

# Vectors and Two Dimensional Motion

Essential idea: Some quantities have direction & magnitude, others have magnitude only, and this understanding is the key to correct manipulation of quantities. This topic will have broad applications across multiple fields within physics & other sciences.

Name some vector quantities

Graphic representation of a vector quantity



Attributes (general characteristics) of a vector

1.

2.

Represented by

Represented by

## Drawing Vectors

1.

2.

3.

Draw the following vectors. State the scale used.

1. A plane flies at 200 m/s,  $65^\circ$  north of east.



2. A dog walks east for 20. m.



3. A box is dragged with a force of 30. N at an angle of  $20.0^\circ$  with the horizontal.



# Adding Vectors

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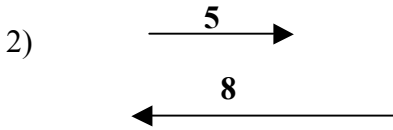
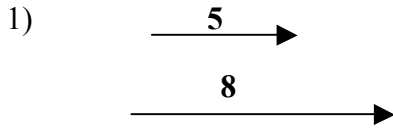
Component Vector:

Resultant Vector:

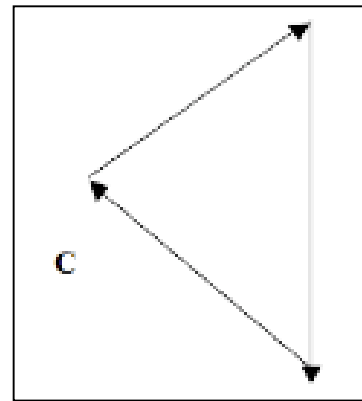
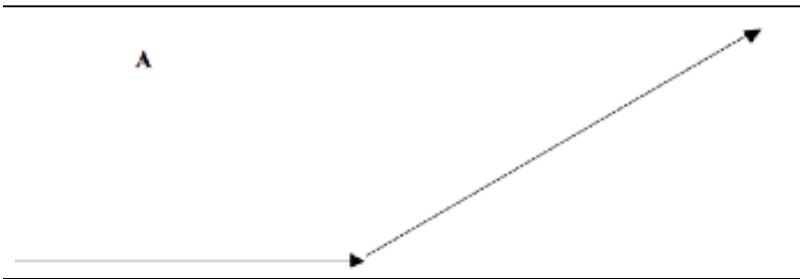
## Components

## Addition of Components

## Resultant



3). Draw the following vectors.



4. When adding vectors . . .

- a)
- b)
- c)
- d)

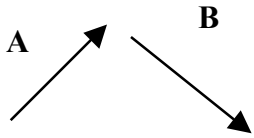
5. A resultant vector is determined by finding its \_\_\_\_\_ and \_\_\_\_\_.

6. Which angle represents the direction of the resultant vector?

7. Compare the placement of the component vectors with the placement of the resultant vector.

Component vectors:

Resultant vector:



$A + B$

$B + A$

$A - B$

$B - A$

[http://mysite.verizon.net/vzeoacw1/velocity\\_composition.html](http://mysite.verizon.net/vzeoacw1/velocity_composition.html)

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3. A man walks 200. m east and then walks 50. m north.

a) How far has he walked?

b) Where is he in relation to where he started?

**Graphical Method**

**Mathematical Method**



Magnitude:

Direction:

Resultant:

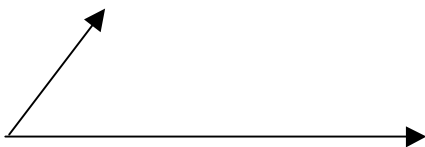
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### Concurrent Vectors

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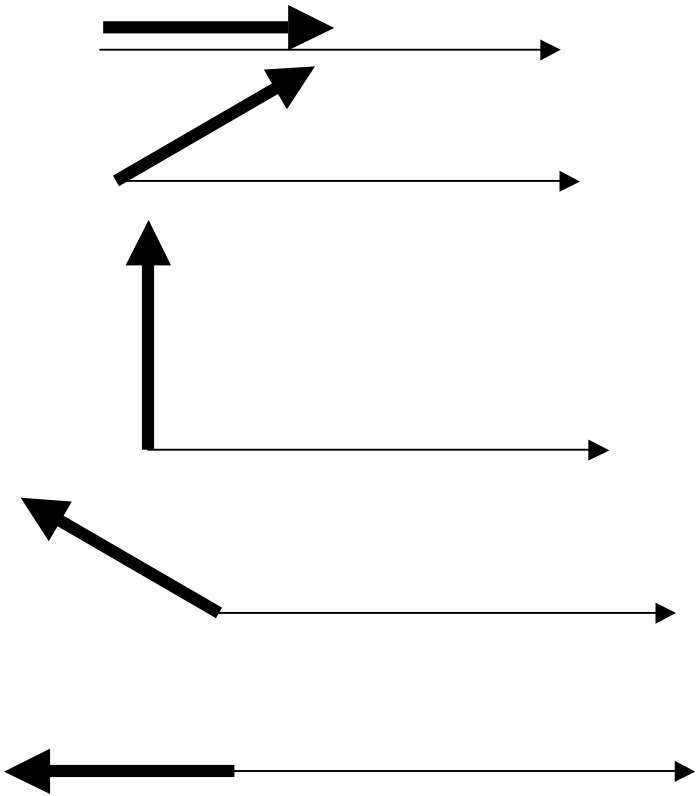
Concurrent Vectors:

Sketch the resultant of the concurrent vectors below.



Conclusion:

Sketch the resultant of the concurrent vectors below.



Based on the results of your drawings at left, answer the following questions:

1. What is the relationship between the magnitude of the resultant and the angle between the concurrent vectors?
2. What angle between concurrent vectors gives a :
  - a) maximum resultant ?
  - b) minimum resultant?

3. Two forces of 12 N and 4 N act concurrently on an object. What are the possible values for the resultant force? Sketch vector diagrams to support your answer.

<http://www.walter-fendt.de/ph11e/resultant.htm>  
<http://physics.bu.edu/~duffy/java/VectorAdd.html>

### Resolving a Vector into Components

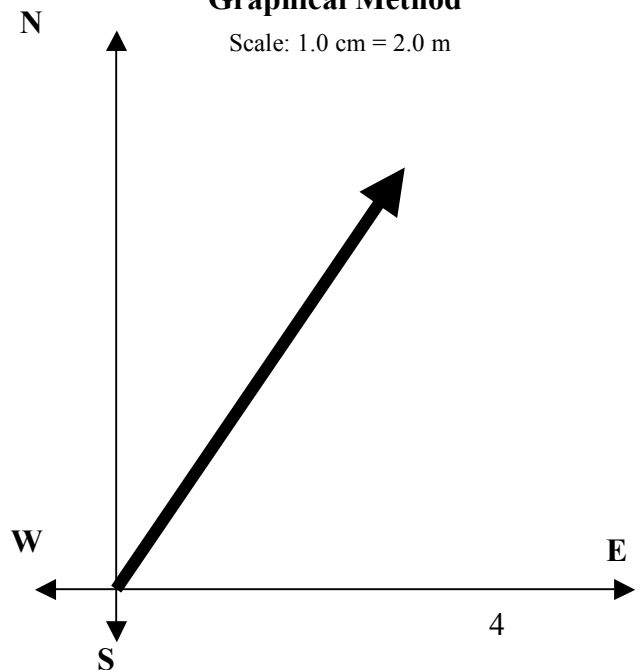
1. Prof. Einstein walked 13.6 m in a direction  $55.0^\circ$  north of east as shown.

- a) How far did he travel north?
- b) How far did he travel east?

#### Mathematical Method

#### Graphical Method

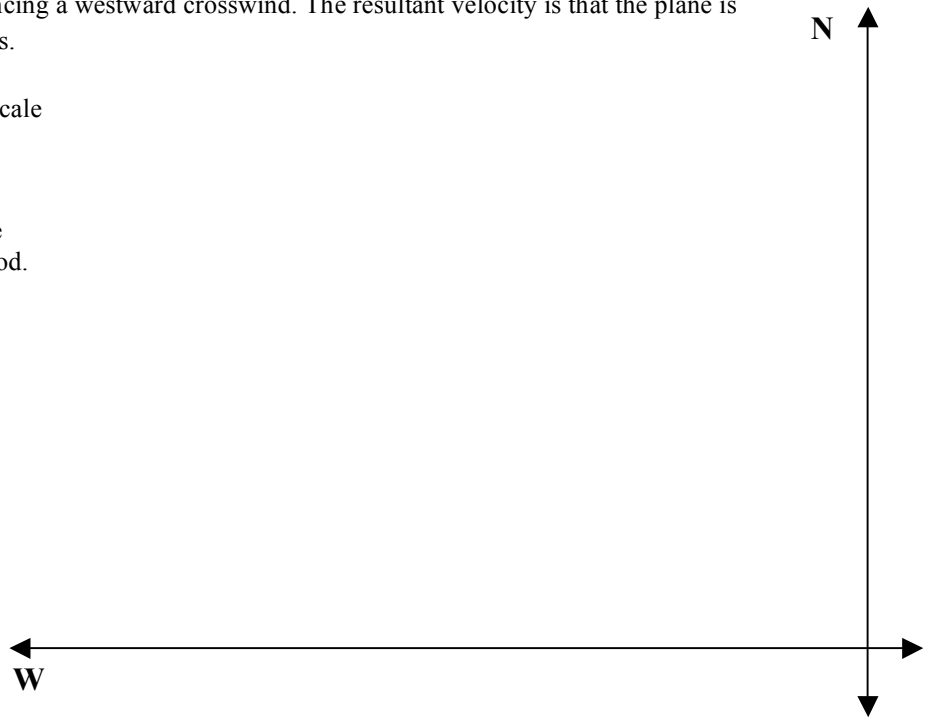
Scale: 1.0 cm = 2.0 m



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2. A plane attempting to head due north is experiencing a westward crosswind. The resultant velocity is that the plane is heading  $40.0^\circ$  north of west at a speed of  $300. \text{ m/s}$ .

- a) Draw the resultant velocity vector using the scale of  $1.0 \text{ cm} = 50. \text{ m/s}$ .
- b) Determine the component velocities (i.e. the plane's speed and the wind's speed) using the graphical method and the mathematical method.



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3. A cannonball is launched with a speed of  $450 \text{ m/s}$  at an angle of  $35^\circ$  above the horizontal.

- a) Sketch an appropriate vector diagram showing the resultant velocity and its horizontal and vertical components. *(Diagram does not need to be drawn to scale but should be roughly to scale.)*
- b) Calculate the horizontal and vertical components of the cannonball's velocity.

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4. A person drags a crate across the floor with a force of  $200. \text{ N}$  at an angle of  $20.^\circ$  above the horizontal as shown (not to scale).

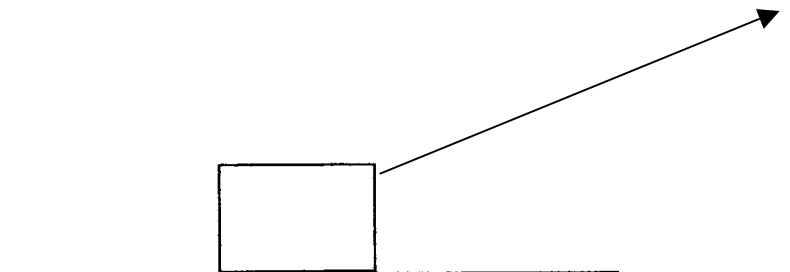
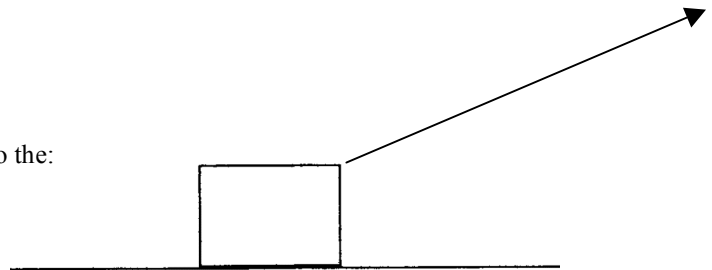
- a) Sketch an appropriate vector diagram showing the horizontal and vertical components of the force.

b) As the angle of the force increases, what will happen to the:

i) resultant force?

ii) horizontal component of the force?

iii) vertical component of the force?



## Relative Velocity

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General Rule:

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1. Two cars are 400 meters apart and traveling toward each other on a long straight road. One car is moving at 30 m/s and the other at 50 m/s. How long will it take before they meet?
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## Independence of Vectors

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2. A motorboat travels at 8.50 m/s, north straight across a river that has a current of 3.80 m/s east.

North shore

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- a) Determine the boat's resultant velocity.
- b) If the river is 100. m wide, how long it will take the boat to cross the river?
- c) How far downstream will the boat be when it reaches the opposite shore?



South shore

- d) How far will the boat actually travel?

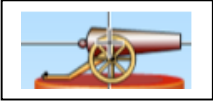
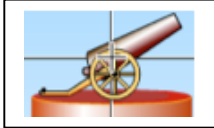
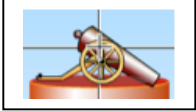
# Projectile Motion

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Projectile:

Trajectory:

1. Predict the trajectory of the cannon ball after it leaves the cannon in each trail, then sketch in its actual trajectory after the demonstration.

Trial A	Trial B	Trial C
		

Describe the shape of the trajectory of the cannon ball.

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Conclusions:

a)

b)

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3. An airplane must drop a rescue package to a person stranded on a desert island. In which position should the airplane be when it drops the package? Sketch the trajectory of the package as it drops.



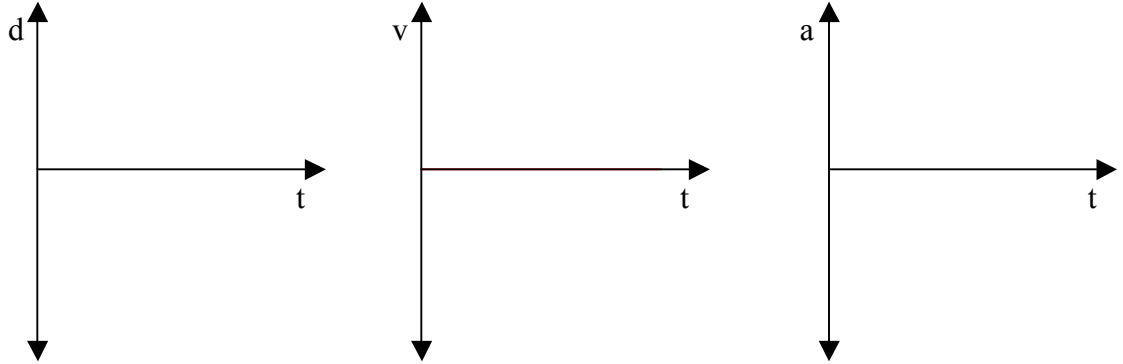
If you were in the airplane watching the package drop from above, how would you describe the trajectory of the package?

4. A sailor drops a ball from the top of the mast of a ship sailing to the right at a constant speed.
- Where does it land?
  - Describe the ball's trajectory as seen by the sailor.
  - Describe the ball's trajectory as seen by an observer on the shore. Sketch it below.

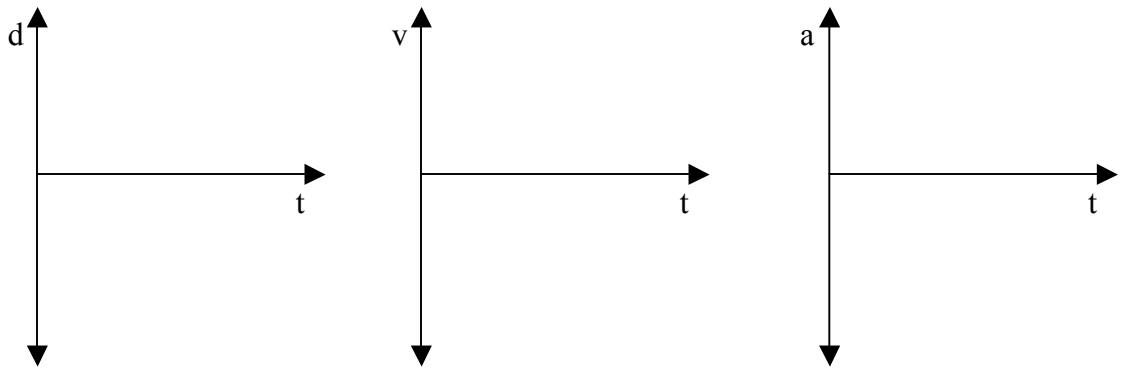


Conclusion:

**Horizontal Motion**



**Vertical Motion**



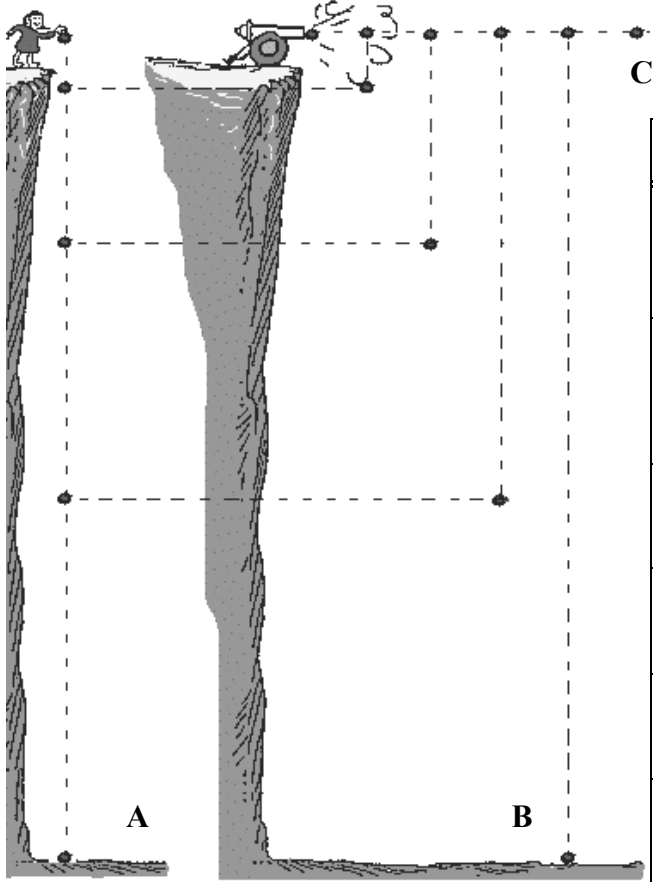
5. A car fires a flare straight upward while traveling at a constant speed. Sketch the position of the flare at each instant.

Where is the flare at each instant? Why?





## Horizontal Projectiles



Ball A is dropped over the edge of the cliff. Ball B is shot horizontally from the same height at 50. m/s. Ball C can be imagined to be the path of ball B if gravity were “turned off.”

Characteristic	A	B	C
Horizontal motion			
Vertical motion			
Initial horizontal velocity			
Initial vertical velocity			
Horizontal acceleration			
Vertical acceleration			

1. If it takes both balls 4.0 seconds to hit the ground, determine:

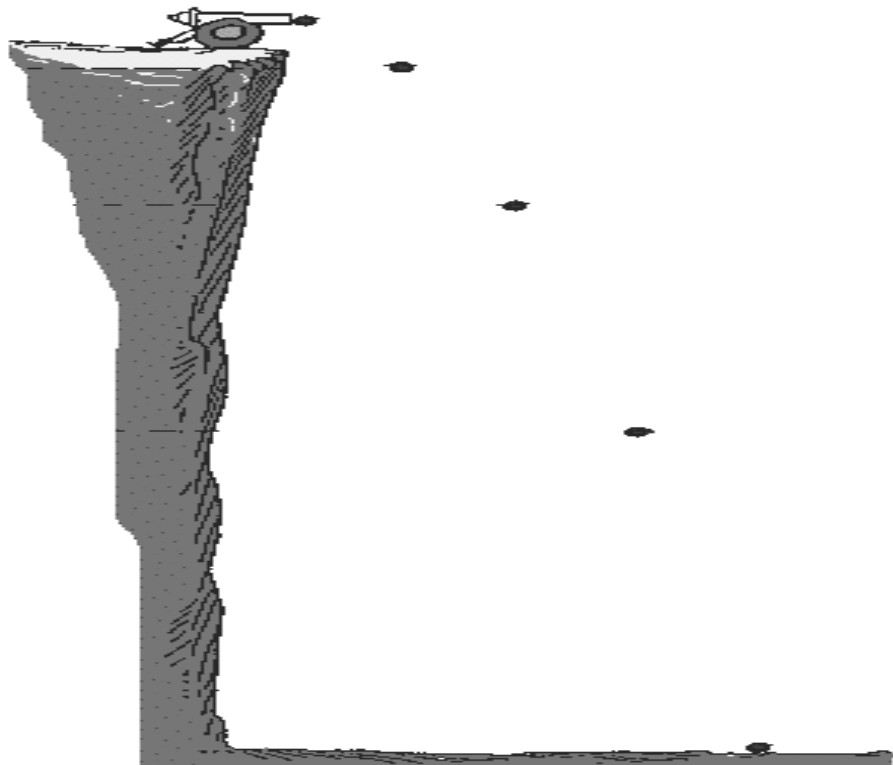
a) the height of the cliff.

b) the distance from the base of the cliff that ball B lands.

c) the impact velocity of ball A.

d) Sketch in the trajectory and the displacement of ball B.

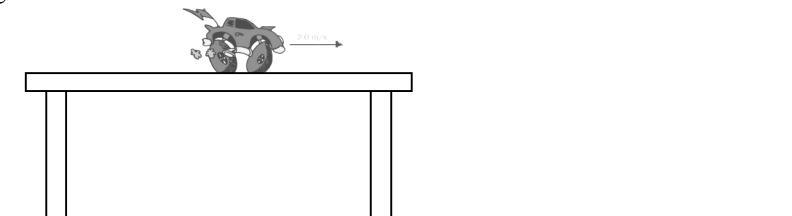
e) Sketch in the instantaneous velocity vectors for ball B at each instant as well as its horizontal and vertical component velocities.



f) Calculate the impact velocity of ball B. How does it compare with that of ball A?

2. A toy car moving at 2.0 m/s runs off a table that is 1.3 m high.  
Determine:

a) the time it takes for the car to hit the ground.



b) how far from the table the toy car lands

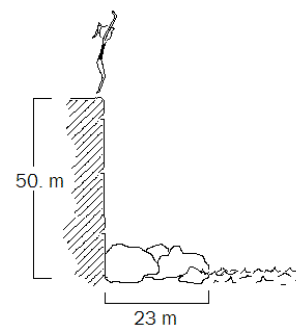
	x	y
d		
t		
a		
$V_i$		
$V_f$		

c) the impact speed of the car

$$\overline{V}$$

3. Cliff divers jump from heights as high as 50 meters.  
Suppose a diver wants to jump off a cliff that has rocks at the base that extend out for 23 m. Determine:

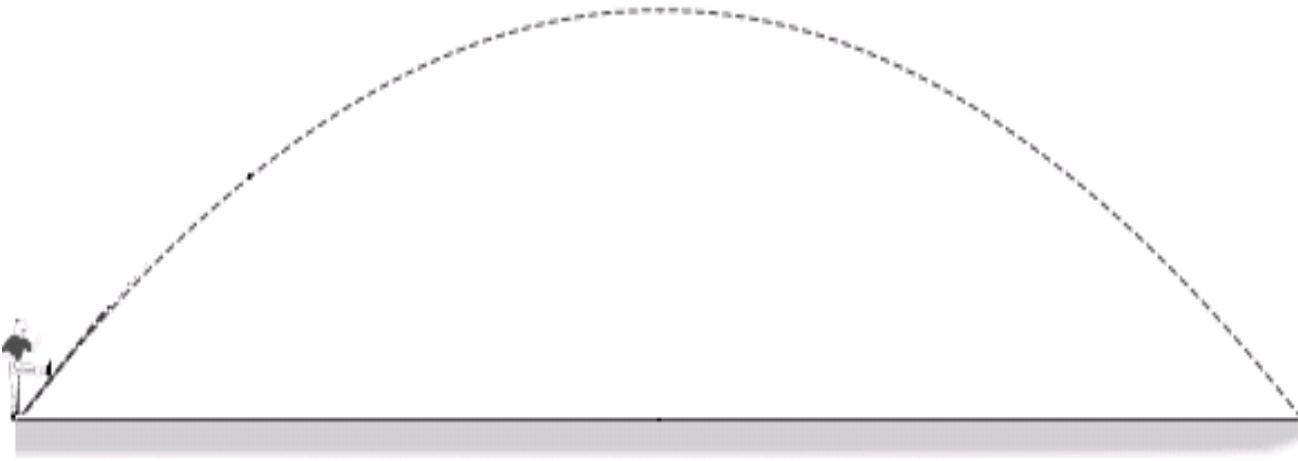
a) how long it will take to hit the water.



b) how fast the diver should run to clear the rocks below?

# Projectiles Launched at an Angle

The opening kick-off of a football game is shown below.

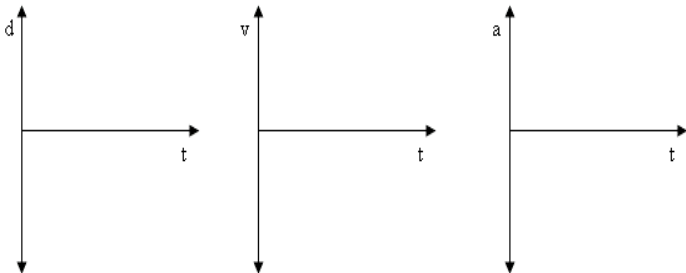


Observations:

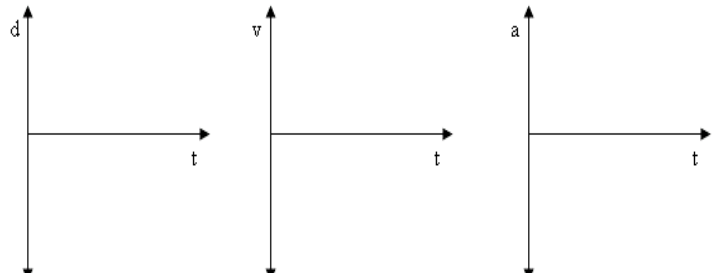
- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

Sketch the graphs below for both the horizontal and the vertical component of the ball's motion.

**Horizontal Component**

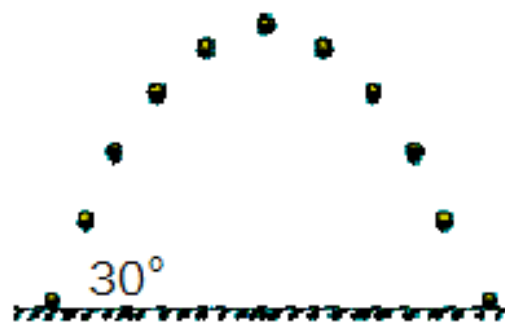


**Vertical Component**



1. A baseball was thrown with a speed of 25.0 m/s at an angle of  $30.0^\circ$ . Determine:

a) Horizontal and vertical components of the initial velocity



b) time taken to reach the top of its flight

c) total time before baseball lands

d) how high the ball went

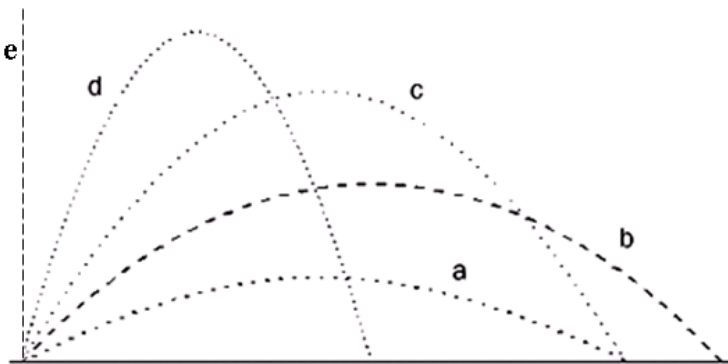
e) how far away the ball landed

	x	y
d		
t		
a		
$v_i$		
$v_f$		

$\bar{v}$

2. A cannon ball is shot at an angle of  $65.0^\circ$  with an initial speed of 330. m/s.  
Determine:
- the components of its initial velocity
  - how long it took to land
  - how far away it landed
  - how high it went

3. The diagram below shows the trajectories of five identical cannonballs all launched with the same speeds but at different launch angles measured from the horizontal.



Cannonball	Launch Angle
a	$30^\circ$
b	$45^\circ$
c	$60^\circ$
d	$70^\circ$
e	$90^\circ$

- Which path shows the projectile having the
  - largest angle of launch?
  - largest initial vertical component of velocity?
  - largest initial horizontal component of velocity?
- As the launch angle increases, what happens to the
  - initial velocity?
  - the components of the initial velocity?
- At what launch angle will the components of the initial velocity be equal?
- What angle of launch will give the largest range?
- Which two projectiles have the same range?
- What angle of launch will give the longest time in the air (flight time)?
- What angle of launch will make the cannonball go the highest?