Ms. Rosenthal **Circular Motion and Gravity** P. 3

IB 11

Uniform Circular Motion: motion in a circle at a constant speed

Cycle: one complete revolution

Period: time for one cycle

Symbol: T

Units: s

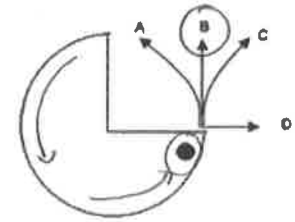
Formula for Period: $\frac{\text{total time}}{\text{\# of cycles}}$

Average Speed

$$v = \frac{d}{t} = \frac{2\pi r}{T} = \frac{\text{circumference (of circle)}}{\text{period}}$$

Which path will the marble take? Why? Explain using one of Newton's Laws.

Newton's 1st Law = an object will stay in motion and remain in constant motion in a straight line unless acted upon by an unbalanced force.



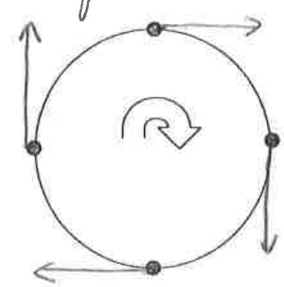
$\vec{v}$ tangent to circle

Instantaneous Velocity (linear velocity, tangential velocity)

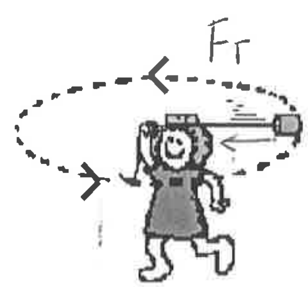
Magnitude: $v = \frac{2\pi r}{T}$

Direction: tangent to the circle

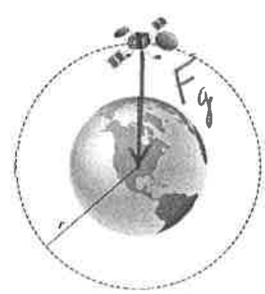
Sketch the instantaneous velocity vectors for the marble at each location shown at right.



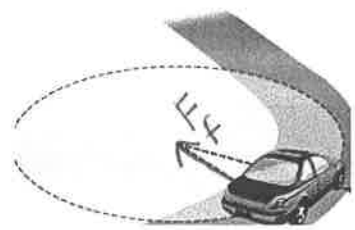
Draw and label the force causing the circular motion in each case below.



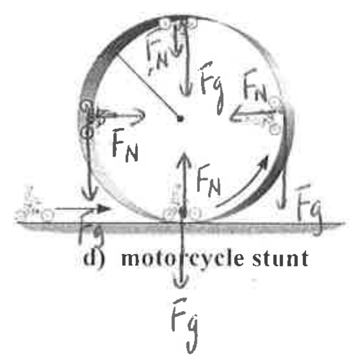
a) rubber stopper on a string



b) satellite circling Earth



c) car rounding a curve



d) motorcycle stunt

1. In each case, what is the direction of the force causing the circular motion?
inward.
2. Are these objects accelerating, even if they are moving at a constant speed?
yes - constantly changing direction
3. In which direction are they accelerating?
inward - matches the direction of force
4. Are objects moving in circular motion ever in equilibrium? Explain.
no because they are changing direction

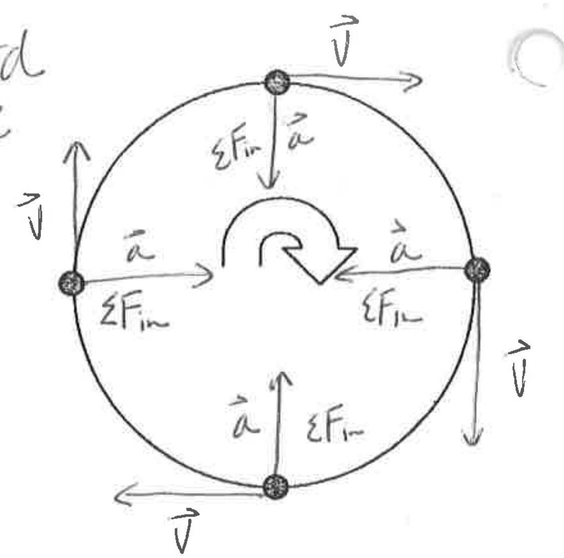
Centripetal Force and Acceleration

Centripetal Force: sum of all the forces acting toward the center of the circle

Symbol: $\Sigma \vec{F}_{in}$

Centripetal Acceleration: acceleration toward center of circle

Symbol: $\vec{a}_c$



Sketch and label the instantaneous velocity, force, and acceleration for the marble at each location in the diagram.

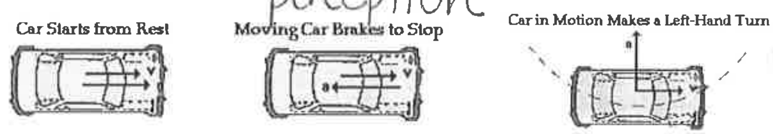
Compare the directions of the instantaneous velocity, force, and acceleration for an object in uniform circular motion.

NOTE: The phrase "centripetal force" does not denote a new and separate force of nature. The phrase is merely another name for the net force pointing toward the center of the circular path. The centripetal force always has another name, such as F_g, F_N, F_T, F_f, or any combination of these.

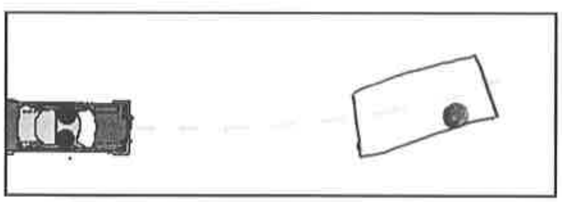
Centripetal force: net inward force

~~Centrifugal~~ force: pseudoforce based on perception

How is the acceleration of a car related to the forces you feel as a rider in the car?



A car traveling down the road makes a quick left-hand turn. Explain, using Newton's laws and a different frame of reference, how any "centrifugal" force you feel can be explained as a centripetal force.



Centripetal Acceleration

$$\vec{a} = \frac{\Delta v}{t}$$

for circles:

$$a = \frac{v^2}{r}$$

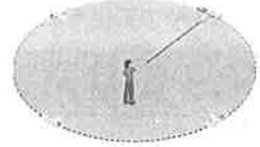
Centripetal Force

$$\Sigma F_{in} = ma$$

Centripetal Force is the Net Force!

$$\Sigma F_{in} = m\frac{v^2}{r}$$

1. A boy flies a 0.750-kg motorized plane on a 2.3 m string in a circular path. The plane goes around 8.0 times in 12.0 seconds. Determine the following:



a) the period of revolution

$$T = \frac{\text{total time}}{\#\text{ of cycles}} = \frac{12.0\,s}{8} = \boxed{1.5\,s}$$

b) the speed of the plane

$$V = \frac{2\pi r}{T} = \frac{2\pi(2.3\,m)}{1.5\,s} = \boxed{9.6\,\frac{m}{s}}$$

c) the acceleration of the plane

$$a = \frac{V^2}{r} = \frac{\left(9.6\,\frac{m}{s}\right)^2}{2.3\,m} = \boxed{40.\,\frac{m}{s^2}}$$

d) the tension in the string

$$\Sigma F_{in} = ma = \frac{mV^2}{r}$$

$$(0.750\,Kg)\left(40.\,\frac{m}{s^2}\right) = \boxed{30.\,N}$$

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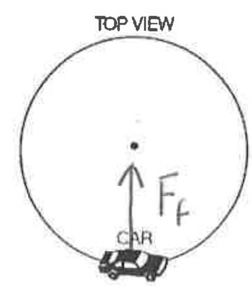
2. A 1.5 kilogram toy car moves on a circular track of 1.3 meter radius at a constant speed of 2.0 meters per second. Determine the following:



a) the time it takes to go around the track once

$$T = \frac{2\pi r}{V} = \frac{2\pi(1.3\,m)}{(2.0\,m/s)} = \boxed{4.1\,s}$$

b) the centripetal acceleration of the cart

$$a_c = \frac{V^2}{r} = \frac{\left(2.0\,\frac{m}{s}\right)^2}{1.3\,m} = \boxed{3.1\,\frac{m}{s^2}}$$

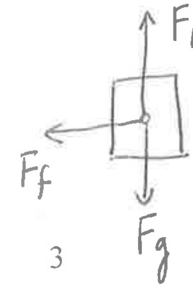
c) the centripetal force acting on the cart

$$\Sigma F_{in} = \frac{mV^2}{r} = \frac{(1.5\,Kg)(2.0\,m/s)^2}{(1.3\,m)} = \boxed{4.6\,N}$$

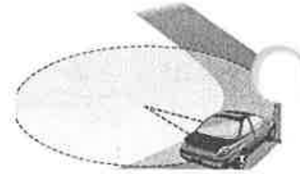
d) What is causing this force?

Force of friction is causing circular motion.

Side view: FBD



3. A 2000. kg car attempts to turn a corner going at a speed of 25 m/s. The radius of the turn is 15 meters.



a) How much friction is needed to negotiate this turn successfully?

$$\Sigma F_{in} = F_f = \frac{mv^2}{r} \qquad \frac{(2000.\,kg)(25\,\frac{m}{s})^2}{(15\,m)} = \boxed{83{,}33\bar{3}\,N}$$

or $83{,}000\,N$

b) If the pavement is dry asphalt, will the car be able to safely turn? Justify your answer.

$$\mu = 1.0 \qquad F_f = \mu F_N \qquad F_N = |F_g| = |mg|$$

$$F_f = (1.0)(2000.\,kg)(9.8\,\tfrac{m}{s^2}) = \boxed{19{,}600\,N}$$

Car does not make the turn :($\qquad F_{f\,max}$

c) Derive an expression for the maximum speed with which a car of mass *m* can safely make a turn around a curve of radius *r*.

$$\Sigma F_{in} = \frac{mv^2}{r} \qquad F_f = \mu F_N \qquad \Sigma F_{in} = F_f$$

$$\frac{mv^2}{r} = \mu F_N$$

$$\frac{\cancel{m}v^2}{r} = \mu \cancel{m}g \qquad \boxed{v = \sqrt{\mu g r}}$$

$$v_{max} = \sqrt{(1.0)(9.8\,\tfrac{m}{s^2})(15\,m)} = \boxed{12\,\frac{m}{s}}$$

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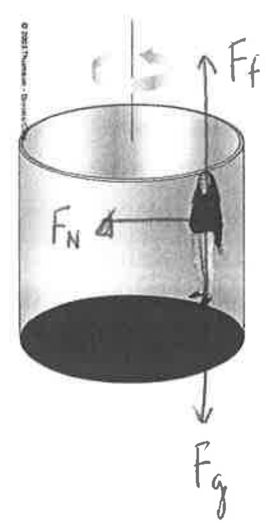
4. At amusement parks, there is a popular ride where the floor of a rotating cylindrical room falls away, leaving the backs of the riders "plastered" against the wall. What is the minimum coefficient of static friction that must exist between a rider's back and the wall, if the rider is to remain in place when the floor drops away?



$$F_f = \mu F_N \qquad \Sigma F_{in} = \frac{mv^2}{r} \qquad \Sigma F_{in} = F_N$$

and in this case: $F_g = F_f$

$$F_N = \frac{F_f}{\mu} \qquad \text{so...} \qquad \frac{F_f}{\mu} = \frac{mv^2}{r}$$

substitute F_g for F_f or $mg = F_f$

$$\frac{\cancel{m}g}{\mu} = \frac{\cancel{m}v^2}{r} \qquad \mu_{minimum} = \frac{gr}{v^2}$$