

Topics: Anything this term with emphasis on up/down Kinematics problems, Newton's Laws, Projectile Motion, Circular Motion, Work/Energy and Impulse/Momentum (many of these include vectors)

I Kinematics - 1D

- 1) A ball is thrown up at 20.0 m/s. \vec{V}_0
a. How far above the ground is it when it is traveling at 10.0 m/s?

$$\vec{V}_f^2 = \vec{V}_0^2 + 2\vec{a}d$$

$$d = \frac{\vec{V}_f^2 - \vec{V}_0^2}{2\vec{a}} = \frac{(10\text{ m/s})^2 - (20\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = \boxed{15.3\text{ m}}$$

- b. When is it traveling at a speed of 10.0 m/s. (There are 2 answers)

$$\vec{V} = \vec{V}_0 + \vec{a}t$$

$$t = \frac{\vec{V} - \vec{V}_0}{\vec{a}} = \frac{(10\text{ m/s}) - (20\text{ m/s})}{-9.8\text{ m/s}^2} = \boxed{1.02\text{ s}} \quad \text{or} \quad \frac{(-10\text{ m/s}) - (20\text{ m/s})}{-9.8\text{ m/s}^2} = \boxed{3.06\text{ s}}$$

- c. How high does it go? ($\vec{V}_f = 0$)

$$\vec{V}_f^2 = \vec{V}_0^2 + 2\vec{a}d$$

$$d = \frac{-\vec{V}_0^2}{2\vec{a}} = \frac{-(20\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = \boxed{20.4\text{ m}}$$

- d. How long does it take (total) to reach the ground? (from the original throw)

$$\vec{d} = \vec{V}_0 t + \frac{1}{2}\vec{a}t^2$$

$$t = \frac{-\vec{V}_0}{\frac{1}{2}\vec{a}} = \frac{-(20\text{ m/s})}{\frac{1}{2}(-9.8\text{ m/s}^2)} = \boxed{4.08\text{ s}}$$

- 2) A man rising at 20.0 m/s in a hot air balloon drops a sand bag from the balloon. If he is 150 m up when he does this:

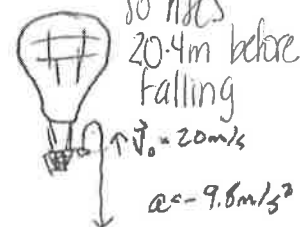
- a. What will the maximum altitude of the sand bag be? Note: $\vec{V}_0 = 20\text{ m/s}$, not zero!

initial = 150 m + ?

$$\vec{V}_f^2 = \vec{V}_0^2 + 2\vec{a}d$$

$$d = \frac{-\vec{V}_0^2}{2\vec{a}} = \frac{-(20\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = \boxed{20.4\text{ m}}$$

max altitude
= 150 m + 20.4 m
= **170.4 m**



- b. How long, from the time of cutting it loose, will the bag take to hit the ground?

solve quadratic or do part d) 1st

$$\vec{V}_f = \vec{V}_0 + \vec{a}t \Rightarrow t = \frac{\vec{V}_f - \vec{V}_0}{\vec{a}} = \frac{-57.8\text{ m/s} - 20\text{ m/s}}{-9.8\text{ m/s}^2} = 7.94\text{ s}$$

- c. How fast will the bag be moving when it hits the ground?

$$\vec{V}_f^2 = \vec{V}_0^2 + 2\vec{a}d$$

$$\vec{V}_f = \pm \sqrt{\vec{V}_0^2 + 2\vec{a}d}$$

$$= -\sqrt{(20\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(-150\text{ m})} = \boxed{-57.8\text{ m/s}}$$

pick negative b/c ball is moving downward

or

$$d = \vec{V}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\left(\frac{1}{2}\vec{a}\right)t^2 + (\vec{V}_0)t + (-d) = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

II. 2d Kinematics

3) A ball is thrown at a speed of 20.0 m/s at an angle of 29.0° to the horizontal.

a. How long did it take to reach its maximum altitude? (use v_{oy}, a_y)

$$\vec{v}_{fy} = \vec{v}_{oy} + \vec{a}_y t \Rightarrow t = -\vec{v}_{oy} / \vec{a}_y$$

$$= -9.70 \text{ m/s} / -9.8 \text{ m/s}^2 = 0.9895 \text{ s}$$

b. What was its maximum altitude? ~~smaller than twice h~~

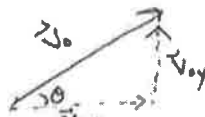
$$d = v_{oy} t + \frac{1}{2} a_y t^2$$

$$= (9.7 \text{ m/s})(0.9895) + \frac{1}{2} (-9.8 \text{ m/s}^2)(0.9895)^2 = 4.80 \text{ m}$$

c. What were the initial velocities in the horizontal and vertical direction?

$$\vec{v}_{ox} = \vec{v}_0 \cos \theta = 20 \text{ m/s} \cos 29^\circ = 17.5 \text{ m/s}$$

$$\vec{v}_{oy} = \vec{v}_0 \sin \theta = 20 \text{ m/s} \sin 29^\circ = 9.70 \text{ m/s}$$



d. How long did it take (from the time it was thrown) to return to its original altitude? (should be twice t_{y2})

use $d_y = 0$

$$d_y = v_{oy} t + \frac{1}{2} a_y t^2$$

$$t = -v_{oy} / \frac{1}{2} a_y = -9.7 \text{ m/s} / \frac{1}{2} (-9.8 \text{ m/s}^2) = 1.98 \text{ s}$$

e. How far horizontally did it travel before returning to its altitude at the throwing point?

$$d_x = v_{ox} t + \frac{1}{2} a_x t^2 = (17.5 \text{ m/s})(1.98 \text{ s}) = 34.65 \text{ m}$$

34.65 m or 34.7 m

4) A projectile is launched at a speed of 30.0 m/s at some angle. It arrives at the ground after 4.00 seconds.

a. How far did it rise? Think: $t_{y2} = 2.00 \text{ s}$

- also, distance up = distance down

- solve for falling distance

$$d_y = v_{oy} t + \frac{1}{2} a_y t^2 = \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 = -19.6 \text{ m} \Rightarrow \text{rose } 19.6 \text{ m}$$

b. How far across the ground did it go?

$$d_x = v_{ox} t + \frac{1}{2} a_x t^2$$

$$= v_0 \cos \theta \cdot t + \frac{1}{2} a_x t^2 = 30 \text{ m/s} \cos 40.8^\circ (4.0 \text{ s}) = 90.8 \text{ m}$$

c. At what angle was it launched?

$$\text{use } t_{y2}: d_y = v_{oy} t + \frac{1}{2} a_y t^2 \Rightarrow v_0 \sin \theta t = d_y - \frac{1}{2} a_y t^2$$

$$v_0 \sin \theta$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{d_y - \frac{1}{2} a_y t^2}{v_0 t} \right)$$

$$= \sin^{-1} \left(\frac{19.6 \text{ m} - \frac{1}{2} (-9.8 \text{ m/s}^2)(2 \text{ s})^2}{(30 \text{ m/s})(2 \text{ s})} \right)$$

d. What was its speed as it hit the ground?

$$\theta = 40.8^\circ$$

$$30 \text{ m/s} \text{ symmetric}$$

	x	y
d	34.65 m	0
v_x	17.5 m/s	17.5 m/s
v_y	17.5 m/s	17.5 m/s
a	0	-9.8 m/s ²
t	1.98 s	1.98 s

	x	y
d	90.8 m	0
v_x	22.7 m/s	19.6 m/s
v_y	22.7 m/s	-19.6 m/s
a	0	-9.8 m/s ²
t	4.00 s	4.00 s



$$t = 4 \text{ s}$$

① same

②

$$v_{fy} = -v_{oy}$$

$$v_{fy} = v_{oy} + a_y t$$

$$-2v_{oy} = a_y t$$

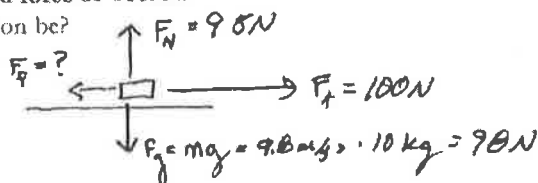
$$v_{oy} = -\frac{a_y t}{2}$$

III. Forces, including friction

5) A 10.0 kg body is slid along a tabletop by an applied force of 100. N.

a. If there were no friction, what would the acceleration be?

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{100. \text{ N}}{10.0 \text{ kg}} = 10.0 \text{ m/s}^2$$



b. If the actual acceleration was only 3.00 m/s², what is the force of friction?

$$\sum \vec{F} = m\vec{a} \rightarrow \vec{F}_f = m\vec{a} - \vec{F}_A = (10.0 \text{ kg})(3.00 \text{ m/s}^2) - (100 \text{ N}) = -70.0 \text{ N}$$

c. What is the coefficient of sliding friction?

$$\mu = \frac{F_f}{F_N} = \frac{70 \text{ N}}{98 \text{ N}} = 0.71$$

6) A 5.00 kg body is pulled by 2 forces as shown.

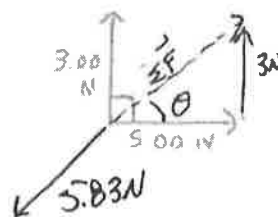
a. What is the net force? (Include direction)

$$|\vec{F}| = \sqrt{(3 \text{ N})^2 + (5 \text{ N})^2} = 5.83 \text{ N}$$

$$\theta = \tan^{-1}(F_y/F_x) = \tan^{-1}(3/5) = 31^\circ$$

b. What is the acceleration?

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{5.83 \text{ N}}{5.00 \text{ kg}} = 1.17 \text{ m/s}^2 \text{ at } \theta = 31^\circ$$

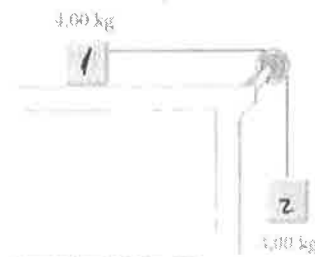


c. Show the direction of the net force on the diagram. Also, draw on the diagram one force that could keep the object from accelerating.

7) Two blocks of ice are suspended as shown in the diagram to the right.

a. Find the acceleration of these two objects in the string as the block slides across a frictionless table.

$$\vec{a} = \frac{\sum \vec{F}}{m_T} = \frac{m_2 g}{m_T} = \frac{4 \text{ kg} \cdot 9.8 \text{ m/s}^2}{8 \text{ kg}} = 4.9 \text{ m/s}^2$$



b. Calculate the tension in the string as the block slides across

Block 1

$$\vec{F}_T = m_1 a$$

$$= 4 \text{ kg} \cdot 4.9 \text{ m/s}^2$$

$$= 19.6 \text{ N}$$

Block 2

$$\sum \vec{F} = m_2 a$$

$$\vec{F}_T + \vec{F}_g = m_2 a$$

$$\vec{F}_T = m_2 a - m_2 g = 4 \text{ kg}(4.9 \text{ m/s}^2) - 4 \text{ kg} \cdot 9.8 \text{ m/s}^2 = -19.6 \text{ N}$$

$$\vec{F}_T \rightarrow m_2 g$$

Circular motion, including gravity

8) A 3.00 kg body is whirled around in a circle at the end of a string. Assume that the radius of the circle was 2.00 m and that one circle was completed each 1.50 s.

a. What was the speed of the body?

$$v = 2\pi r / T = \frac{2\pi \cdot 2.00\text{m}}{1.50\text{s}} = \boxed{8.38\text{m/s}}$$

b. What was the centripetal acceleration?

$$a_c = v^2 / R = \frac{(8.38\text{m/s})^2}{2.00\text{m}} = \boxed{35.1\text{m/s}^2} \quad (\text{in towards center})$$

c. What was the centripetal force?

$$\Sigma F_{in} = \frac{mv^2}{r} = \frac{(3.00\text{kg})(8.38\text{m/s})^2}{2.00\text{m}} = \boxed{105\text{N}} \quad (\text{in towards center})$$

9) This question is about the earth and the moon.

a. If our moon were twice as far from us as it is now, how would the force of earth's gravity on the moon compare with the current force?

$$\frac{1}{4} F_g = \frac{GMm}{(2r)^2} \Rightarrow \frac{1}{4}x$$

b. If the moon were twice as heavy as it is, and it remained where it is now, how would the force of earth's gravity compare with the current force?

$$2 F_g = \frac{GM^2m}{r^2} \Rightarrow 2x$$


c. Calculate the force between the earth and the moon. Relevant and irrelevant data: mass of the earth = $5.97 \times 10^{24}\text{kg}$, radius of earth = $6.23 \times 10^6\text{m}$, mass of moon = $7.34 \times 10^{22}\text{kg}$, radius of moon $1.74 \times 10^6\text{m}$, moon's orbital radius around earth = $3.8 \times 10^8\text{m}$.


$$F_g = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot 7.34 \times 10^{22}\text{kg} \cdot 5.97 \times 10^{24}\text{kg}}{(3.8 \times 10^8\text{m})^2} = \boxed{2.02 \times 10^{20}\text{N}}$$

Work/Energy

10) Two people, each pushing with a force of 150. N can make a 1200. kg car move at a constant speed of 1.00 m/s on a level road. (hint: this tells you something about the force of friction. Draw a free body diagram). If they now each push with 200. N and move the car 50.0 m:

a. How much work do they do on the car?

1st (to find friction)  } constant friction at 300 N

2nd (for problem) 

$$W = F_{\text{net}} \cdot d = 400 \text{ N} \cdot 50 \text{ m} = \boxed{20,000 \text{ J}}$$

b. How much work does friction do on the car?

$$W = F_f \cdot d = -300 \text{ N} \cdot 50 \text{ m} = \boxed{-15,000 \text{ J}}$$

c. If the car starts from rest, how much kinetic energy does the car acquire during the 50.0 m trip? How fast was it going?

$$W = \Delta E = 5000 \text{ J}$$

assuming all work goes into KE,

$$KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \cdot 5000 \text{ J}}{1200 \text{ kg}}} = \boxed{2.89 \text{ m/s}}$$

11) A 758 g mass swings as a pendulum. The mass is lifted 60.0 cm from its equilibrium point and then released.

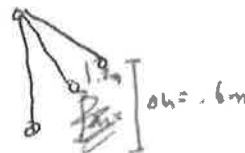
a. How much potential energy did it have when released? (Use the bottom of the motion as the zero for potential energy).

$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh}$$

$$mgh = (0.758 \text{ kg})(9.8 \text{ m/s}^2)(0.60 \text{ m}) = \boxed{4.46 \text{ J}}$$

b. How fast would it be going at the bottom?

$$v = \sqrt{2 \cdot 9.8 \cdot 0.60 \text{ m}} = \boxed{3.43 \text{ m/s}}$$



c. When it had fallen 20.0 cm, so it was still 40.0 cm above the bottom, how fast was it going?

$$v = \sqrt{2g\Delta h} = \sqrt{2 \cdot 9.8 \text{ m/s}^2 \cdot (0.20 \text{ m})} = \boxed{1.98 \text{ m/s}}$$

12 A spring is found to have a force constant of 20.0 N/

m. A 1.00 kg mass is hung on the spring.

a. How much force is exerted on the spring?

$$F_s = kx$$

at equilibrium: $mg = kx_e \Rightarrow mg = 9.8 \text{ N}$

b. How far does it stretch? (The place it stretches to is called the equilibrium point below.)

$$x_e = \frac{mg}{k} = \frac{9.8 \text{ N}}{20 \text{ N/m}} = 0.49 \text{ m}$$

c. How much potential energy is stored in the spring.

$$\frac{1}{2} kx_e^2 = \frac{1}{2} (20 \text{ N/m}) (0.49 \text{ m})^2 = 2.40 \text{ J}$$

d. The mass is lifted 0.200 m above this point and dropped. How far does it drop?

Drops 0.400 m (symmetric bounce)

e. How much PEs ($\frac{1}{2}kx^2$) is there at the top of the oscillation?

$$x_T = (0.49 \text{ m}) - (0.20 \text{ m}) = 0.29 \text{ m}$$

$$\frac{1}{2} kx_T^2 = \frac{1}{2} \cdot 20 \text{ N/m} \cdot (0.29 \text{ m})^2 = 0.84 \text{ J}$$

$$\frac{1}{2} kx_T^2 + mgh_{\text{top}}$$

f. How much PEs ($\frac{1}{2}kx^2$) is there at the bottom of the oscillation?

$$x_B = (0.49 \text{ m}) + (0.20 \text{ m}) = 0.69 \text{ m}$$

$$\frac{1}{2} kx_B^2 = \frac{1}{2} \cdot 20 \text{ N/m} \cdot (0.69 \text{ m})^2 = 4.76 \text{ J}$$

$$\frac{1}{2} kx_B^2 + mgh$$

g. Calculate the change in PEs (mgh) from the top to the bottom of the "bounce". hint: it should be the difference between e. and f. Do you see why?

$$mgh = 1 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 0.400 \text{ m} = 3.92 \text{ J}$$

$$\text{check: } 4.76 \text{ J} - 0.84 \text{ J} = 3.92 \text{ J}$$

h. As it is falling, how fast is it moving as it passes through the equilibrium point? $x_e = 0.49 \text{ m}$ $h_e = 0.20$

$$\text{use } E_{\text{total}} = 4.76 \text{ J} \quad E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx_e^2 + mgh$$

$$v = \sqrt{(E_T - \frac{1}{2}kx_e^2 - mgh) / \frac{1}{2}m} = \sqrt{(4.76 \text{ J} - \frac{1}{2} \cdot 20 \text{ N/m} \cdot (0.49 \text{ m})^2 - 1 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 0.20 \text{ m}) / \frac{1}{2} \cdot 1 \text{ kg}}$$

$$= 8.94 \text{ m/s}$$

12) A spring is fastened to a wall and a mass is slid into it with an initial speed of 4.00 m/s. The spring is strong enough that a force of 1.00 N will stretch the spring 0.500 m. (See diagram)

a. What is the force constant of the spring?

$$F = kx \Rightarrow k = \frac{F}{x} = \frac{1 \text{ N}}{0.5 \text{ m}} = 2 \text{ N/m}$$

b. How far will the mass compress the spring?

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{mv^2/k} = \sqrt{2 \text{ kg} \cdot (4 \text{ m/s})^2 / 2 \text{ N/m}} = 4 \text{ m}$$

c. How much potential energy is stored in the spring when it is compressed 2.00 m?

$$\frac{1}{2}kx^2 = \frac{1}{2} \cdot 2 \text{ N/m} \cdot (2 \text{ m})^2 = 4.00 \text{ J}$$

d. How fast is the mass traveling at this point?

$$E_T = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2} \cdot 2 \text{ N/m} \cdot (4 \text{ m})^2 = 16 \text{ J}$$

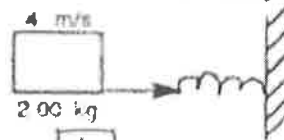
at 2 m pt.

$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = 12 \text{ J}$$

$$v = \sqrt{\frac{2 \cdot 12 \text{ J}}{m}} = \sqrt{\frac{2 \cdot 12 \text{ J}}{2 \text{ kg}}} = 3.46 \text{ m/s}$$

Diagram for Problem 1



- 14 A 0.110 tin can is resting on top of a 1.70 m high fence post. A 0.00200 kg bullet is fired horizontally at the can. It strikes the can with a speed of 900. m/s, passes through it, and emerges with a speed of 720. m/s. (Ignore friction between the can and the post).

a) What is the velocity of the can as it leaves the fence post? (take care with direction)

collision: cons. of \vec{p}

$$m_b v_{b0} + m_c v_{c0} = m_b v_{bf} + m_c v_{cf}$$

$$v_{cf} = \frac{m_b v_{b0} - m_b v_{bf}}{m_c} = \frac{(0.002 \text{ kg})(900 \text{ m/s}) - (0.002 \text{ kg})(720 \text{ m/s})}{0.110 \text{ kg}} = 0.327 \text{ m/s}$$

b) How long does it take the can to fall to the ground?

$$v_{y0} = 0 \quad a_y = -9.8 \text{ m/s}^2 \quad d_y = v_{y0} t + \frac{1}{2} a_y t^2$$

$$d_y = -1.70 \text{ m} \Rightarrow \frac{1}{2} t = \sqrt{\frac{2 d_y}{a_y}} = \sqrt{\frac{2 \cdot (-1.70 \text{ m})}{-9.8 \text{ m/s}^2}} = 0.589 \text{ s}$$

c) How far does the can travel from the fence post in this time?

$$d_x = v_{ox} t + \frac{1}{2} a_x t^2 = (0.327 \text{ m/s})(0.589 \text{ s}) = 0.193 \text{ m}$$

- 15 A piece of high explosive hanging from a thread suspended from a tree limb explodes, breaking into three pieces. Two pieces go off at right angles to each other, a 1.00 kg piece at 100. m/s and a 2.00 kg piece at 80.0 m/s. if the mass of the third piece is 3.00 kg, determine its speed and direction. (hint: what is conserved here? Draw a diagram showing the pieces. Remember that momentum is a vector.)

1) Find $\Sigma \vec{p}$ of 1st two pieces

2) \vec{p}_3 must be opposite

$$\sqrt{(100 \text{ kg} \cdot 100 \text{ m/s})^2 + (160 \text{ kg} \cdot 80 \text{ m/s})^2} = 199 \text{ kg} \cdot \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{160}{100}\right) = 58.0^\circ$$

- 16 A car of mass 1200. Kg is moving west at 20.0 m/s. The brakes are applied and the car stops.

a. How much momentum did the car have before stopping?

$$\vec{p} = m\vec{v} = 1200. \text{ kg} \cdot 20.0 \text{ m/s} = 24000 \text{ kg} \cdot \text{m/s}$$

b. What was the change in momentum? (Define west as positive)

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = 0 - 24000 \text{ kg} \cdot \text{m/s} = -24000 \text{ kg} \cdot \text{m/s}$$

c. What was the impulse?

$$\vec{J} = \Delta \vec{p} = -24000 \text{ N} \cdot \text{s}$$

d. If the braking time was 5.00s, what was the average force?

$$\vec{J} = \vec{F}_{\text{avg}} \cdot t \Rightarrow \vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{-24000 \text{ N} \cdot \text{s}}{5.00 \text{ s}} = -480 \text{ N}$$

- 17 A 10.0 gram bullet is fired into a 100. g block of wood at rest on a horizontal surface. After impact, the block slides 8.00 m before coming to rest. If the coefficient of friction $\mu = 0.600$

a. Find the force of friction acting on the block as it is sliding. What was the acceleration?

$$F_f = \mu \cdot F_n = \mu mg = 0.600 \cdot 0.110 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 0.647 \text{ N}$$

b. Find the original velocity of the block after being hit by the bullet

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}d$$

$$v_0 = \sqrt{-2ad} = \sqrt{-2(-5.88 \text{ m/s}^2)(8.00 \text{ m})} = 9.70 \text{ m/s}$$

c. Find the velocity of the bullet before impact.

$$m_b v_{b0} + m_B v_{B0} = m_B v_{Bf}$$

$$v_b = \frac{m_B v_{Bf}}{m_b} = \frac{0.110 \text{ kg} \cdot 9.70 \text{ m/s}}{0.01 \text{ kg}} = 107 \text{ m/s}$$