$$\int x e^{(3+x^2)}\,dx = \int e^{(3+x^2)} \cdot \underline{x\,dx}$$

$u = 3 + x^2$

$\frac{1}{2}\,du = \cancel{\tfrac{1}{2}2}x\,dx$

[redacted]

$\frac{1}{2}\,du = x\,dx$

$$= \int e^{u} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int e^{u}\,du$$

$$= \frac{1}{2}e^{u} + C$$

$$= \frac{1}{2}e^{(3+x^2)} + C$$

2) $$\int \frac{3x^2-4}{x^3-4x}\,dx = \int \frac{1}{x^3-4x} \cdot \underbrace{(3x^2-4)\,dx}$$

$u = x^3 - 4x$

$du = \underbrace{(3x^2-4)\,dx}$

$$\frac{A}{B} = \frac{1}{B}\cdot\frac{A}{1} = \frac{1}{B}\cdot A$$

$$= \int \frac{1}{u}\cdot du$$

$$= \ln|u| + C$$

$$= \ln|x^3-4x| + C$$

3) $$\int x^3 \sin(5+4x^4)\,dx = \int \sin(5+4x^4)\cdot x^3\,dx$$

$u = 5 + 4x^4$

$du = 16\overbrace{x^3\,dx}$

$\frac{1}{16}\,du = x^3\,dx$

$$= \int \sin u \cdot \frac{1}{16}\,du$$

$$= \frac{1}{16}\int \sin u\,du$$

$$= \frac{1}{16}(-\cos u) + C$$

$$-\frac{1}{16}\cos(5+4x^4) + C$$

Section 7.2 Integration by Parts

ex: $\int xe^x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$$= xe^x - \int e^x dx$$

$$\boxed{= xe^x - e^x + C}$$

Choose u and dv

$u = x \qquad dv = e^x dx$

derivative $\qquad$ antiderivative

$du = dx \qquad v = e^x$

Check: $\frac{d}{dx}\left(xe^x - e^x + c\right)$

$$\underbrace{xe^x + \cancel{e^x \cdot 1}}_{\text{Product}} - \cancel{e^x} + 0$$

$$xe^x$$

How to choose u and dv:

- Whatever you let dv be, you need to be able to find v.
- It helps if du is simpler than u.
- It helps if v is simpler than dv.

$$\qquad\qquad u \cdot v - \int v \cdot du$$

ex: $\int \theta \cos\theta \, d\theta = \theta \cdot \sin\theta - \underbrace{\int \sin\theta \, d\theta} = \boxed{\theta\sin\theta + \cos\theta + C}$

$u = \theta \qquad dv = \cos\theta \, d\theta$

$du = d\theta \qquad v = \sin\theta$

ex: $\int x^3 \ln x \, dx$

Whenever there's an $\ln x$, let $u = \ln x$

$u = \ln x \qquad dv = x^3 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4}$

$\int$ v $\qquad$ du

$$= \underset{u}{\ln x} \cdot \overset{v}{\frac{x^4}{4}} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3\,dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\cdot\frac{x^4}{4} + C$$

$$= \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C$$

ex: $\int x^4 \sin x\,dx = x^4(-\cos x) - \int (-\cos x)\,4x^3\,dx$

$u = x^4 \qquad dv = \sin x\,dx$

$du = 4x^3\,dx \qquad v = -\cos x$

$$= -x^4\cos x + \underbrace{\int 4x^3 \cos x\,dx}_{\text{uh-oh gotta do parts again}}$$

ALTERNATE PARTS METHOD – TABULAR METHOD

sign	derivative u	antideriv. dv
$+$	x^4	$\sin x$
$-$	$4x^3$	$-\cos x$
$+$	$12x^2$	$-\sin x$
$-$	$24x$	$\cos x$
$+$	24	$\sin x$
$-$	0	$-\cos x$

$$x^4(-\cos x) - 4x^3(-\sin x) + 12x^2\cos x - 24x\sin x + 24(-\cos x) + C$$

$$-x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + C$$

DON'T USE TABULAR FOR THE LN PROBLEMS

3) $\int x^2 e^{5x}\,dx$

$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

sign	u	dv
$+$	x^2	e^{5x}
$-$	$2x$	$\frac{1}{5}e^{5x}$
$+$	2	$\frac{1}{25}e^{5x}$
$-$	0	$\frac{1}{125}e^{5x}$

$x^2 \cdot \frac{1}{5}e^{5x} - 2x \cdot \frac{1}{25}e^{5x} + 2 \cdot \frac{1}{125}e^{5x} + C$

$\frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C$

Try:

1) $\int x\sin x\,dx$

2) $\int x^2\cos(3x)\,dx$

3) $\int x^9 \ln x\,dx$