

WARMUP - Copy these formulas into your notes

### Antiderivatives of Trig Functions

- 1)  $\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$
- 2)  $\int \csc \theta d\theta = -\ln |\csc \theta + \cot \theta| + C$
- 3)  $\int \sin \theta d\theta = -\cos \theta + C$
- 4)  $\int \cos \theta d\theta = \sin \theta + C$
- 5)  $\int \sec^2 \theta d\theta = \tan \theta + C$
- 6)  $\int \csc^2 \theta d\theta = -\cot \theta + C$
- 7)  $\int \tan \theta d\theta = -\ln |\cos \theta| + C$
- 8)  $\int \cot \theta d\theta = \ln |\sin \theta| + C$
- 9)  $\int \sec \theta \tan \theta d\theta = \sec \theta + C$
- 10)  $\int \csc \theta \cot \theta d\theta = -\csc \theta + C$

### Useful Derivatives

$$\frac{d}{d\theta} [\tan \theta] = \sec^2 \theta$$

$$\frac{d}{d\theta} [\sec \theta] = \sec \theta \tan \theta$$

### Pythagorean Identities

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$\frac{1}{\sin \theta} = \csc \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

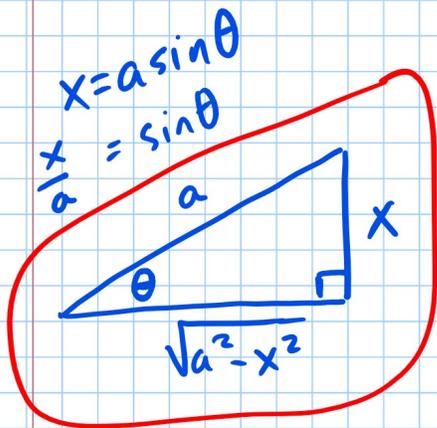
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

# TRIG SUBSTITUTION

CASE 1: For integrals involving  $\sqrt{a^2 - x^2}$ , let  $x = a \sin \theta$

$$\begin{aligned}\sqrt{a^2 - x^2} &= \sqrt{a^2 - (a \sin \theta)^2} = \sqrt{a^2 - a^2 \sin^2 \theta} \\ &= \sqrt{a^2(1 - \sin^2 \theta)} \\ &= \sqrt{a^2 \cos^2 \theta}\end{aligned}$$

$$\sqrt{a^2 - x^2} = a \cos \theta$$



$$\begin{aligned}\text{adj}^2 + x^2 &= a^2 \\ \text{adj}^2 &= a^2 - x^2 \\ \text{adj} &= \sqrt{a^2 - x^2}\end{aligned}$$

ex:  $\int \frac{1}{x^2 \sqrt{9 - x^2}} dx$

$x = 3 \sin \theta$   
 $dx = 3 \cos \theta d\theta$   
 $\sqrt{9 - x^2} = 3 \cos \theta$

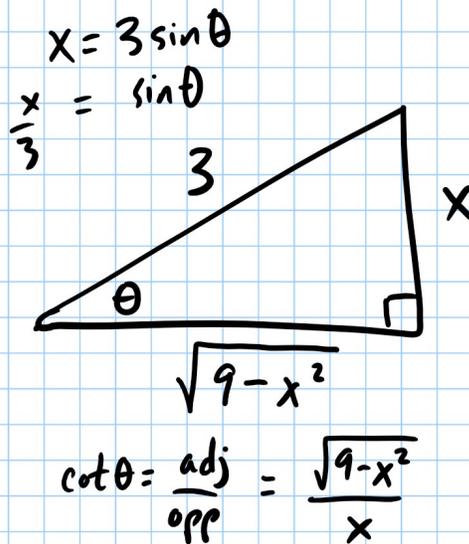
$$= \int \frac{1}{(3 \sin \theta)^2 \cdot 3 \cos \theta} \cdot 3 \cos \theta d\theta$$

$$= \int \frac{1}{9 \sin^2 \theta} d\theta$$

$$= \frac{1}{9} \int \frac{1}{\sin^2 \theta} d\theta$$

$$= \frac{1}{9} \int \csc^2 \theta d\theta$$

$$= \frac{1}{9} (-\cot \theta) + C$$



$$= -\frac{1}{9} \cot \theta + C$$

$$= -\frac{1}{9} \cdot \frac{\sqrt{9-x^2}}{x} + C$$

$$\boxed{= -\frac{\sqrt{9-x^2}}{9x} + C}$$

Now try:  $\int \frac{1}{x\sqrt{25-x^2}} dx$

$$x = 5 \sin \theta$$

$$dx = 5 \cos \theta d\theta$$

$$\sqrt{25-x^2} = 5 \cos \theta$$

$$= \int \frac{1}{5 \sin \theta \cdot 5 \cos \theta} \cdot 5 \cos \theta d\theta$$

$$= \frac{1}{5} \int \frac{1}{\sin \theta} d\theta$$

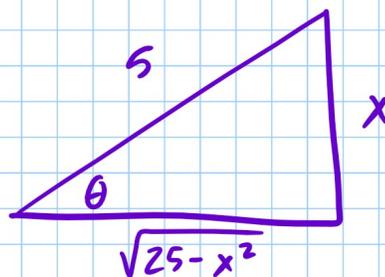
$$= \frac{1}{5} \int \csc \theta d\theta$$

$$= -\frac{1}{5} \ln | \csc \theta + \cot \theta | + C$$

$$= -\frac{1}{5} \ln \left| \frac{5}{x} + \frac{\sqrt{25-x^2}}{x} \right| + C$$

$$= -\frac{1}{5} \ln \left| \frac{5 + \sqrt{25-x^2}}{x} \right| + C$$

$$\sin \theta = \frac{x}{5}$$



CASE 2: For integrals involving  $\sqrt{a^2+x^2}$  (or  $\sqrt{x^2+a^2}$ )

let  $\boxed{x = a \tan \theta}$

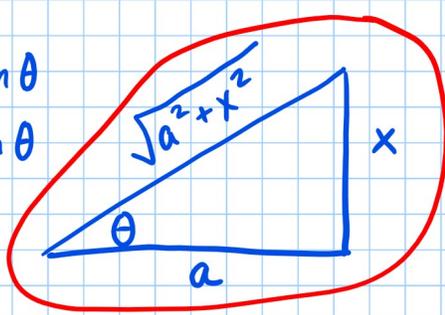
$$\sqrt{a^2+x^2} = \sqrt{a^2+a^2 \tan^2 \theta} = \sqrt{a^2(1+\tan^2 \theta)}$$

$$= \sqrt{a^2 \sec^2 \theta}$$

$$\boxed{\sqrt{a^2+x^2} = a \sec \theta}$$

$$x = a \tan \theta$$

$$\frac{x}{a} = \tan \theta$$



ex:  $\int \frac{1}{(16+x^2)^{3/2}} dx$

$$x = 4 \tan \theta$$

← nowhere to put this

$$dx = 4 \sec^2 \theta d\theta$$

$$= \int \frac{1}{\left(\frac{\sqrt{16+x^2}}{\sqrt{4^2+x^2}}\right)^3} dx$$

$$\sqrt{16+x^2} = 4 \sec \theta$$

$$= \int \frac{1}{(4 \sec \theta)^3} \cdot 4 \sec^2 \theta d\theta$$

$$x = 4 \tan \theta$$

$$\frac{x}{4} = \tan \theta$$

$$= \int \frac{1}{16 \sec^3 \theta} \cdot \cancel{4 \sec^2 \theta} d\theta$$

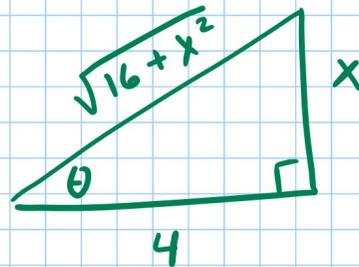
$$= \frac{1}{16} \int \frac{1}{\sec \theta} d\theta$$

$$= \frac{1}{16} \int \cos \theta d\theta$$

$$= \frac{1}{16} \sin \theta + C$$

$$= \frac{1}{16} \cdot \frac{x}{\sqrt{16+x^2}} + C$$

$$= \frac{x}{16 \sqrt{16+x^2}} + C$$



$$\frac{1}{\cos \theta} = \sec \theta$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\int \frac{x}{\sqrt{49+x^2}} dx = \sqrt{49+x^2} + C$$