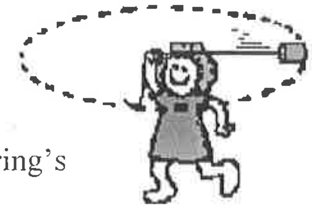
Proportional Reasoning

1. A student swings a rubber stopper around on a string at a constant speed with a centripetal acceleration of 6.0 m/s^2, as shown. What would happen to the acceleration if:



a) the speed is doubled? $a_c = \frac{v^2}{r}$

$a = \frac{(2v)^2}{r} =$ 4 times $\vec{a}_i$

b) the speed is halved?

$a = \frac{(\frac{1}{2}v)^2}{r}$ $\frac{1}{4}$ initial $\vec{a}$

c) the string's length is tripled?

$a = \frac{v^2}{3r} = \frac{1}{3}$ initial $\vec{a}$

d) the speed is doubled and the string's length is halved?

$a = \frac{(2v)^2}{(\frac{1}{2})r} =$ 8 times the $\vec{a}$

e) the mass of the stopper is doubled?

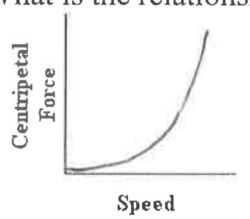
stays the same

f) What would happen to the tension in the string if the mass is doubled and the speed is halved?

$F_T = \frac{mv^2}{r} = \frac{2m(\frac{1}{2}v)^2}{r} = \frac{1}{2}F_T$

Graphical Relationships

2. What is the relationship between centripetal force and speed?



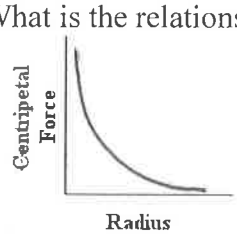
quadratic

$\Sigma F_{in} \propto v^2$

$\Sigma F_{in} = \frac{mv^2}{r}$

Control: m, r

3. What is the relationship between centripetal force and radius?

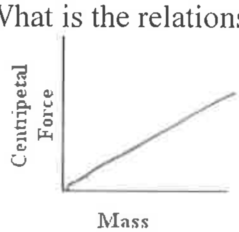


inverse

$\Sigma F_{in} \propto \frac{1}{r}$

Control: m, v

4. What is the relationship between centripetal force and mass?



direct

$\Sigma F_{in} \propto m$

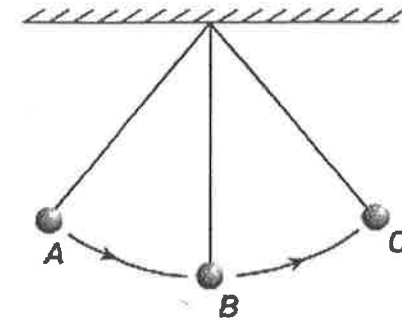
Control: v, r

Vertical Circles

1. Is a swinging pendulum in equilibrium? Explain.

   not in equilibrium = changing direction

2. Is a swinging pendulum in uniform circular motion? Justify your answer.



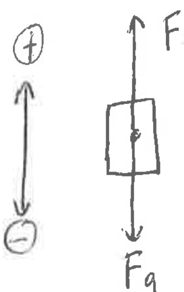
No! not a full circle,
speeds up, slows down, stops + changes direction
$B = V_{max}$ $\quad A + C = V_{min}$

3. Compare the tension in the pendulum's string as it swings.

   Greatest tension is at "B" when going at max speed.

4. Compare the tension in a swinging pendulum to one that is hanging motionless. Sketch appropriate diagrams to aid your explanation.

Hanging

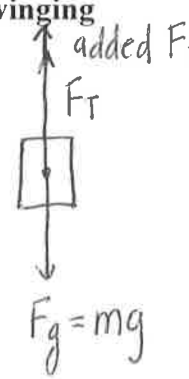


$\Sigma F = \emptyset N$

$F_T + F_g = \emptyset N$

$\vec{a} = \emptyset \frac{m}{s^2}$

Swinging

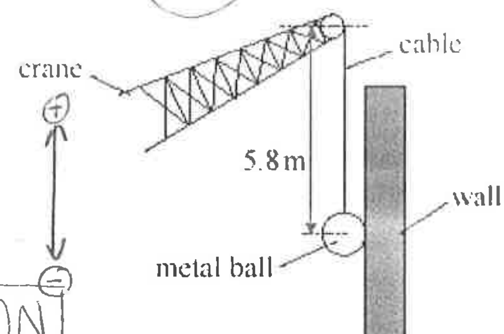


$\Sigma F_{in} = \frac{mv^2}{r}$

$F_g + F_T = \frac{mv^2}{r}$

extra tension required for swing

5. A 2100-kg demolition ball is attached to the end of a 5.8-m cable.



a) Determine the tension in the cable as the ball hangs motionless.

$\Sigma F = F_g + F_T = \emptyset N$

$\underset{(mg)}{F_g} = -F_T = (2100 kg)(-9.8 m/s^2) = \boxed{21{,}000 N}$

b) The ball is pulled back and released. At the lowest point of the swing, the ball is moving at a speed of 7.6 m/s. Determine the tension in the cable upon impact with the wall.

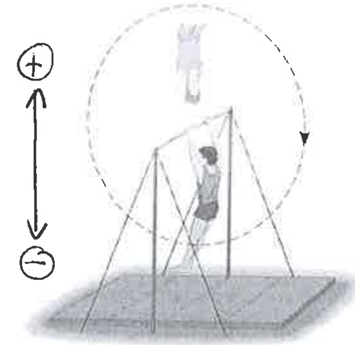
$\Sigma F_{in} = \frac{mv^2}{r} = F_T + F_g$ $\qquad \frac{(2100 kg)(7.6 \frac{m}{s})^2}{5.8 m} - (2100 kg)(-9.8 \frac{m}{s^2})$

$F_T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg$ $\qquad = \boxed{41{,}493 N}$

41,000N

* Physics classroom multi-media Circular Motion

6. How much force does this 55 kilogram gymnast ~~deed~~ need to hold onto the bar as they swing through the bottom of their swing at 3.4 meters per second? Assume their center of mass is approximately 0.80 meter from their outstretched hands.



$r = 0.80 m$

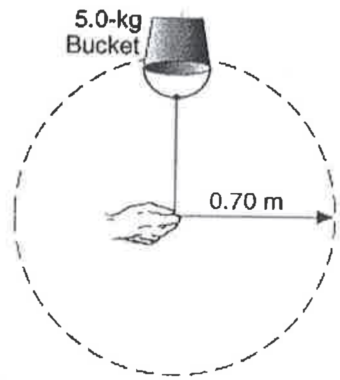
$$\Sigma F_{in} = \frac{mv^2}{r} = F_T + F_g$$

$$\oplus \quad \ominus$$

$$F_T = \frac{mv^2}{r} - F_g \qquad \frac{(55 kg)(3.4 \frac{m}{s})^2}{0.80 m} - (55 kg)(-9.8 \frac{m}{s^2})$$

$$F_T = 1333 N \rightarrow \boxed{1300 N}$$

7. a) What is the maximum speed that this bucket can have at the bottom of its swing if the breaking strength of the rope is 100. newtons?



$F_{T\,max} = 100. N$ at the bottom $v_{max} = ?$

$$F_{in} = \frac{mv^2}{r} = F_T + F_g$$

$$\boxed{v = \sqrt{\frac{(F_T + F_g) r}{m}}} = \sqrt{\frac{[(100. N) + (5.0 kg)(9.8 \frac{m}{s^2})] 0.70 m}{(5.0 kg)}} \quad \boxed{v_{max} = 2.7 \frac{m}{s}}$$

b) What is the minimum speed the bucket must have at the top of its swing to make it around without the water falling out?

at the top $v_{min} = ?$

If $F_T < F_g$ at the top, the bucket will collapse.

F_T must be $\geq F_g$ to spin.

If exceed v_{max} - rope will break.

If less than v_{min} - rope will collapse.

Minimum speed does not depend on mass.

$$\Sigma F_{in} = F_g = F_T \text{ for } v_{min}$$

so... $$\frac{mv^2}{r} = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\boxed{v_{min} = \sqrt{rg}} \; \boxed{= 2.6 \frac{m}{s}}$$