WARMUP

Use the chain rule to calculate $f'(x)$.

1) $f(x) = (x^2+3)^{10}$

$f'(x) = 10(x^2+3)^9 \cdot 2x = 20x(x^2+3)^9$

so $\int 20x(x^2+3)^9 dx = (x^2+3)^{10} + C$

2) $f(x) = (5x-1)^{15}$

$f'(x) = 15(5x-1)^{14} \cdot 5$

$= 75(5x-1)^{14}$

3) $f(x) = \sqrt{3x+10}$

$f(x) = (3x+10)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(3x+10)^{-1/2} \cdot 3$

$= \dfrac{3}{2\sqrt{3x+10}}$

Section 7.1 Integration By Substitution

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$

$$\int (3x+1)^9 dx = \int u^9 \cdot \frac{1}{3} du$$

Let $u = 3x+1$

$\cancel{dx} \cdot \dfrac{du}{\cancel{dx}} = 3\,dx$ ←

$\frac{1}{3} du = \frac{1}{\cancel{3}}\cancel{3}\,dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3}\int u^9 du$

$= \frac{1}{3} \cdot \frac{u^{10}}{10} + C$

$$\boxed{= \frac{(3x+1)^{10}}{30} + C}$$

ex: $\int x^3(5x^4-3)^{10}\,dx = \int (5x^4-3)^{10}\cdot x^3\,dx$

$u = 5x^4-3$

$du = 20x^3\,dx$

$\frac{1}{20}du = x^3\,dx$

$= \int u^{10}\cdot\frac{1}{20}\,du$

$= \frac{1}{20}\int u^{10}\,du$

$= \frac{1}{20}\cdot\frac{u^{11}}{11} + C$

$= \frac{(5x^4-3)^{11}}{220} + C$

ex: $\int x^6(3x^7+5)^{12}\,dx = \int (3x^7+5)^{12}x^6\,dx$

$u = 3x^7+5$

$du = 21\underbrace{x^6\,dx}$

$\frac{1}{21}du = x^6\,dx$

$= \int u^{12}\cdot\frac{1}{21}\,du$

$= \frac{1}{21}\int u^{12}\,du$

$= \frac{1}{21}\cdot\frac{u^{13}}{13} + C$

$= \frac{(3x^7+5)^{13}}{273} + C$

ex: $\int \frac{x}{\sqrt{x^2+7}}\,dx = \int x(x^2+7)^{-1/2}\,dx = \int (x^2+7)^{-1/2}\cdot x\,dx$

$u = x^2+7$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int u^{-1/2}\cdot\frac{1}{2}\,du$

$= \frac{1}{2}\int u^{-1/2}\,du$

$$= \cancel{\frac{1}{2}} \cdot \frac{u^{1/2}}{\cancel{1/2}} + C$$

$$= (x^2+7)^{1/2} + C$$

$$= \sqrt{x^2+7} + C$$

ex: $\displaystyle\int \frac{e^x+1}{e^x+x}\,dx$

$u = e^x + x$

$du = (e^x+1)\,dx$

$$\int \underbrace{\frac{1}{e^x+x}} \cdot \underbrace{(e^x+1)\,dx}$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln|e^x + x| + C$$

when we substitute, the result has to look like one of these

$$\begin{cases} \int u^n\,du = \frac{u^{n+1}}{n+1} + C \\ \int \frac{1}{u}\,du = \ln|u| + C \\ \int e^u\,du = e^u + C \\ \int \cos u\,du = \sin u + C \\ \int \sin u\,du = -\cos u + C \end{cases}$$

ex: $\displaystyle\int \frac{x}{x^2+4}\,dx = \int \frac{1}{x^2+4} \cdot \underline{x\,dx}$

$u = x^2 + 4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int \frac{1}{u} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(x^2+4) + C$$

ex: $\int (\theta+2)\cos(\theta^2+4\theta)\,d\theta = \int \cos(\underbrace{\theta^2+4\theta}_{u}) \cdot \underbrace{(\theta+2)\,d\theta}_{\frac{1}{2}du}$

$u = \theta^2 + 4\theta$

$\frac{du}{2} = \frac{(2\theta+4)}{2}\,d\theta$

$\frac{1}{2}du = (\theta+2)\,d\theta$

$= \frac{1}{2}\int \cos u \, du$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(\theta^2+4\theta) + C$

Assignment

1) $\int x^2(5x^3+3)^7\,dx$

2) $\int \frac{x+2}{x^2+4x+19}\,dx$

3) $\int x^2\sin(x^3+1)\,dx$