

Section 6.7 Solving Trig Equations

An equation like $\sin\theta = \frac{1}{2}$ is asking for what angles does the $\sin\theta = \frac{1}{2}$?

ex: Solve $\sin\theta = \frac{1}{2}$ for $0 \leq \theta < 2\pi$
radians

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

If we were asked to give general solution (to solve for all real numbers)

$$\theta = \frac{\pi}{6} + 2\pi k \quad (k \text{ is any integer})$$

$$\theta = \frac{5\pi}{6} + 2\pi k$$

ex: Solve $-2\cos\theta = \sqrt{3}$. Give general solutions.

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{6} + 2\pi k$$

$$\theta = \frac{7\pi}{6} + 2\pi k$$

ex: Solve $\tan\theta = -1$. Give general solutions.

The period of $\tan\theta$ is π

$$\left. \begin{aligned} \theta &= \frac{3\pi}{4} + \pi k \\ \theta &= \frac{7\pi}{4} + \pi k \end{aligned} \right\} \begin{array}{l} \text{stating both answers is} \\ \text{redundant since } \frac{3\pi}{4} \\ \text{and } \frac{7\pi}{4} \text{ are } \pi \text{ apart} \end{array}$$

So answer is $\frac{3\pi}{4} + \pi k$

ex: $\cos(2\theta) = \frac{1}{2}$ Solve on $0 \leq \theta < 2\pi$

Period = $\frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ ←

$$\frac{1}{2} \cdot 2\theta = \frac{\pi}{3} \cdot \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} + \pi = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$$

$$\frac{1}{2} \cdot 2\theta = \frac{5\pi}{3} \cdot \frac{1}{2} \Rightarrow \theta = \frac{5\pi}{6} + \pi = \frac{5\pi}{6} + \frac{6\pi}{6} = \frac{11\pi}{6}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin\left(\frac{1}{2}\theta\right) = \frac{\sqrt{2}}{2}$$

$$\cancel{2} \cdot \frac{1}{2}\theta = \frac{\pi}{4} \cdot 2$$

$$\theta = \frac{\pi}{2}$$

$$2 \cdot \frac{1}{2}\theta = \frac{3\pi}{4} \cdot 2$$

$$\theta = \frac{3\pi}{2}$$

Assignment

For 1-4 find general formula

1) $\tan\theta = \frac{\sqrt{3}}{3} (+ \pi k)$

2) $3\sin\theta + 3 = 0$

3) $2\sec\theta = 4$

} $+ 2\pi k$

For 5-8 Solve on
 $0 \leq \theta < 2\pi$

5) $2\cos\theta + 3 = 2$

6) $\sin(2\theta) = \frac{\sqrt{3}}{2}$

7) $\cos^2\theta = \frac{1}{4}$

$$4) 2 \sin \theta + 1 = 0$$

$$8) \tan \theta = \sqrt{3}$$