WARMUP

Find the general antiderivative for each

1) $f(x) = 6x^2 + 6x - 1$
2) $g(x) = 5\sin x$
3) $h(x) = \frac{4}{x} - 3e^x$

1) $2x^3 + 3x^2 - x + C$
2) $-5\cos x + C$
3) $4\ln x - 3e^x + C$

Section 6.3 Differential Equations

ex: $f'(x) = 3x^2 + 6x - 5$ with $f(0) = 7$.

It's asking you to find $f(x)$

$f(0) = 7$ is the initial condition

STEP 1: Calculate the antiderivative of $f'(x)$. This gives us $f(x)$.

$$f(x) = \frac{3x^3}{3} + \frac{6x^2}{2} - 5x + C$$

$$f(x) = x^3 + 3x^2 - 5x + C$$

STEP 2: Plug in the initial condition and solve for C.

$$f(0) = 0^3 + 3\cdot 0^2 - 5\cdot 0 + C = 7$$

$$C = 7$$

STEP 3: Write $f(x)$

$$f(x) = x^3 + 3x^2 - 5x + 7$$ "Particular Solution"

ex: $f'(x) = 5\cos x$ with $f\left(\frac{\pi}{2}\right) = -3$

$f(x) = 5\sin x + C$

$f\left(\frac{\pi}{2}\right) = 5\sin\frac{\pi}{2} + C = -3$

$$5\cdot 1 + C = -3$$
$$C = -8$$
$$f(x) = 5\sin x - 8$$

ex: $\frac{dP}{dt} = 6e^t + 7 \quad P(1) = -2$

like $P'(t)$

$$P = 6e^t + 7t + C$$
$$P(1) = 6\cdot e^1 + 7\cdot 1 + C = -2$$
$$C = -9 - 6e$$
$$C \approx -25.31$$

$$\boxed{P(t) = 6e^t + 7t - 25.31}$$

Motion Problems

ex: A ball is thrown upward from a height of 6 ft with an initial velocity of 60 ft/sec. How high will the ball go? Use $a(t) = -32$ ft/sec^2

Acceleration is the derivative of velocity

Velocity is the derivative of height

So velocity is the antiderivative of acceleration.

so $v(t) = -32t + C$

$$v(0) = 60$$
$$v(0) = -32\cdot 0 + C = 60$$
$$C = 60$$
$$v(t) = -32t + 60$$

$h(t)$ is the antiderivative of $v(t)$.

$$h(t) = -\frac{32t^2}{2} + 60t + C$$

initial height $= 6$

$$h(t) = -16t^2 + 60t + 6$$

highest height

$$v(t) = 0$$
$$-32t + 60 = 0$$
$$-32t = -60$$
$$t = 1.875 \text{ sec.}$$

so $h(1.875)$ is highest height

$$h(1.875) = -16 \cdot 1.875^2 + 60 \cdot 1.875 + 6$$
$$= 62.25 \text{ ft}$$

p277 1-11 odd, 15, 16, 18

5) $\frac{dy}{dx} = 1 + \cos x$

$y = x + \sin x + C$

7) $\frac{dP}{dt} = 10e^t \quad P(0) = 25$

$P = 10e^t + C$

$P(0) = 10e^0 + C = 25$

$10 \cdot 1 + C = 25$

$C = 15$

$P = 10e^t + 15$

11) $a(t) = -9.8 \text{ m/sec}^2$

a) $v(t) = -9.8t + 40$

$h(t) = -4.9t^2 + 40t + 25$

b) $v(t) = 0$

$-9.8t + 40 = 0$

$t = \frac{-40}{-9.8} = 4.08 \text{ sec.}$

$h(4.08) = -4.9(4.08)^2 + 40(4.08) + 25$

$= 106.63 \text{ m}$

c) 8.75 sec

15) 0 to 80 in 6 seconds

$$\frac{80 \text{ miles}}{\cancel{\text{hr}}} \cdot \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} = 0.022 \frac{\text{miles}}{\text{sec}}$$

$$a = \frac{0.022 - 0}{6 - 0} = \frac{11}{3000} \frac{\text{miles}}{\text{sec}^2} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 19.36 \text{ ft}/\text{sec}^2$$

16) $a(t) = -0.6t + 4$ for $0 \le t \le 12$

$v(t) = -0.3t^2 + 4t$

$x(t) = -0.1t^3 + 2t^2$

$x(12) - x(0)$

$-0.1(12)^3 + 2(12)^2 - 0$

115.2 m

$-0.1t^3 + 2t^2 = 100$

$-0.1t^3 + 2t^2 - 100 = 0$

10 seconds