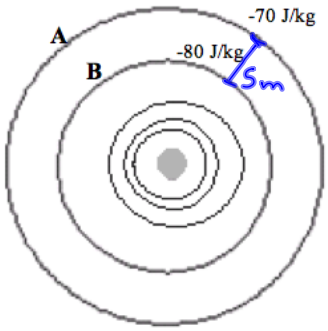
Gravitational Potential Gradient

gradient: rate of change with respect to something
that is, "slope" or "derivative"

gravitational potential gradient:
the gravitational field is the negative gradient of the gravitational potential with respect to distance

Formula

$$g = -\frac{\Delta V}{\Delta r}$$

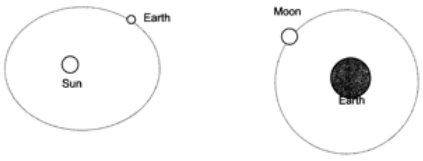


$$\vec{g} = \frac{-\Delta V}{r} = \frac{10\ \frac{J}{kg}}{5\ m}$$

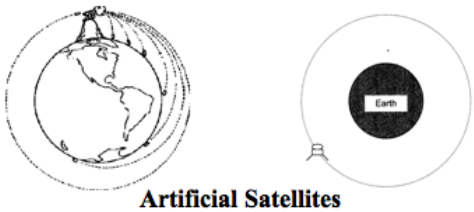
What is the average gravitational field strength between equipotential surfaces A and B if they are 5.0 m apart?

Satellite Motion

IB 2



Natural Satellites



Artificial Satellites

1. Astronauts in the space shuttle float about as if there is no force of gravity acting on them. Is there?

yes...

2. What is "weightlessness?"

sensation of no weight (really no normal force)

3. What causes the sensation of weightlessness in:

a) free fall? no normal force - floor falling w/same accl

b) orbital motion? still in free fall - no normal force

c) deep space? no normal force b/c no gravity

Period of a satellite:

$T^2 = kr^3$

$k = \frac{4\pi^2}{GM}$

Orbital speed of a satellite:

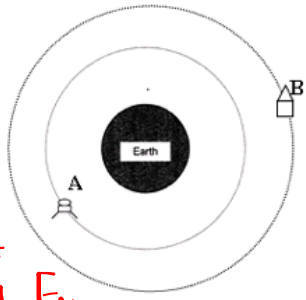
$V = 2\pi r / T$

Acceleration of a satellite:

$a = V^2/r = 4\pi^2 r / T^2$

$g = \frac{GM}{r^2}$

4. Compare the motions and energies of satellites A and B.



B: Slow, Low E_K, High E_P

A: Fast, High E_K, Low E_P

Energy of Orbiting Satellites

Gravitational Potential Energy $E_p = -\frac{GMm}{r}$

Kinetic Energy

$\Sigma F = \frac{mv^2}{r}$

$\frac{1}{2}\frac{GMm}{r^2} = \frac{1}{2}\frac{mv^2}{r}$

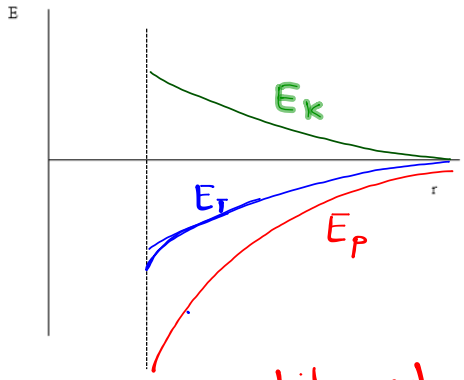
$-\frac{1}{2}E_p = E_k$

Total Energy

$E_T = -E_k$ $\quad E_T = E_k + E_p$

$E_T = \frac{1}{2}E_p$

Graphs of the energies of an orbiting satellite



Comparisons:

orbit speed

$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$

$v_{orb} = \sqrt{\frac{GM}{r}}$

$v_{orb} = \sqrt{-V}$

1. A 1500 5000 kg satellite is to be put into orbit around the Earth at an altitude of 200 km.

a) How much potential energy will the satellite have at this altitude?

$E_{P_f} = -3.04\times10^{11}\ J$

$E_{P_o} = -3.13\times10^{11}\ J$ $\Big)\ \Delta E_p$

b) How much kinetic energy will the satellite need to orbit at this altitude?

$E_k = -\frac{1}{2}E_{P_f} = +1.52\times10^{11}\ J$

c) What is the total amount of energy the satellite has at this altitude?

$E_T = \frac{1}{2}E_p = -E_k = -1.52\times10^{11}\ J$

d) What is the orbital speed of this satellite?

$E_k = \frac{1}{2}mv^2$

$v_{orb} = \sqrt{\frac{2(1.52\times10^{11}\,J)}{5000\,kg}} = 7.8\times10^{3}\,m/s$

$= \sqrt{GM_E/r}$

e) What is the minimum amount of energy needed to lift the satellite from the surface of the Earth to this altitude?

$\Delta E_p = +9\times10^{9}\,J$

f) What is the minimum amount of energy needed to put the satellite in orbit at this altitude?

$E_{needed} = \Delta E_p + E_k$

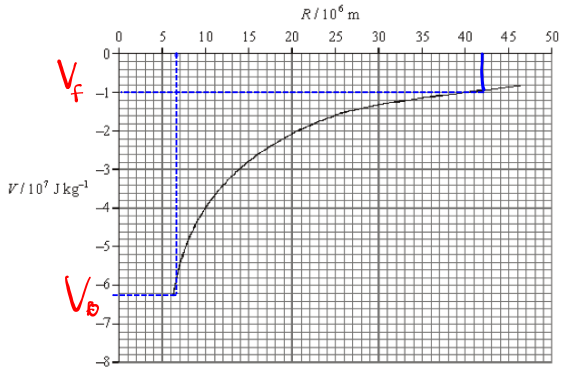
$\Delta E_p = 9\times10^{9}\,J$

$E_k = 1.52\times10^{11}\,J$

g) Why would more energy be needed in real life than that calculated in the previous part?

air resistance at lower altitudes

2. The graph below shows the variation of gravitational potential V due to the Earth with distance R from the centre of the Earth. The radius of the Earth is 6.4 x 10^6 m. The graph does not show the variation of potential V within the Earth.



Use the graph to determine the minimum energy required to put a satellite of mass 1.0 x 10^4 kg into an orbit at a height of 3.6 x 10^7 m above the surface of Earth.

$r = 4.2 \times 10^7$ m

Energy to lift: $\Delta E_p = m\Delta V = 10^4 kg\left(-1\times10^7 J/kg - (-6.3\times10^7 J/kg)\right)$

$= 5.3\times10^{11} J$

$E_K = -\frac{1}{2}E_p = \frac{1}{2}mV_f = \frac{1}{2}\cdot 10^4 kg \cdot (10^7 J/kg) = .5\times10^{11} J$

$E_{needed} = 5.3\times10^{11} J + .5\times10^{11} J = 5.8\times10^{11} J$

More energy than this is needed because: