel at the same speed, the flashes arrive at him simultaneously.

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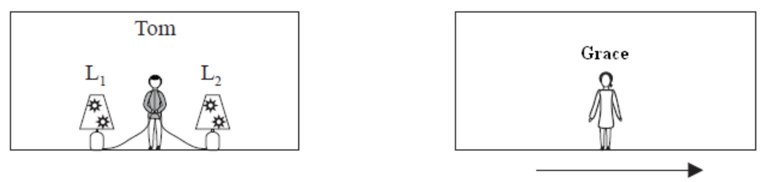
3. Tom and Grace are two observers each in a separate reference frame. The reference frames are moving relative to each other in the same straight line with constant velocity.



direction of motion of Grace's reference frame relative to Tom's reference frame

Two lamps L_1 and L_2 are operated by the same switch. Tom is at the mid-point between the lamps as measured in his frame of reference. The lamps and the switch are at rest relative to Tom.

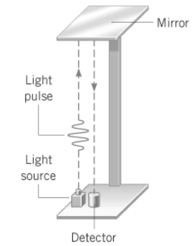
Tom switches on the lamps and to him they light simultaneously. Explain why the lamps will not light simultaneously, according to Grace.

Electrical signals (electric fields, electricity) travel at the speed of light. According to Grace, Tom is moving to the left. Hence the signal will reach lamp L_2, first since it is moving towards the signal, and then lamp L_1, since it is moving away from the signal. So lamp L_2 will light first, according to Grace.

But as the light flash emitted by lamp L_2 travels towards Tom, Tom is moving away from it and towards the light flash emitted from lamp L_1. Grace will thus predict that Tom will report that the lamps lit simultaneously since L_2 lit first but had a longer distance to travel (at the same speed) than the flash from L_1 which was emitted later but had a shorter distance to travel.

B. Time Dilation

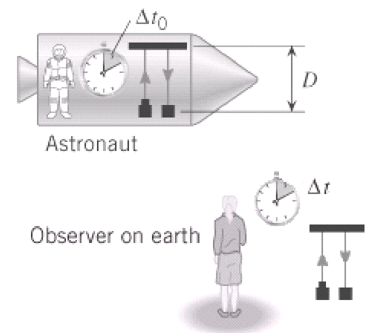
Light Clock:



Beginning Event:

Ending Event:

Each observer uses a light clock to measure the time, as seen from their frame of reference, between the pulse being emitted and detected. When the space ship is at rest with respect to the observer on Earth, the two clocks measure the same amount of time.

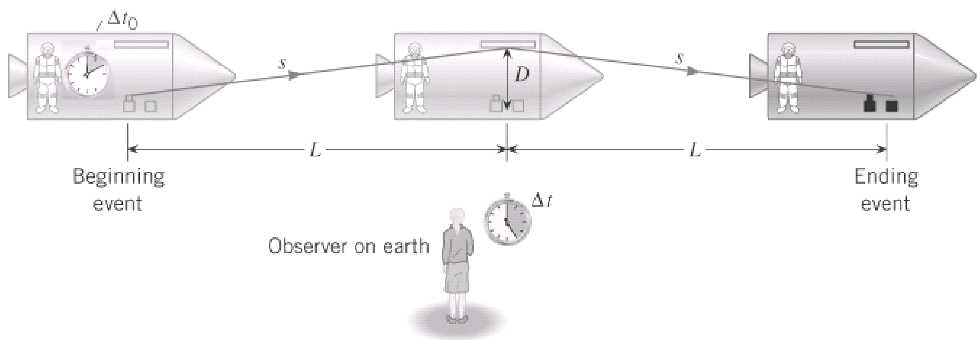


Δt_0 =

Δt =

If the two frames of reference are at rest with respect to one another, then

If the spaceship moves to the right with a speed *v*, the observer on Earth sees the light pulse travel a greater distance between the two events. Since each observer measures the same speed for the light pulse, if it traveled a greater distance then it must have taken a longer time. The observer on Earth thus measures a greater time interval between the two events than the astronaut does.



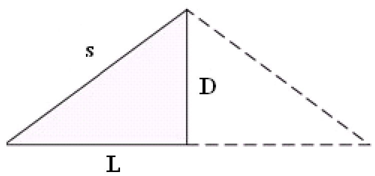
If the astronaut's frame of reference is moving with respect to the observer on Earth, then

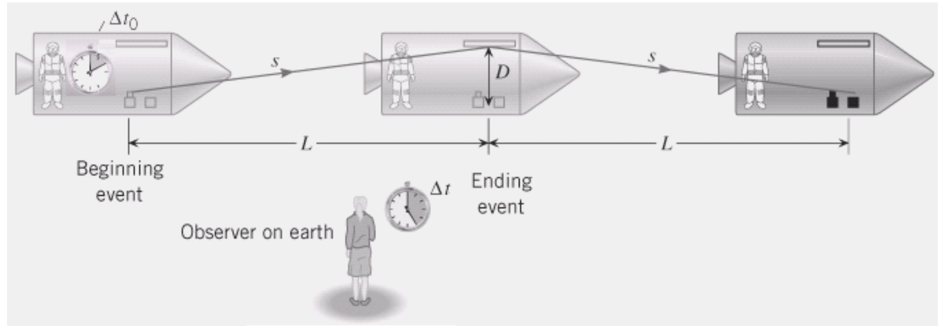
Time dilation:

NOTE:

Proper time interval (Δt_0):

NOTE:





Derivation of time dilation formula:

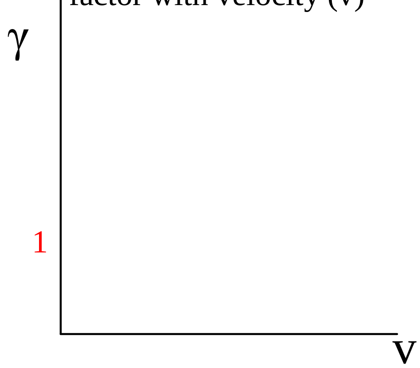
Observer sees ship move: Observer sees light pulse move: Astronaut sees light pulse move:

Lorentz gamma factor

Note:

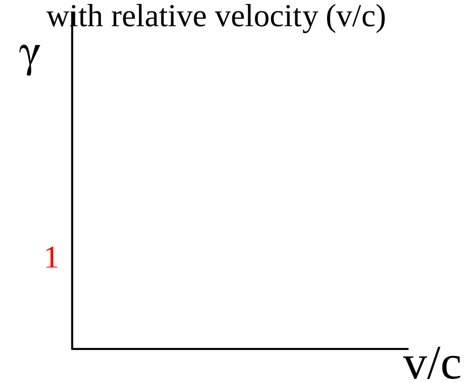
Lorentz factor	For an object at rest:	At low (non-relativistic) velocities:	At high (relativistic) velocities:

Variation of Lorentz gamma factor with velocity (v)



Relative velocity:

Variation of Lorentz gamma factor with relative velocity (v/c)



Example: A certain particle created in an experiment has a lifetime of 2.2μs when measured in a reference frame in which the particle is at rest.

a) Describe a reference frame in which the particle could be considered at rest.

b) What is the "proper lifetime?"

c) In another experiment, the particle is accelerated in a "particle accelerator" to a speed of 2.7×10^8 m/s. This is the speed of the particle as measured relative to a stationary frame of reference. Give an example of such a frame of reference.

d) Calculate the Lorentz factor for this particle.

e) Calculate the lifetime of the particle as measured in the stationary reference frame.

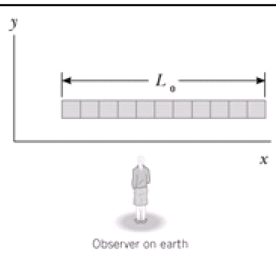
f) What would be its lifetime if it traveled at 0.98c?

C. Length Contraction

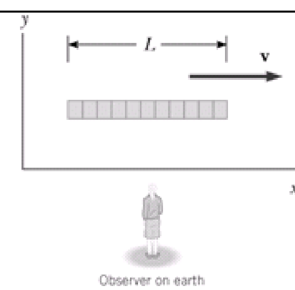
Because of Special Relativity, observers moving at a constant velocity relative to each other measure different time intevals between two evetnts. Bt if *speed = distance/time* and the speed is the same for each observer, then the two observers must measure different distances or lengths as well. This effect is known as **length contraction.**

length contraction:

For example, a ruler at rest appears to have a length of L_0. This is known as its **proper length.**



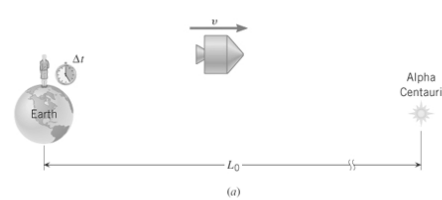
For a stationary observer on Earth, a moving ruler would appear to be shorter but just as thick. It only shrinks in the horizontal direction.



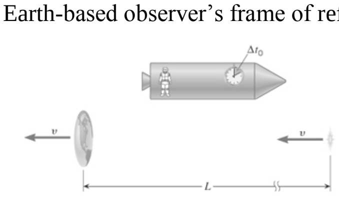
proper length:

NOTE:

A rocket travels to Alpha Centauri at a speed of v = 0.95c, as measured by an Earth-based observer. Both observers agree on the relative speed since, to the astronaut, the Earth observer is moving the other way at v = 0.95c. There is no preferred inertial frame of reference from which to measure absolute speed. However, to the Earth observer, the clock on board the space ship will appear to run more slowly and the ship will appear to shrink in the direction of motion. The situation is reversed for the astronaut. Relative to the astronaut, the clock on Earth will appear to run slowly and the width of the Earth, as well as the distance to the star, will appear to shrink. Both observers will agree, however, on the diameter of the ship and "height" of the Earth.



Earth-based observer's frame of reference

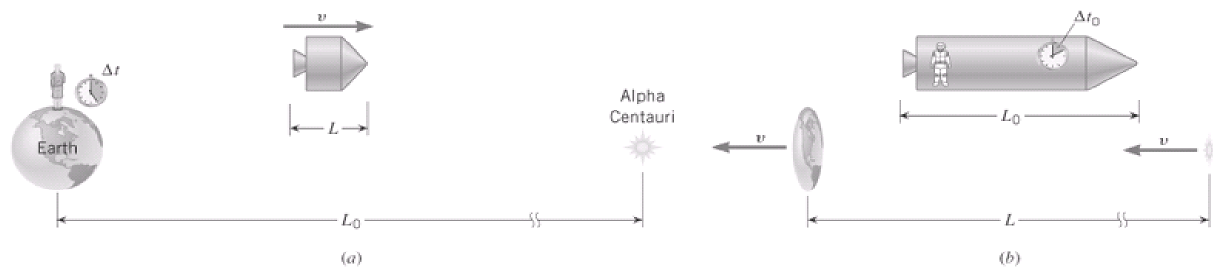


astronaut's frame of reference

A note on proper time and proper distance

The proper time in this example is the time recorded by the astronaut because only in the astronaut's frame of reference do the two events (leaving Earth and arriving at the star) occur at the same location (the door of the ship). To the astronaut, it's as if the ship is at rest and the Earth and star are in motion in the other direction and pass by the door of the ship as they move.

The correct frame of reference in which to measure the proper length, however, depends on what is being measured. If the distance from Earth to the star is being measured, then the correct frame of reference is the Earth-based observer's since both the star and the Earth are at rest relative to this person. But if the length of the ship is to be measured, then the correct frame of reference is the astronaut's since the ship is at rest relative to the astronaut.



EXAMPLE: An astronaut is set to go on a journey to Alpha Centauri, a nearby star in our galaxy that the astronaut measures from her observatory to be 4.07 x 10^{16} m away. The astronaut boards the ship at rest on Earth before take-off and uses a meter stick to measure the length of the ship as 82 m and the diameter as 21 m. After take-off, an observer on Earth notices the space ship traveling past him at a speed of v= 0.950c in route to Alpha Centauri.

a) How long does the trip to Alpha Centauri take as measured by:

i) the Earth bound observer?

ii) the moving astronaut?

b) What is the distance between Earth and the star as measured by:

i) the Earth bound observer?

ii) the moving astronaut?

c) While the ship is on its journey, what is the length of the ship as measured by:

i) the Earth bound observer?

ii) the moving astronaut?

d) While the ship is on its journey, what is the diameter of the ship as measured by:

i) the Earth bound observer?

ii) the moving astronaut?

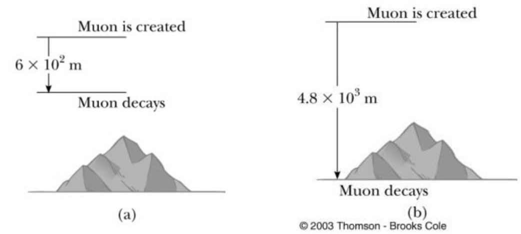
Light-year (ly): Distance can also be measured in light-years which is the distance light will travel in one year.

e) If the distance to Alpha Centauri is 4.3 ly, how long will it take the spaceship:

i) as measured by the astronomer on Earth? ii) as measured by the astronaut in the ship?

Cosmic Ray Muon Experiment

Muon:



Experiment:

1)

2)

3)

Question: Why do so many muons reach the ground before decaying?

From Earth frame of reference:

From muon's frame of reference:

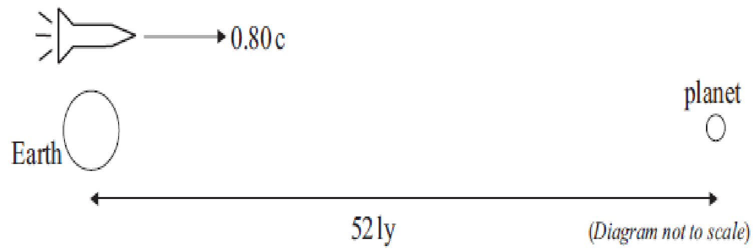
EXAMPLE: A muon having a lifetime of 2.2 μs as measured in its own frame of reference is created in the upper atmosphere and travels toward Earth at a speed of 0.99c.

1. How far can a muon travel before it decays, as measured in its own frame of reference?

2. What is the lifetime of the muon, as measured from the Earth?

3. How far will the muon travel through the atmosphere, as measured from the Earth?

EXAMPLE: A spacecraft leaves Earth at a speed of 0.80 c as measured by an observer on Earth. It heads towards, and continues beyond, a distant planet. The planet is 52 light years away from Earth as measured by an observer on Earth. When the spacecraft leaves Earth, Amanda, one of the astronauts in the spacecraft, is 20 years old.



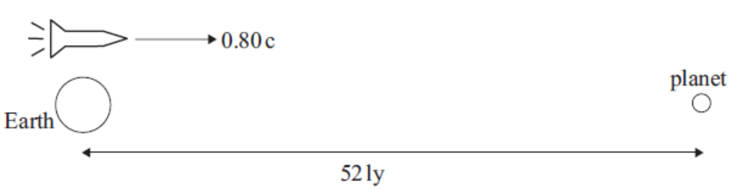
a) Calculate the time taken for the journey to the planet as measured by an observer on Earth.

b) Calculate the distance between Earth and the planet, as measured by Amanda.

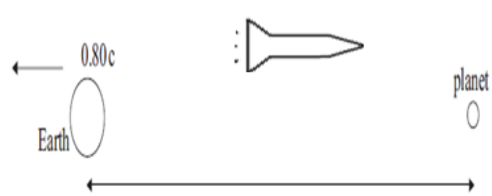
c) Calculate Amanda's age as the spacecraft goes past the planet, according to:

i) an observer on Earth.

ii) Amanda.



Earth-based observer's frame of reference

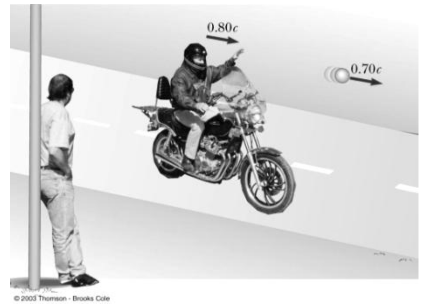


Astronaut's frame of reference

d) As the spacecraft goes past the planet, Amanda sends a radio signal to Earth. Calculate, as measured by the spacecraft observers, the time it takes for the signal to arrive at Earth.

Lorentz Transformations and Relativistic Formulas for Addition of Velocities

1. A motorcyclist drives past a stationary observer at a speed of 0.80*c* and throws a ball forward at 0.70*c*, as shown. The stationary observer is 30,000. m from a stoplight. After 3 seconds:
   -how far is the motorcyclist from the stoplight?
   -how fast is the ball moving relative to the stationary observer?



$\mathbf{x}^{/}$ = position of an object as measured in moving frame
x = position of an object as measured in stationary frame
$\mathbf{u_x}^{/}$ = velocity of object in x-direction as measured in moving frame
$\mathbf{u_x}$ = velocity of object in x-direction as measured in stationary frame
v = velocity of frame 2 in x-direction as measured in stationary frame
c = speed of light in a vacuum

Galilean transformation:

Relativistic transformation:

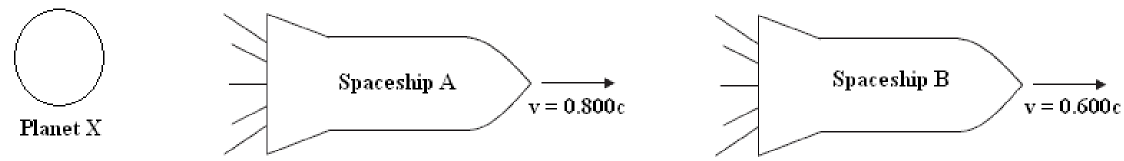
Relativistic transformation formula:

NOTE:

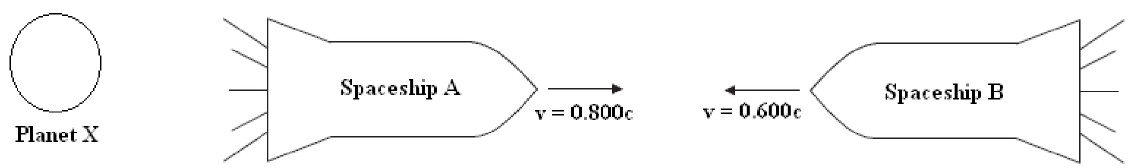
2. Suppose the motorcyclist in the above example shines a flashlight ahead of him. How fast does the stationary observer see the light beam travel?

3. Two bicyclists approach each other at a speed of 0.60*c*. What is their relative velocity of approach? What is the velocity of approach as measured by someone at rest with respect to the ground?

4. Two spaceships are moving with the speeds indicated, as measured by an observer on Planet X. Calculate the relative velocity of approach, as measured in the frame of reference of one of the spaceships.



5. Two spaceships are moving with the speeds indicated, as measured by an observer on Planet X. Calculate the relative velocity of approach, as measured in the frame of reference of one of the spaceships.



Spacetime Diagrams and World Lines

One implicit assumption in our traditional motion diagrams is that time is the same for all frames of reference. We can visualize relativistic motion, but we need a more sophisticated diagram.

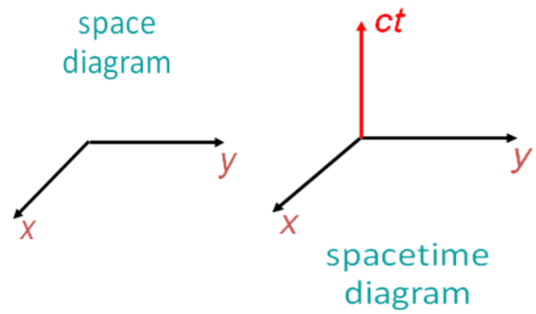
Spacetime diagrams are a very clear and illustrative way to show graphically how different observers in relative motion to each other have measurements that differ from each other.

1. As an example of the difference between a spacetime diagram and a traditional space diagram, contrast the plots of a particle in uniform circular motion in the x-y plane.

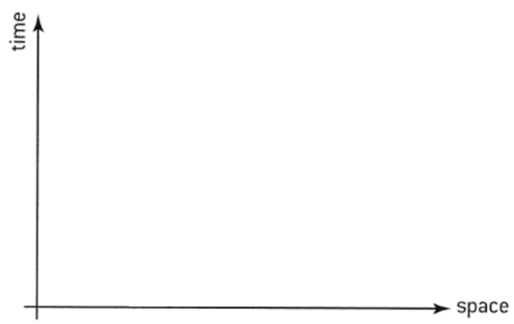
a. In the traditional diagram note that the particle

b. In the spacetime diagram the particle **<u>never</u>** repeats its coordinates.

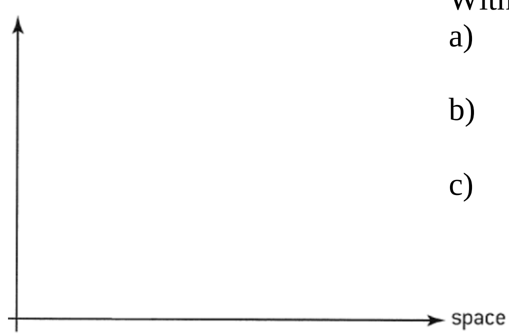
c. It as regularly as the in the traditional diagram,
but coordinate.



World Lines



Alternate vertical axis:



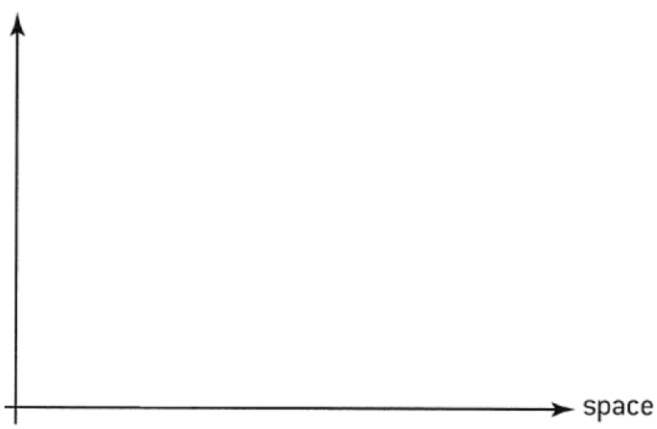
With a ct-x axis:

a)

b)

c)

Marking spacetime diagrams



Consider yourself with a radar gun and a friend w/ a mirror (similar to a light clock)

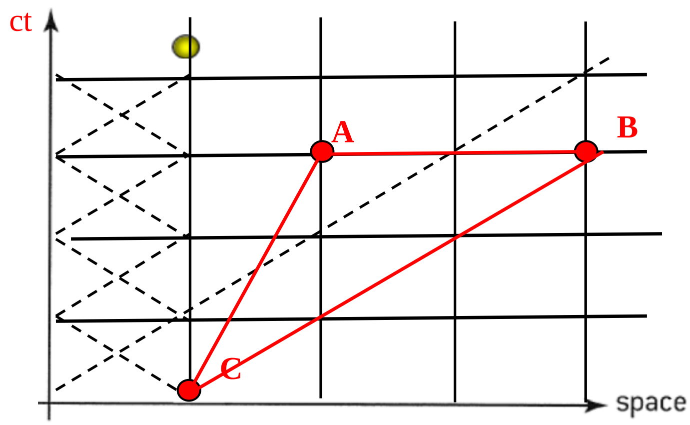
- Your friend moves down the x axis and you fire the radar gun, which measures the time it takes the light to travel to and from your friend's mirror($\Delta t = 2D / c$, where D=desired unit)
- For ex. you want D=1m, each meter is $\Delta t = 2(1) / 3\times10^8$ seconds in elapsed time.

2) Now consider spots **A**, **B**, and **C**

a) the square of the spacetime interval $(\Delta s)^2$ between each pair of pts.

b) Which paths are "legal" paths for a particle to follow?

Marking spacetime diagrams



Nature of science: Visualization of models- The visualization of the description of events in terms of spacetime diagrams is an enormous advance in understanding the concept of spacetime.

3) Find the shortest distance between A and B.

- If we go directly from A to B the spacetime distance is 2.83 m.
- However, if we go from from A to C (2.00 m) and then from C to B (0.00 m) we cut off 0.83 m!
-

- Minkowski's view of space is non-Euclidean. In Euclidean space, which can also be four-dimensional, the distance formula would be

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 + (c\Delta t)^2.$$

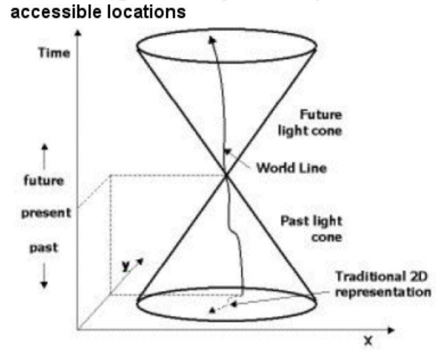
- It is precisely the **subtraction** of the $(c\Delta t)^2$ term in

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2,$$

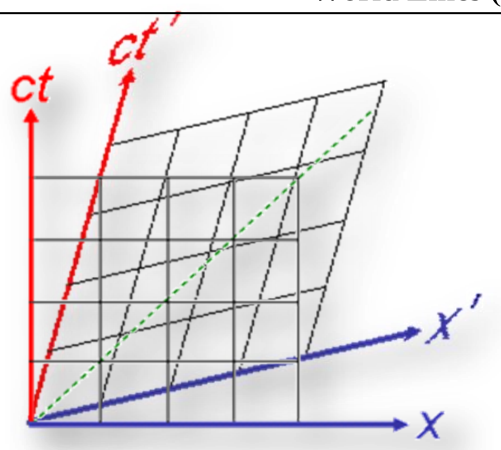
required by Einstein's 2nd postulate, that makes Minkowski spacetime non-Euclidean.

We cal this spacetime interval

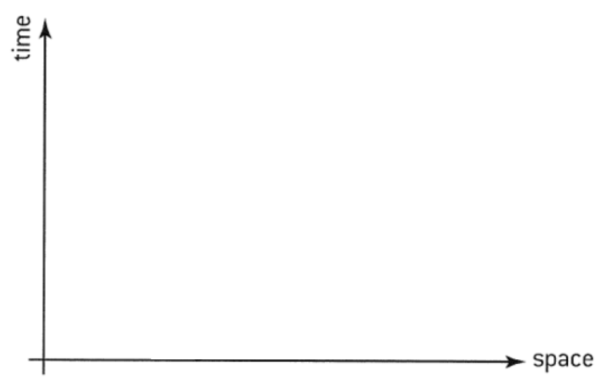
Minkowski visualised space and time as a light cone defining a boundary between past and future accessible locations



World Lines (x) with another IRF (x')



Your Turn



Note:

a)

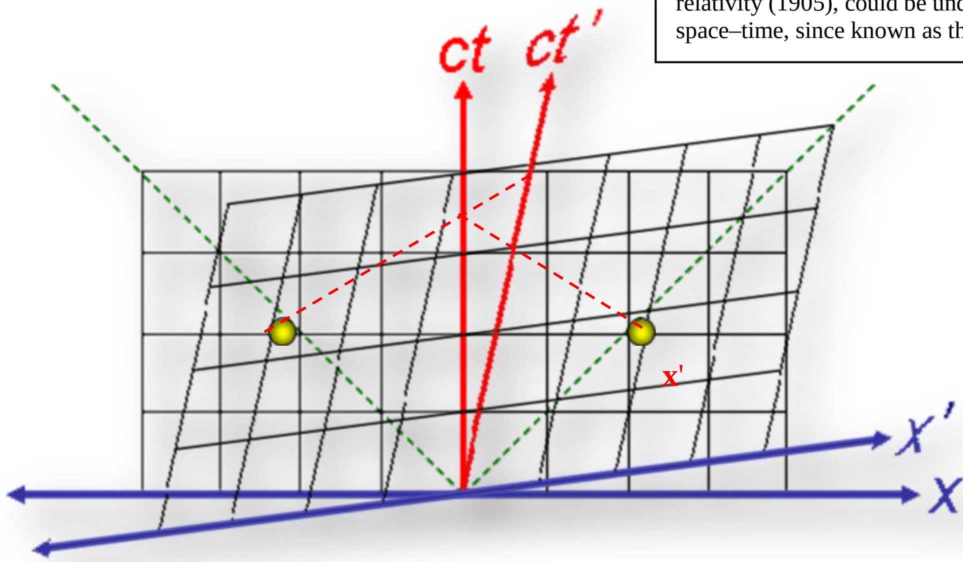
b)

c)

d)

Revisiting Simultaneity

Hermann Minkowski (1864 – 1909) was a German-Jewish mathematician and professor. In 1907 he showd that his former student Albert Einstein's special theory of relativity (1905), could be understood as four-dimensional space–time, since known as the "Minkowski spacetime".

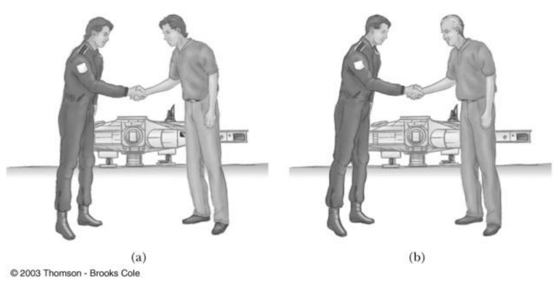


-An observer in IRF S is in the exact center of a train car. At the same instant, lights at each end of the car are turned on.
-Above, the world line of the same lights in a spacetime diagram for an observer in S', another IRF moving to the right relative to S. The two observers are directly opposite to each other at the instant the lights are turned on.
1) For the moving observer, which photon will be observed 1st?
2) Find the velocity of S' relative to S, in terms of *c*.

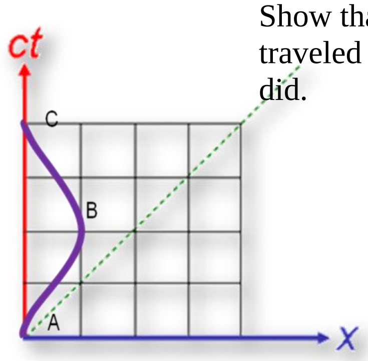
The Twin Paradox

According to special relativity, there is no preferred inertial reference frame so the time dilation effect is the same for all observers. Since each observer sees the other as moving past at a constant speed, each observer measures the other's clock as running slowly – the effect is symmetric. But what about this?

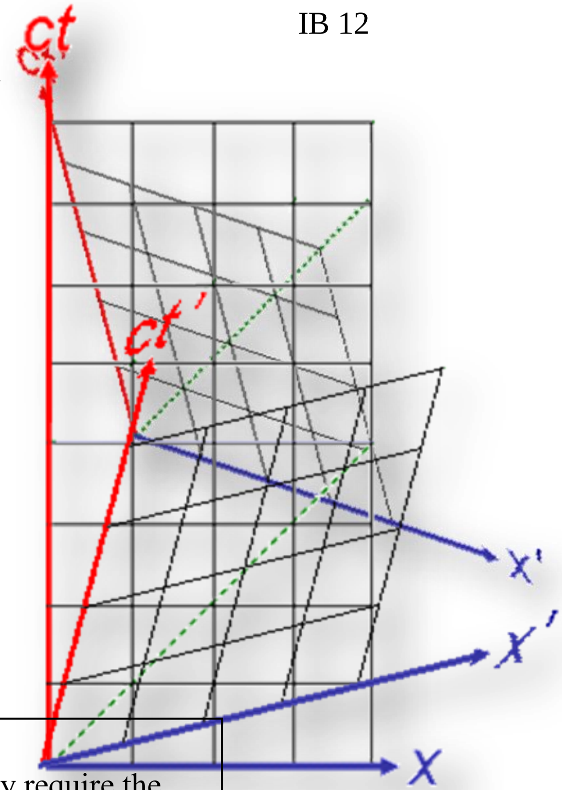
Two twins, Ein and Stein, grow up. Ein becomes and astronaut and Stein becomes a physics teacher. One day, Ein says goodbye to his brother and leaves on a space voyage to a distant star. Some time later, when he returns home, he meets his brother again. However, by now his brother is 30 years older than he is. You might think that this is because of relative motion. The clock in the space ship runs more slowly than the clock on the Earth, so Ein has aged less. But what about the symmetry of the time dilation effect? According to astronaut Ein, his ship was at rest while brother Stein and the Earth moved in the other direction. Since Stein's clock is now the moving one, shouldn't his clock run more slowly and Ein return to Earth as the older brother? Whose view of the situation is correct? In fact, shouldn't the brothers still be the same age since there is no preferred inertial frame of reference?



Explanation:



Show that in spacetime geometry, Einstein traveled a shorter distance than his twin on Earth did.



Above, the Twin Paradox scenario from the perspective of the twin on Earth (S). **Note:**

•The accelerations (curves) of Einstein's spacetime trajectory in S at the beginning, the turn-around, & the ending of his rocket trip.

•At no point is the slope of the tangent of Einstein's trajectory less than 1 (his speed is always $v < c$)

Theory of knowledge:
• Can paradoxes be solved by reason alone, or do they require the utilization of other ways of knowing?

Evidence to Support Special Relativity

1. Michelson-Morley experiment

Evidence of:

2. Hafele-Keating experiment

In 1971, experimenters J.C. Hafele and R.E. Keating from the U.S. Naval Observatory undertook an experiment to test time dilation. They made two flights around the world aboard commercial airliners, once east and once west, with each circuit taking about three days. They carried with them four cesium beam atomic clocks, accurate to within $\pm 10^{-9}$ s. The researchers expected that the relative motion of the clocks would produce a measurable time dilation effect ("moving clocks run slow").

In a frame of reference at rest with respect to the center of the earth,

a) the clock aboard the plane moving eastward, in the direction of the earth's rotation, is moving faster than a clock that remains on the ground, therefore it should

b) while the clock aboard the plane moving westward, against the earth's rotation, is moving slower than a clock that remains on the ground, therefore it should

When they returned, they compared their clocks with a ground based clock at the Observatory in Washington, D.C. The time intervals measured by the clocks that had traveled on the aircraft differed from those time intervals measured by the ground based clocks and provided confirmation of the time dilation effects of relativity.

Note that in this experiment, both time dilation due to motion or kinematics (special relativity) and time dilation due to gravity (general relativity) are significant and had to be taken into account.

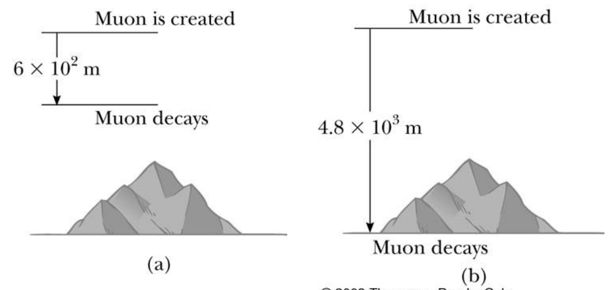
Evidence of:

Experimental Results:

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

3. Muon Decay experiments

A muon is an elementary particle that, like an electron, is classified as a lepton. It is unstable and decays into other particles with an average lifetime of about 2.2 microseconds. Muons are created naturally by collisions of incoming cosmic radiation from outer space with particles in the earth's upper atmosphere. Hence, this type of muon is known as a "cosmic ray muon."



The question is: Why do so many muons make it to the bottom of the mountain?
The answer lies in special relativity.

From the frame of reference of Earth, time runs slowly for the muon so it has time to reach the ground before decaying.

Evidence of:

From the muon's frame of reference, the height of the atmosphere contracts so the muon has very little distance to travel. It can easily cross this distance within its 2.2 μs lifetime.

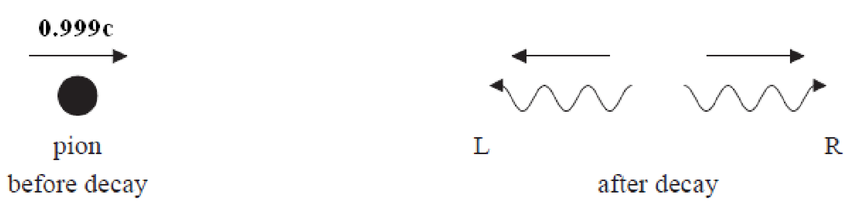
Evidence of:

1) A muon traveling at 0.99c has a gamma factor of 7.1.

   a) What is the lifetime of the muon, as measured from Earth?

4. Pion Decay experiments

A pion (short for pi meson) is an unstable subatomic particle (denoted $\pi+$, π^-, or π^o). Experiments at CERN in 1964 showed that fast moving pions decayed into two high energy gamma-ray photons moving in opposite directions. Measurements of the speed of the photons showed that they were still moving at 3.00 x 120^8 m/s no matter how fast the original pion was moving.



Evidence of:

F. Relationship Between Mass and Energy

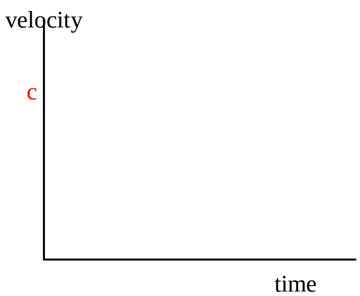
Newton mechanics: A constant force produces a constant acceleration.

Implication:

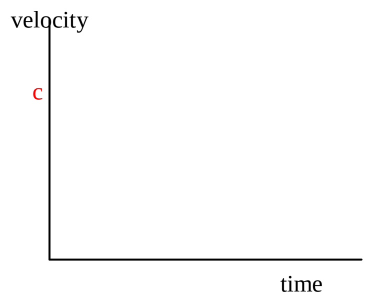
Relativistic mechanics: As the object's speed approaches the speed of light, the acceleration decreases even if the force is constant.

Implication: .

Newtonian mechanics



Relativistic mechanics



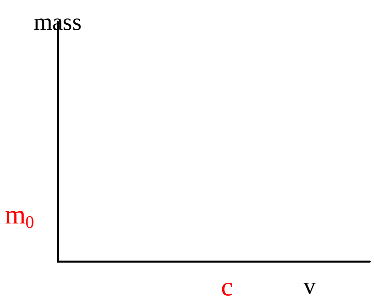
Rest mass (m_0):

NOTE:

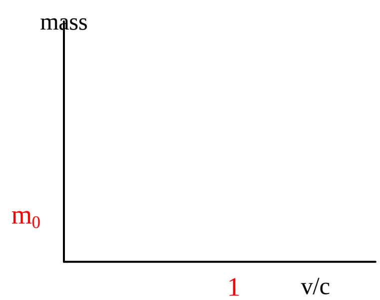
Mass (m):

Relationship:

Mass versus actual speed



Mass versus relative speed



Consequence:

Explanation:

Rest energy (E_0):

Relationship:

1. What is the energy equivalent of a 0.20 kg golf ball at rest?

2. What is the rest energy of an electron?

Alternate units for energy

1 eV =

1 eV =

1 MeV =

3. What is the rest energy of an electron?

Alternate units for mass

4. What is the rest mass of an object whose energy equivalent is 1 MeV?

5. What is the rest mass of an electron?

Quantity	**Units**	**Units**
Energy		
Mass		
Momentum		

Formula representing equivalence of mass and energy:

For an object at rest:

For an object in motion:

6. What is the energy equivalent of an electron accelerated to a speed of 0.90*c*?

Total energy of a moving object =

Derivation:

Relativistic kinetic energy formula:

7. What is the kinetic energy of an electron accelerated to a speed of 0.90*c*? The rest mass of an electron is 0.51 MeV c^{-2}.

8. A proton is accelerated to a speed of 0.95*c*. Determine its energy, rest energy, and kinetic energy.

Particle acceleration

units for charge:

9. An electron is accelerated through a potential difference of 2.0×10^6 V. Calculate its energy, kinetic energy, and speed.

G. Relativistic Momentum and Energy

Newtonian momentum and kinetic energy	
Relativistic momentum and kinetic energy	
units for Newtonian momentum	units for relativistic momentum
Relativistic total energy	

1. a) A proton is at rest. Determine its:

i) mass

ii) energy

iii) momentum

b) The proton is accelerated through a potential difference V until it reaches a speed of 0.900c. Determine:

i) its kinetic energy

ii) the potential difference

iii) its mass

iv) its energy

v) its momentum

2. A proton is accelerated through a potential difference of 3.0×10^9 V.

a) Calculate the energy of the proton after its acceleration.

b) Calculate the final momentum of the proton.

3. "Pair production" is a process by which antimatter pairs of particles are produced from energy. This can happen when a high energy gamma ray photon is in the vicinity of a heavy nucleus. For example, if a gamma photon is near a lead atom, the reaction pictured at right might occur, where the photon creates an electron-positron pair. If the energy of the photon is 3.20 MeV, calculate the following quantities. (Neglect the recoil of the lead atom and assume the energy is shared equally between the particles.)

$$\gamma \rightarrow e^- + e^+$$

a) The energy and kinetic energy of each particle.

b) The speed of each particle.

c) The mass of each particle.

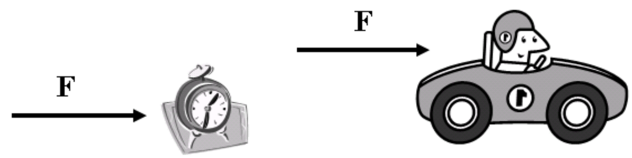
d) The momentum of each particle.

General Relativity

General Theory of Relativity: a more general theory of relativity that takes into account non-inertial (accelerating) reference frames and relates them to the effects of gravity

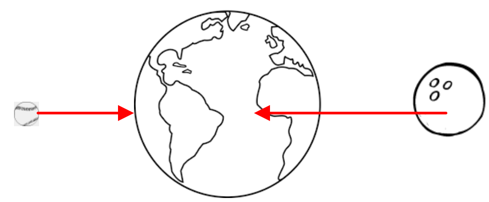
1. **Inertial Mass** – A property of an object that determines how much it resists accelerating.

   Different masses have different accelerations when the same net force acts on them.



2. **Gravitational Mass** – the property of an object that determines how much gravitational force it feels when near another object.

   Different masses have different gravitational forces acting on them them.



Observation: All experiments to measure each type of mass for an object have shown that, within the experimental uncertainty,

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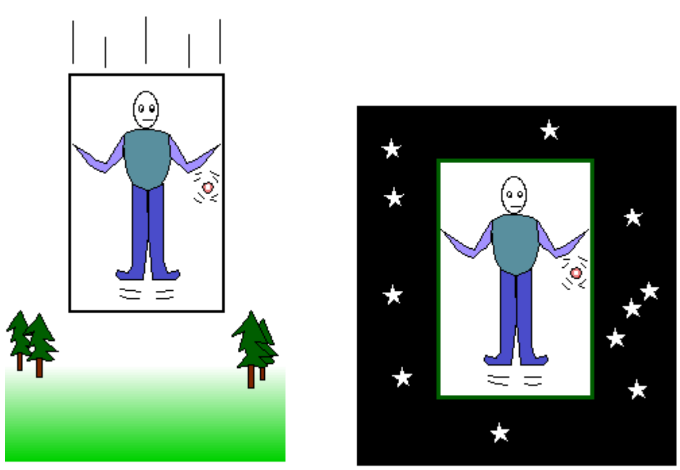
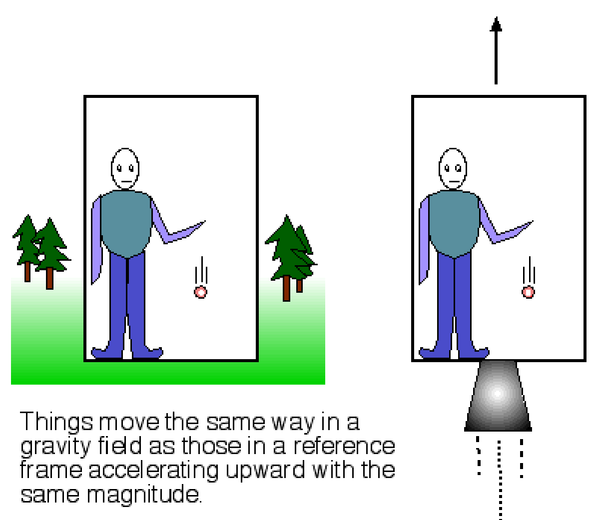
Einstein's Principle of Equivalence:



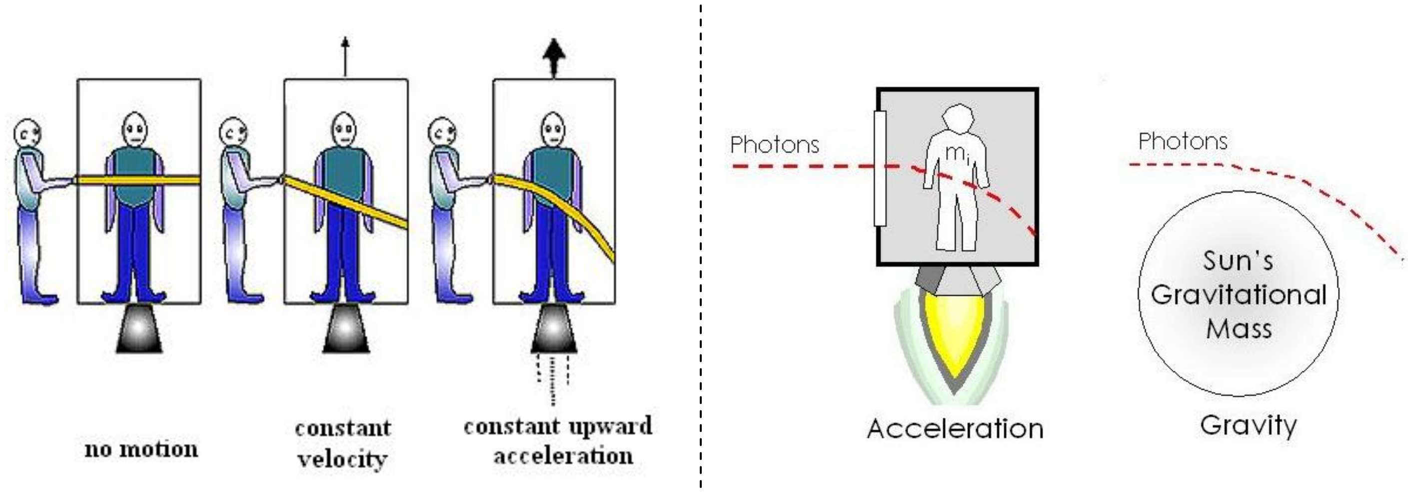
Things falling freely in a gravity field all accelerate by the same amount, so they move the same way as if they were in a region of zero gravity — ``weightlessness''!

Inertial frames of reference that are equivalent

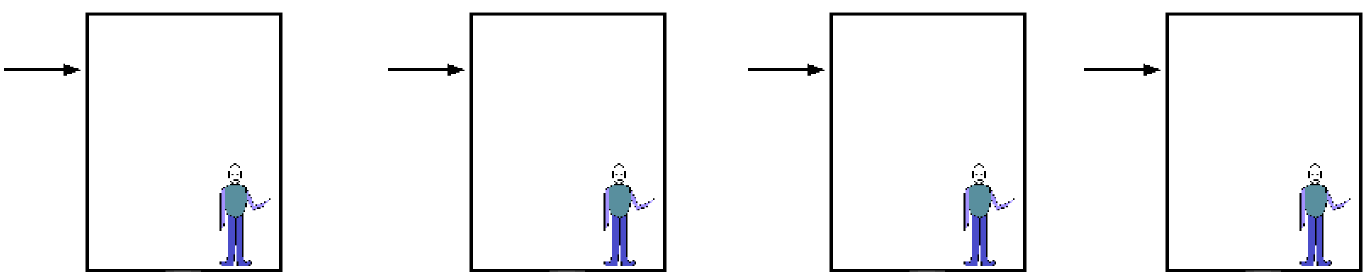


Non-inertial frames of reference that are equivalent

Based on the principle of equivalence, Einstein predicted that . . .
.



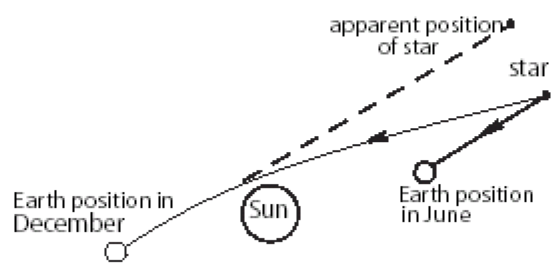
Sketch the path of the light beam across each elevator.



a) floating in space | b) in free-fall on Earth | c) accelerating upward in space | d) at rest on Earth's surface

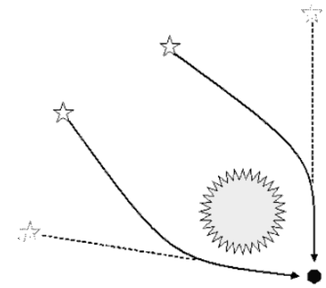
Eddington's solar eclipse measurements

The positions of several stars were measured against a background of fixed stars. Six months later, those stars were hidden behind the Sun due to Earth's new position in its yearly revolution. It was predicted by General Relativity that these stars should still be visible if the gravitational field of the Sun bent the light rays around it and deflected the light rays toward Earth. However, these "hidden" stars would still not be visible due to the glare of the Sun.



But an expedition led by Sir Arthur Eddington sent to the island of Principe sought to measure the deflection of these light rays during a total eclipse of the Sun in 1919 when the stars would be briefly visible. He measured the new positions of the stars against the background of fixed stars and found that they had apparently shifted position.

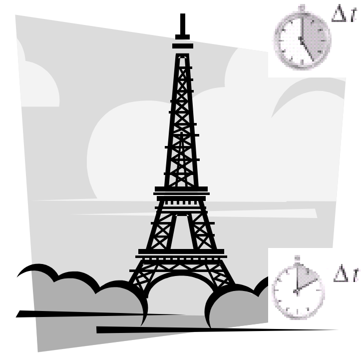
Importance:



Gravitational lensing: Massive galaxies can deflect the light from distant sources of light so that the rays bend around the galaxy. The galaxy acts like a lens so that observers on Earth can see multiple images of the source.

Importance:

Based on the principle of equivalence, Einstein predicted that . . . .



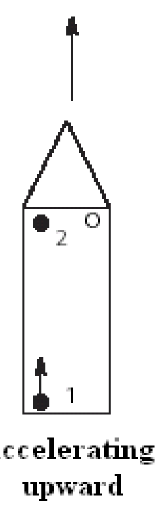
Clocks near the surface of the Earth . . .

Gravitational Red Shift: the increase in wavelength (or decrease in frequency) of light when it moves from a stronger to a weaker gravitational field

Explanation: The frequency of vibration of an object or of an electromagnetic wave (a photon) is essentially a measurement of time. Slowing the frequency of vibration means that time is running slower. The decrease in frequency of vibration in a gravitational field can be explained in two different ways.

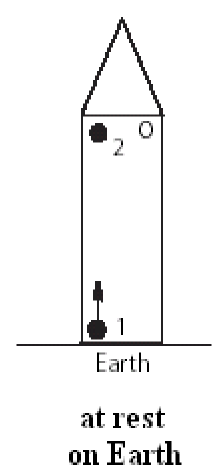
Doppler Shift

Consider a rocket with a light source at the bottom (1) and a detector at the top (2), as shown in (a). If the rocket accelerates upward, the detector will be accelerating away from the light source. Thus, the light waves from 1 will reach the detector at 2 less frequently, hence the received frequency will now be less than the emitted frequency, that is, the frequency will be shifted to a lower (redder) frequency.



Energy Loss

Consider a rocket at rest on the surface of Earth. As a photon moves from the bottom (1) to the top (2), it gains potential energy due to the gravitational field. Since its total energy must remain constant, it will lose "kinetic" energy. Since the energy of a photon is $E_{photon} = hf$ this means that its frequency will decrease.



Result: Since a lower frequency means a higher period, in both cases it will appear to an observer at the top that clocks at the bottom are running slower.

Gravitational Red-Shift
Frequency Formula:

Assumption:

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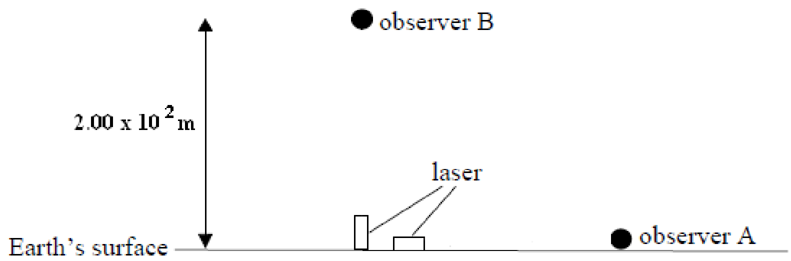
1. Two identical lasers are at rest on the Earth's surface, each pointed towards a different observer. Observer A measures the frequency of the laser to be 6.89×10^{14} Hz.

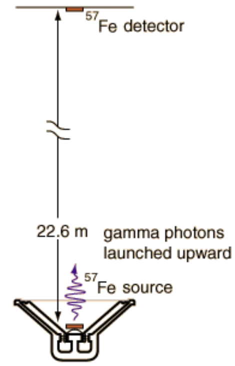


a) Does observer B measure a frequency that is greater, less or the same as that measured by observer A?

b) Calculate the difference in frequencies as measured by the two observers.

c) If this experiment was repeated in a rocket ship in space far from any massive bodies, how fast would the rocket have to accelerate for the results to be the same? Justify your answer.

Evidence to support gravitational red-shift effect



1. **Experiment**: Pound-Rebka experiment

   In the early 60's physicists Pound, Rebka,and Snyder at the Jefferson Physical Laboratory at Harvard measured the shift in gamma rays emitted from iron-57 by placing a source at the base of Harvard Tower and a detector at its top, a distance of 22.6 m higher. They were able to measure the shift in frequency of the photons and the results agreed with the predicted value to within 1%.

2. **Experiment**: Shapiro time delay experiment

   The delay in time taken for a radar pulse to travel to a nearby planet (Venus or Mercury) and return due to gravitational field of the Sun was measured in 1960s. Results agreed with general relativity predictions.

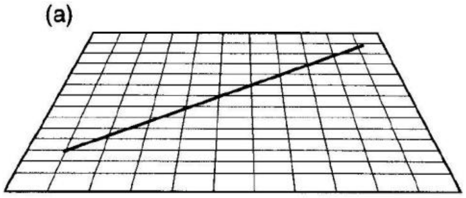
Spacetime:

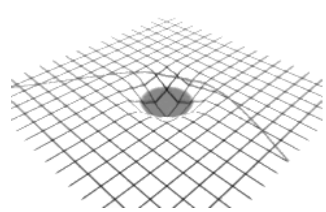
Curvature of spacetime

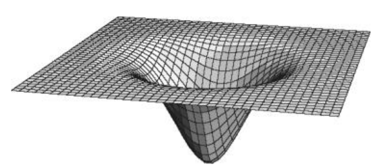
a) Spacetime of empty space (with no mass nearby) appears flat.

b) Mass (matter) warps spacetime so that the geometry of space does not follow the usual rules of Euclidean geometry. Light and particles with no forces on them follow paths through spacetime, called geodesics, which appear curved against the background of flat space.

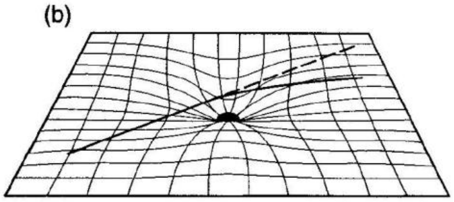
(a)

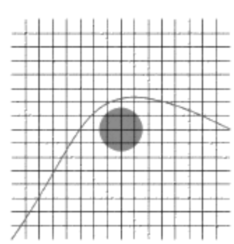


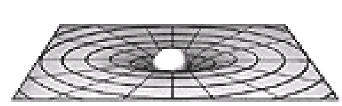




(b)







NOTE:

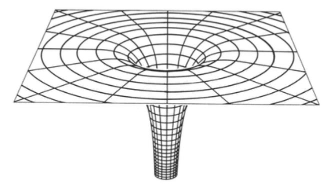
1) An object moving with constant velocity follows the shortest path through spacetime which is a straight line.

2) Since any mass (such as the Sun or a planet) will warp spacetime,

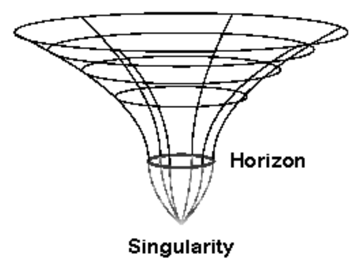
   a) a projectile follows the shortest path in spacetime which is a curve (parabola)

   b) a planet or satellite in orbit follows the shortest path in spacetime which is a curve (circle or ellipse)

3) A black hole causes such an extreme curvature of spacetime that any light leaving the surface of a black hole will be bent back to the surface and not escape.



Black Hole: Since matter warps spacetime, a black hole is a region of such extreme curvature in spacetime that not even light can escape it.

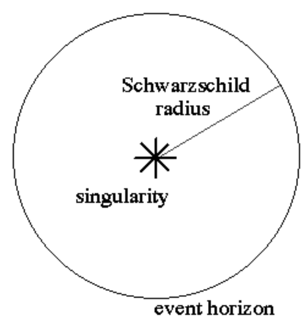


Center of a Black Hole (singularity) – the single point to which all mass would collapse

Surface of a Black Hole (event horizon) – where the escape speed is equal to *c* and within this surface, mass has "disappeared" from the universe

Schwarzschild Radius (R_S):

- the distance from the center of a black hole where the escape speed is the speed of light
- the distance from the singularity to the event horizon of a black hole
- the distance from a singularity at which light can no longer escape



Means of detecting black holes:

1) matter falling into black holes radiates
2) its gravitational influence on other objects
3) observing gravitational lensing effects
4) emission of Hawking raditation

Schwarzchild radius formula

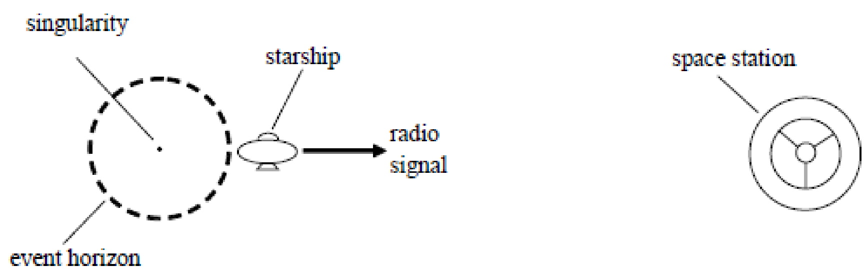
1. Calculate the size of a black hole that has the same mass as our Sun (m = 1.99 x 10^{30} kg).

Gravitational Red-Shift (Time Dilation) near a Black Hole

As one gets close to a black hole . . .

At the event horizon . . .

1. A starship is stationary outside the event horizon of a black hole. A space station is also stationary and is located far away from the black hole and any other massive object.



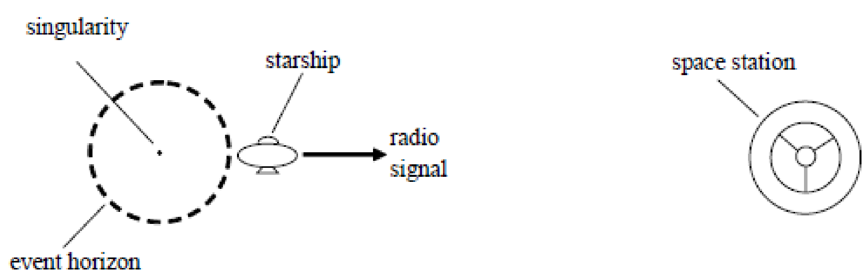
a) The starship transmits a radio signal to the space station. Compare the frequency transmitted by the starship to the frequency received by the space station.

b) The radio signal is sent once every minute for an hour, as measured by an observer on the starship. Compare this to what is received at the space station.

Gravitational time dilation near a black hole (gravitational red shift)

Δt_0	Time measured by an observer near the black hole
Δt	Time measured by an outside observer far away from the black hole
R_s	Schwarzschild radius (distance from singularity to event horizon)
r	How far observer near the black hole is to the singularity

2. The starship now moves to a distance of three times the Schwarzschild radii from the event horizon and sends out the radio signal once every minute, as measured by an observer on the starship.



a) How much time elapses between each radio signal as measured by an observer at the space station?

b) If the frequency of the radio signal is measured at the starship to be 5.0×10^{14} Hz, what frequency is received at the space station?