THE CHESS AND MATHEMATICS CONNECTION: *More Than Just a Game*

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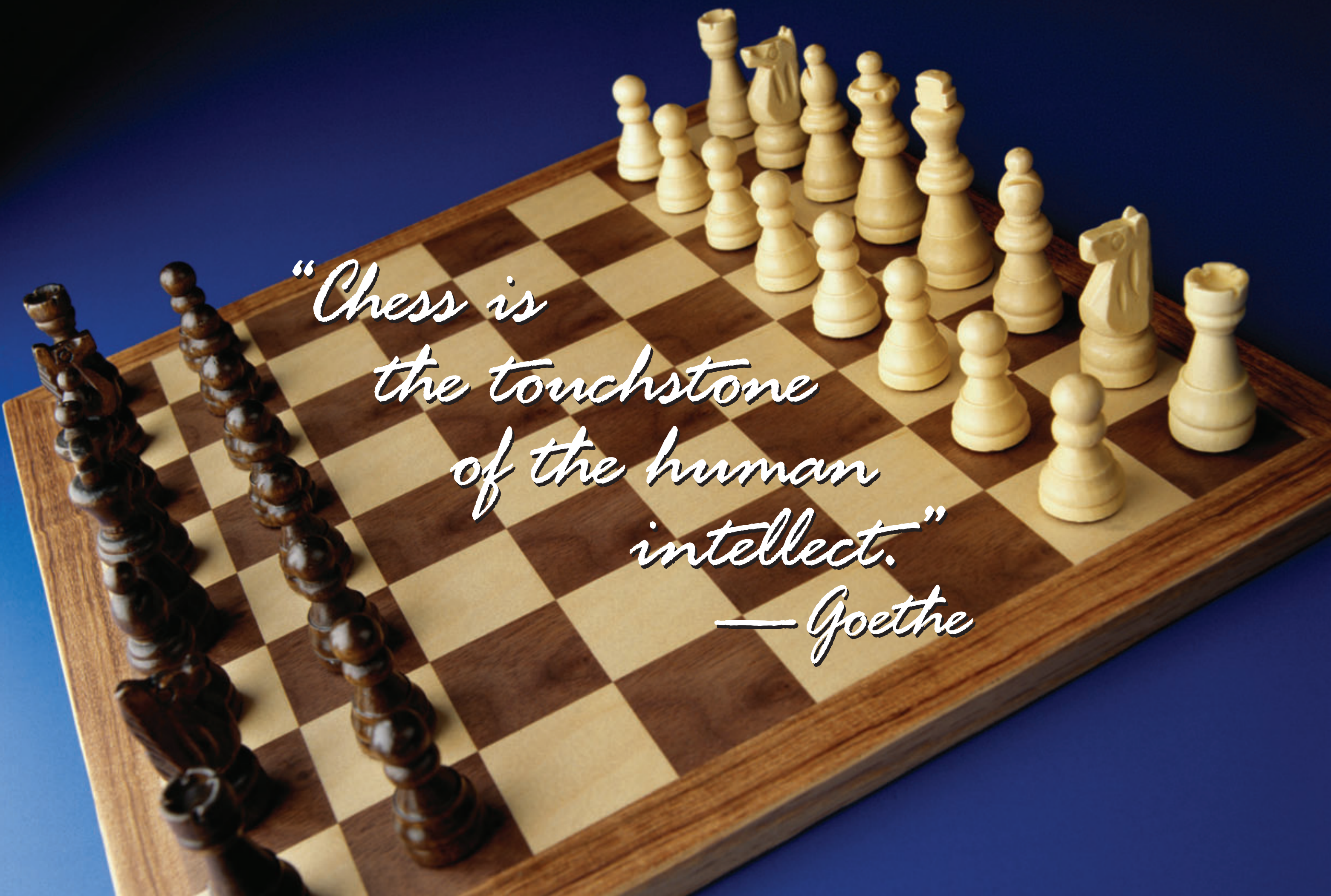

"Chess is the touchstone of the human intellect."
—Goethe

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MATHEMATICS EDUCATORS HAVE KNOWN for a long time that competitive games can play a dual purpose in advancing our work. One type of game can act as a vehicle for mastering a certain skill, like "multiplication war," in which each player lays down two cards and compares the products to determine who wins. A second type is useful for developing thinking strategies that have applications to mathematics. A simple game like "poison apple," in which each child takes turns removing one or two apples from a tree until one person is stuck with the last apple, helps develops planning, strategizing, and pattern-recognition skills.

Chess, however, belongs to a class of games that serves both functions simultaneously. As a strategy game, it promotes higher-order thinking skills that are helpful in solving mathematical problems. These skills include predicting the outcome of a move, planning ahead, and looking at the correlation between two variables (for example, deciding which piece is better to sacrifice at a given point in a game). It is also a game of geometry in that pieces move in predefined paths around the chessboard on diagonals, orthogonals, or some combination of the two.

At the same time, chess is a game of mathematical skill, in which pieces occupy and defend territory that is defined by a simple coordinate system. Chess requires continuous calculations, as the sums of the pieces are totaled only to be traded or sacrificed. The decision to fight on or resign can hinge on whether the player is ahead or behind by a lowly pawn.

Studies have shown that playing chess can have a profound influence on children's behavior (Margulies and Speeth 1999), not to mention reading and mathematics scores (Ferguson 1995). In one study, children who studied chess just fifty minutes per week actually improved their reading scores more than those who just took a remedial reading class (Margulies 1991).

Unlike the Mozart Effect, referring to children who listened to classical music for a short period of time showing improved low-level arithmetic skills, the "Chess Effect" works on many levels and can last a lifetime. Successful chess players must continually scrutinize problems in multiple ways, predict the outcome of their actions, plan several steps ahead, and use visual information. Thus, analyzing a chess position has much in common with solving a mathematics problem.

In addition to the rules and strategy, chess can be used as a context for developing other kinds of mathematical activities, including algebra, combinatorics, geometry, and set theory. Using chess as a context to study mathematics can enhance students' interests in both mathematics and chess, as well as give them an entirely new understanding of what takes place on the sixty-four squares of a chessboard.

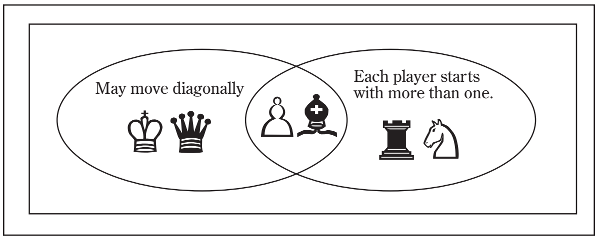


Fig. 1 A Venn diagram helps organize the chess exploration.

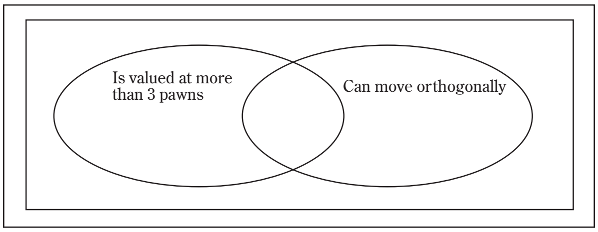


Fig. 2 Students decide which pieces belong to each set.

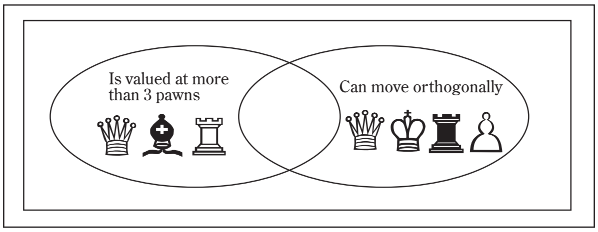


Fig. 3 The outer loops are filled in.

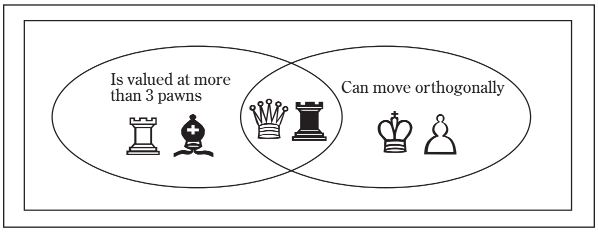


Fig. 4 Overlapping items are filled in.

The game of chess provides fertile ground for creating activities around set theory, as the pieces share similar attributes. The queen is similar to the bishop in that it may move diagonally, but it also shares a similarity to the rook in the ability to move orthogonally (that is, up and down and side to side). The king is similar to the queen in its ability to move in any direction, but its distance is limited to one space, much like the pawn after its initial move. A knight is unique because it may jump over other players on the board, and like the pawn, it can be the first piece moved during a game. Using this information, we can create a Venn diagram to observe the relationships of game pieces. See **figure 1.**

With this understanding, students are now ready to work on blank puzzles using actual chess pieces, or picture cards. We may have a diagram that starts out like **figure 2.** The student can work in two steps, first filling in the outer "loops," as in **figure 3.** Then, students can complete the puzzle by looking for pieces that show up in both loops (see **fig. 4**). I find that students are far more creative in determining different attributes than I. For example, my students observed that only two pieces "wear a crown," which refers to the king and queen. Other clues that my students found include whether the piece could be the first to move in a game or whether it could take part in castling. (*Castling* is a special type of chess move in which a player can simultaneously move the king and a rook under certain circumstances.)

In a similar mode, one can also look at the journey of yet another chess piece, the rook, which can only move up and down and side to side. This problem is better known in the context of someone traveling from one point to another on a grid. In the context of chess, the issue is more powerful, since it gets to the heart of one basic strategy for moving a rook: It is most powerful when the rows or columns (in chess, they are called *ranks* and *files*) are clear. The following activity helps explain why.

Suppose that you started out with a rook at one corner of the board and wanted to capture an opponent's piece at the exact opposite side. (See **figure 5**'s illustration of rank and file moves.) This task seems easy

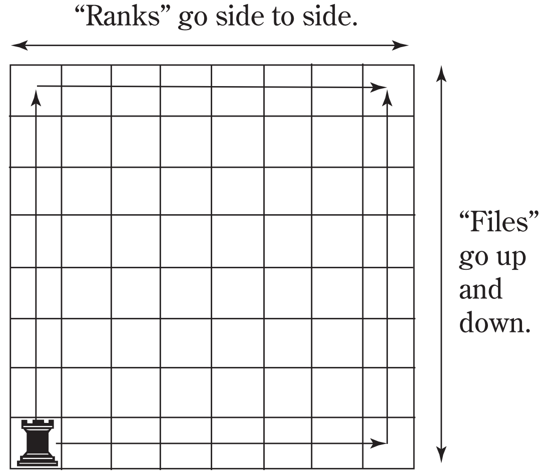


Fig. 5 Illustrating columns and rows, which are known in chess as ranks and files.

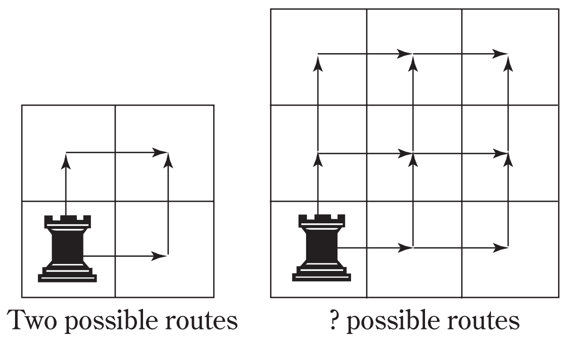


Fig. 6 Students begin with simpler cases to examine the number of paths available to the rook.

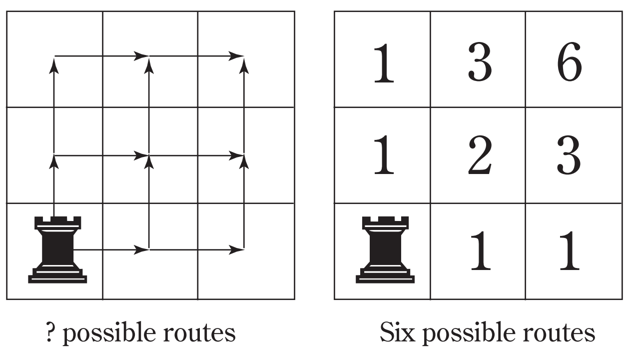


Fig. 7 Numbering the possible routes makes it easier to follow.

1	8	36	120	330	792	1716	3432
1	7	28	84	210	462	924	1716
1	6	21	56	126	252	462	792
1	5	15	35	70	126	210	330
1	4	10	20	35	56	84	120
1	3	6	10	15	21	28	36
1	2	3	4	5	6	7	8
	1	1	1	1	1	1	1

(a)

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

(b)

Fig. 8 The total number of moves for a rook is shown in (a); (b) shows that the numbers replicate the same sequence as Pascal's triangle.

Basic Rules of the Game of Chess

1. The game is for two players, traditionally playing as either black or white.
2. The white side always moves first.
3. Each player has 16 pieces: 1 king, 1 queen, 2 rooks (castles), 2 bishops, 2 knights, and 8 pawns
4. The game is played on an 8 × 8 chessboard, totaling 64 black-and-white squares.
5. Each player takes turns moving one piece at a time, following the rules of each piece.
6. The object is to defeat the opponent by outmaneuvering pieces and checking the opponent's king so that it cannot move.

Types of Pieces and Movement

King: The king can move one square in any direction, never into check.

Queen: The queen can move in one direction per move: forward, backward, straight, diagonal, or sideways without limit.

The Rook, or Castle: This piece can move as many squares as the player wishes, either horizontally or vertically, but it cannot jump over any other chess piece.

Bishop: Each bishop is allowed to move diagonally either forward or backward.

The Knight: The knight is unique in that it can change direction and does not move in a straight line. The knight moves in an L shape, either one or two squares in any straight direction (not diagonally) then either one or two squares sideways. It must have traveled three squares, no more no less, which enables it to "jump" pieces.

Pawn: The pawn can only move forward one square at a time. It can move two squares if never before moved, but once only. The pawn is allowed to move diagonally one square if it is attacking an opponent's chess piece.

Other Various Rules

Here are just a few of the most important rules of the game.

Pawn Promotion: If a pawn successfully moves to the chess opponent's back row and can travel no farther, it may be promoted to any other piece that the player chooses, other than a king. Since the queen is the most powerful chess piece, a player will usually choose the queen as the promotion piece.

Castling: This maneuver is the only time in a game of chess when a player may move two chess pieces in a single move under certain conditions:

- Neither the king nor the rook has moved during the chess game thus far.
- The king is not in check and is not moving into check.
- The squares between the king and rook must be empty.
- All the in-between squares must be free from check even if the end result is acceptable.
- The king moves two squares toward the rook's position; the rook then moves over the king and lands on the far side of, but adjacent to, the king.

Checkmate: The opponent's king is not allowed to be taken, so when a player threatens this piece, the checked king is required to move into a safe position or the defending player must remove the threat by positioning a piece between the king and the opposing piece. Checkmate occurs when the defending player is unable to move or protect the king either because no pieces are suitable to block the threat or because any move the king could make would still leave it in check.

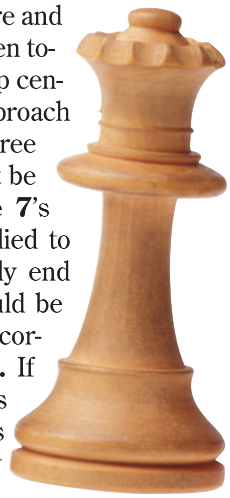
Stalemate: If all pieces are lost or blocked so that the only piece capable of moving is the king (and if all other moves would place the king in check), then the game is a draw and the position is said to be stalemate.

Source: "Chess Rules—Basic," www.chess-sets-of-england.co.uk/chess_rules.htm

enough: Move the rook straight ahead to the last rank, then move it sideways to the last file. Unfortunately, more often than not, one path will be blocked, and the player will need to take a different route. Maybe moving sideways first would be easier, but the situation is no different, unless it is relatively late in the game and few pieces are on the board to block the path.

This question now arises: How many paths can the rook take from one end of the board to the other? There are many ways to approach this problem. The simplest way might be to try again with an easier problem, say, a 2 × 2 board, then work up to more difficult boards. Unfortunately, keeping track of these journeys starts to get cumbersome after the simplest case. See **figure 6,** which compares a 2 × 2 board with a 3 × 3 board.

To better organize this approach, number each route at the corner that shows that the rook can approach a square from two directions: either from the left or from underneath. These approaches add up: If there are two ways to get to the center square and one way to get to the square to the left of it, then together there are three ways to arrive at the top center square. Since there are three ways to approach the upper-right corner from the left and three ways to get to it from underneath, there must be six ways to travel to it altogether. See **figure 7**'s routes and numbering. If this pattern is applied to successively larger boards, we will eventually end up with a diagram that shows that there would be 3432 ways for a rook to travel to the opposite corner of a chessboard, as shown in **figure 8a.** If these numbers appear vaguely familiar, it is because they replicate the same sequence as Pascal's triangle, which is more commonly written in triangular form (see **fig. 8b**).



All this information provides excellent territory for exploring, identifying, and classifying patterns and progressions, such as that 1, 3, 6, 10, and 15 is an increasing arithmetic progression. In addition, the triangle itself can be continued to predict how many journeys a rook might take on a larger board, such as a 10×10.

I used these as well as many other chess-based activities with students who were enrolled in an English-as-a-Second-Language (ESL) program in a school that I worked in several years ago. The children enjoyed the activities because the context of chess united them in a common language where they could contribute insights in many ways. They enjoyed finding their own attributes for the chess pieces and creating puzzles of their own that they could share with other class members. Although they found the second activity to be a great deal more challenging, the patterns that emerged gave them a feeling of accomplishment.

When all the mathematics is set aside, chess is just another game, with its own history, rules, strategies, and tactics. When viewing it through the lenses of geometry, algebra, or set theory, however, it takes on an entirely new dimension. Benjamin Franklin wrote about the subject in *The Morals of Chess,* once describing chess as being more than an idle amusement: he stated that it taught players foresight, circumspection, and caution. In effect, to Franklin, chess was a lot like life: "We have often points to gain, and competitors or adversaries to contend with, and in which there is a vast variety of good and ill events that are, in some degree, the effect of prudence, or the want of it." By using chess in a context to enhance our understanding of mathematics, this ancient game can live up to Franklin's promise, as well as much more.

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