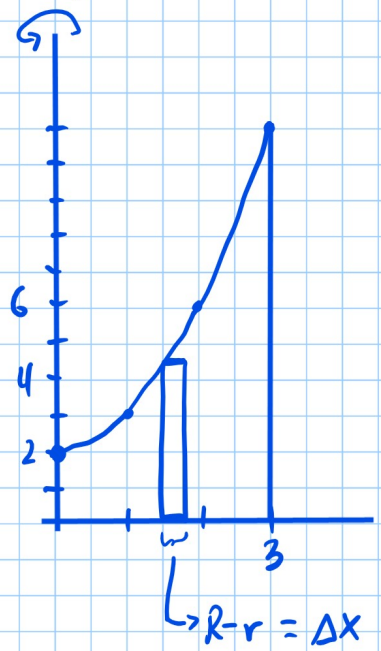
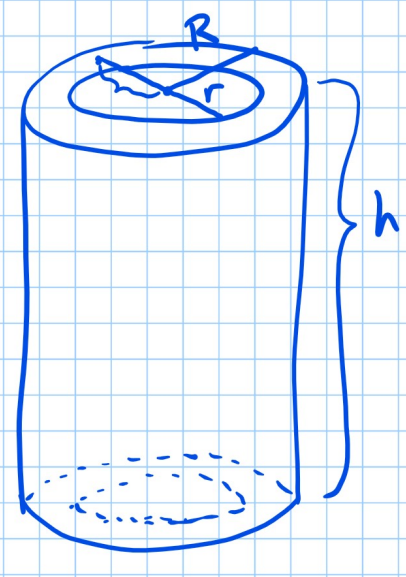
# Volume By Cylindrical Shells

Suppose you rotate the region bounded by $y = x^2 + 2$, $x = 0$, $x = 3$, and the x-axis about the y-axis.




avg radius $= \frac{R+r}{2}$




$$V = \pi R^2 h - \pi r^2 h$$

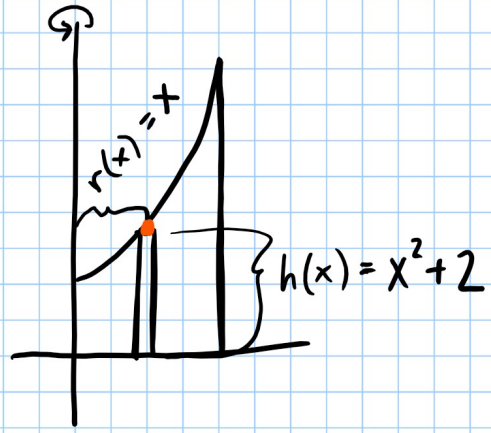
$$V = \pi (R^2 - r^2) h$$

$$V = \pi (R+r)(R-r) h$$

$$V = \pi\, 2 \underbrace{\left(\frac{R+r}{2}\right)}_{\text{avg radius} = r} dx \cdot h$$

$$V = 2\pi\, r(x) \cdot h(x)\, dx$$

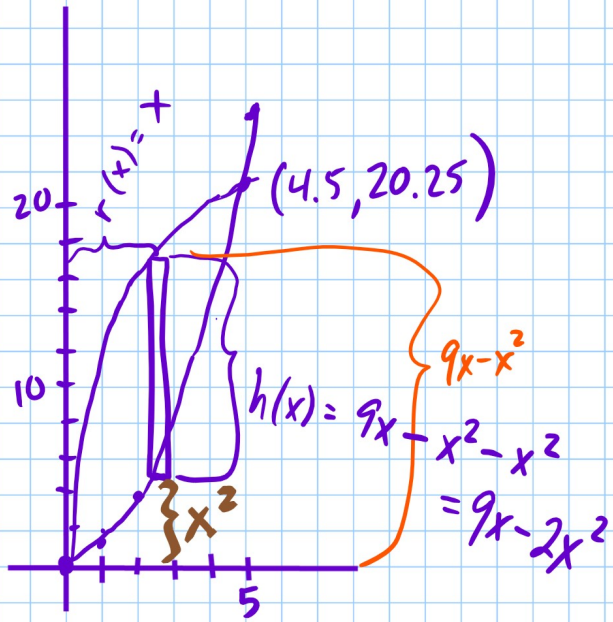
$$\boxed{\text{Shells: } V = 2\pi \int_a^b r(x) \cdot h(x)\, dx}$$




$$V = 2\pi \int_0^3 x\left(x^2+2\right)\,dx = \frac{117\pi}{2}$$

2*fnInt(x(x²+2),X,0,3) ▶FRAC

ex: Graph the region bounded by $y = 9x - x^2$ and $y = x^2$

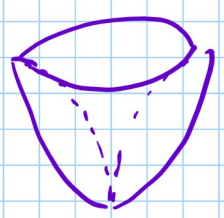


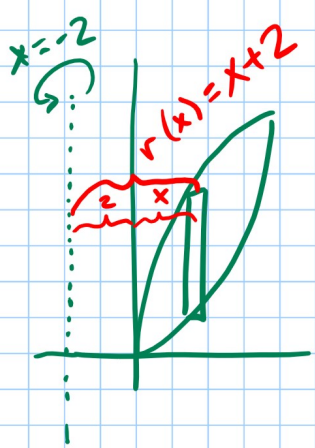

a) Volume around the y-axis

$$V = 2\pi \int_0^{4.5} x\left(9x - 2x^2\right)dx$$

2*fnInt(x(9x−2x²),X,0,4.5) ▶FRAC

$$= \frac{2187\pi}{16}$$





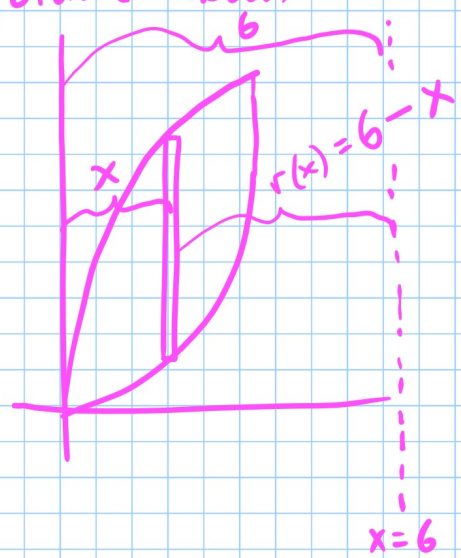

b) Volume around $x = -2$

$$V = 2\pi \int_0^{4.5} (x+2)\left(9x - 2x^2\right)dx$$

2*fnInt((x+2)(9x−2x²),X,0,4.5) ▶FRAC

$$= \frac{4131\pi}{16}$$

c) Volume about the line $x=6$




$$V = 2\pi \int_0^{4.5} (6-x)(9x-2x^2)\,dx$$

$$= \frac{3645\pi}{16}$$

2*fnInt((6-x)(9x-2x²), X, 0, 4.5)

▶ FRAC

## Volume/Area Assignment

Sketch the region bounded by $y=-x^2+6x-4$ and $y=x$

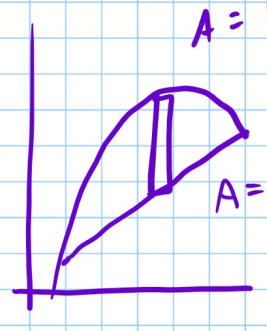
1) Find its area.

2) Find volume when region is rotated about

A) the y-axis

B) the line $x=-3$

C) the line $x=10$



$$A = \int_1^4 (-x^2+6x-4)-x\,dx$$

$$A = \int_1^4 \underbrace{(-x^2+5x-4)}_{h(x)}\,dx$$