WARMUP

Establish the identity:

19) $\sec\theta - \tan\theta = \dfrac{\cos\theta}{1+\sin\theta}$

$$\sec\theta - \tan\theta = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{(1-\sin\theta)}{\cos\theta} \cdot \frac{(1+\sin\theta)}{1+\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$\sin^2\theta + \cos^2\theta = 1$

$$= \frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

Assignment:

p481 21, 27, 34, 39, 51, 65, 72

21) $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$

$3\sin^2\theta + 4\cos^2\theta = \sin^2\theta + \sin^2\theta + \sin^2\theta + \cos^2\theta + \cos^2\theta + \cos^2\theta + \cos^2\theta$

$= 3 + \cos^2\theta$

$3(1-\cos^2\theta) + 4\cos^2\theta$

$3 - 3\cos^2\theta + 4\cos^2\theta$

$3 + \cos^2\theta$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1}$

27) $\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = 2\tan\theta$

$\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = \frac{(1/\cos\theta)}{(1/\sin\theta)} + \frac{\sin\theta}{\cos\theta} =$

$\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 2\frac{\sin\theta}{\cos\theta} = 2\tan\theta$

34) $1 - \frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

$1 - \frac{\sin^2\theta}{1+\cos\theta} = 1 - \frac{1-\cos^2\theta}{1+\cos\theta} =$

$1 - \frac{(1-\cos\theta)\cancel{(1+\cos\theta)}}{\cancel{1+\cos\theta}} = 1 - (1-\cos\theta)$

$1 - 1 + \cos\theta = \cos\theta$

39) $\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \sec\theta$

$$\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\sin\theta(1+\sin\theta) + \cos\theta\cdot\cos\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$$

$$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cancel{\sin\theta + 1}^{\,1}}{\cos\theta\,\cancel{(1+\sin\theta)}}$$

$$= \sec\theta$$

51) $\dfrac{\sec\theta - \csc\theta}{\sec\theta\csc\theta} = \sin\theta - \cos\theta$

$$\frac{5-4}{6} = \frac{5}{6} - \frac{4}{6}$$

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\frac{\sec\theta - \csc\theta}{\sec\theta\csc\theta} = \frac{\cancel{\sec\theta}^{\,1}}{\cancel{\sec\theta}\csc\theta} - \frac{\cancel{\csc\theta}^{\,1}}{\sec\theta\,\cancel{\csc\theta}}$$

$$= \frac{1}{\csc\theta} - \frac{1}{\sec\theta}$$

$$= \sin\theta - \cos\theta$$

65) $\dfrac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \cos^2\theta$

$$\frac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$$

$$\frac{10}{11} - \frac{3}{11} = \frac{10-3}{11}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$$

$$= \cancel{(\cos^2\theta - \sin^2\theta)} \cdot \frac{\cos^2\theta}{\cancel{\cos^2\theta - \sin^2\theta}}$$

$$= \cos^2\theta$$