

Section 7.3 Table of Integrals

ex: $\int \sin(7x) \sin(3x) dx$ $b^2 - a^2 = 3^2 - 7^2 = 9 - 49 = -40$

$\overbrace{a=7}^1 \quad \overbrace{b=3}^1$

which formula?

$$\begin{aligned} \#10) \quad \int \sin(ax) \sin(bx) dx &= \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C \\ &= -\frac{1}{40} [7 \cos(7x) \sin(3x) - 3 \sin(7x) \cos(3x)] + C \end{aligned}$$

Ex: $\int (x^2 - 3x + 2) e^{3x} dx$

$$\begin{aligned} \#14) \quad \int p(x) e^{ax} dx &= \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots \\ &= \frac{1}{3} (x^2 - 3x + 2) e^{3x} - \frac{1}{3^2} (2x - 3) e^{3x} + \frac{1}{3^3} (2) e^{3x} - \cancel{\frac{1}{3^4} (0)}^0 \\ &= \frac{1}{3} (x^2 - 3x + 2) e^{3x} - \frac{1}{9} (2x - 3) e^{3x} + \frac{2}{27} e^{3x} + C \end{aligned}$$

ex: $\int \sin^6 x dx$

$$\begin{aligned} \#17) \quad \int \sin^n x dx &= -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx \\ \int \sin^6 x dx &= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \int \sin^4 x dx \\ &= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left[-\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x dx \right] \\ &= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5}{8} \int \sin^2 x dx \\ &= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5}{8} \left[-\frac{1}{2} \sin x \cos x + \frac{1}{2} \int \sin^0 x dx \right] \\ &= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} \int 1 dx \\ \int \sin^6 x dx &= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C \end{aligned}$$

$\sin^2 x + \cos^2 x = 1$
 $\sin^2 x = 1 - \cos^2 x$
 $\cos^2 x = 1 - \sin^2 x$

~~check:~~

$$\begin{aligned}
 & \frac{d}{dx} \left[-\frac{1}{6} \sin^5 x \cos x + \frac{-5}{24} \sin^3 x \cos x + \frac{-5}{16} \sin x \cos x + \frac{5}{16} x + C \right] \\
 &= -\frac{1}{6} \sin^5 x (-\sin x) + \cos x \cdot \left(-\frac{5}{6} \sin^4 x \cdot \cos x \right) + \left(\frac{-5}{24} \sin^3 x \right) (-\sin x) + \cos x \left(-\frac{5}{8} \sin^2 x \cdot \cos x \right) \\
 &\quad + \left(-\frac{5}{16} \sin x \right) (-\sin x) + \cos x \left(-\frac{5}{16} \cos x \right) + \frac{5}{16} + 0 \\
 &= \frac{1}{6} \sin^6 x - \frac{5}{6} \sin^4 x \cos^2 x + \frac{5}{24} \sin^4 x - \frac{5}{8} \sin^2 x \cos^2 x + \frac{5}{16} \sin^2 x - \frac{5}{16} \cos^2 x + \frac{5}{16} \\
 &= \frac{1}{6} \sin^6 x - \frac{5}{6} \sin^4 x (1 - \sin^2 x) + \frac{5}{24} \sin^4 x - \frac{5}{8} \sin^2 x (1 - \sin^2 x) + \frac{5}{16} \sin^2 x \\
 &\quad - \frac{5}{16} (1 - \sin^2 x) + \frac{5}{16} \\
 &= \underline{\frac{1}{6} \sin^6 x} - \underline{\frac{5}{6} \sin^4 x} + \underline{\frac{5}{6} \sin^6 x} + \underline{\frac{5}{24} \sin^4 x} - \underline{\frac{5}{8} \sin^2 x} + \underline{\frac{5}{8} \sin^4 x} + \underline{\frac{5}{16} \sin^2 x} \\
 &= \left(\frac{1}{6} + \frac{5}{6} \right) \sin^6 x + \left(-\frac{5}{6} + \frac{5}{24} + \frac{15}{24} \right) \sin^4 x + \left(-\frac{10}{16} + \frac{5}{16} + \frac{5}{16} \right) \sin^2 x \\
 &= \boxed{\sin^6 x}
 \end{aligned}$$

p 308 2, 11, 15, 21, 26

2) $\int x^5 \ln x \, dx$

11) $\int \sin 3\theta \cos 5\theta \, d\theta$

15) $\int x^4 e^{3x} \, dx$

21) $\int e^{5x} \sin 3x \, dx$

26) $\int \frac{1}{x^2 + 4x + 3} \, dx$