WARMUP

Find a function whose derivative is each of the following:

1) $f'(x) = 3x^2 \Rightarrow f(x) = x^3$

2) $f'(x) = 6x^5 \Rightarrow f(x) = x^6$

3) $g'(x) = x^3 \Rightarrow g(x) = \frac{1}{4}x^4$

4) $h'(x) = \cos x \Rightarrow h(x) = \sin x$

5) $f'(x) = \frac{1}{x} \Rightarrow f(x) = \ln x$

6) $g'(x) = e^x \Rightarrow g(x) = e^x$

Section 6.2 Constructing Antiderivatives Analytically

indefinite integral →

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

it's asking for an antiderivative

$$\int k \, dx = kx + C$$

ex: $$\int (3x^2 + 5x^1 + 3)\, dx = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{5x^2}{2} + 3x + C$$

$$= x^3 + \frac{5}{2}x^2 + 3x + C$$

ex: $$\int \frac{7}{x^2}\, dx = \int 7x^{-2}\, dx = \frac{7x^{-1}}{-1} + C$$

$$= \frac{-7}{x} + C$$

$\frac{8}{5} + 1$

ex: $\int 4\sqrt[5]{x^8}\,dx = \int 4x^{8/5}\,dx = \dfrac{4x^{13/5}}{\frac{13}{5}} + C$

$\sqrt[n]{b^m} = b^{m/n}$

$= \dfrac{5}{13}\cdot\dfrac{4}{1}x^{13/5} + C$

$= \dfrac{20}{13}\sqrt[5]{x^{13}} + C$

ex: $\int \underbrace{(x^2+5)^2}_{\text{FOIL}}\,dx = \int (x^4 + 10x^2 + 25)\,dx$

$= \dfrac{x^5}{5} + \dfrac{10x^3}{3} + 25x + C$

$(x^2+5)(x^2+5)$

$x^4 + 5x^2 + 5x^2 + 25$

ex: $\int \dfrac{x^7 + 3x^5 - 7x^4}{x^2}\,dx = \int \left(\dfrac{x^7}{x^2} + \dfrac{3x^5}{x^2} - \dfrac{7x^4}{x^2}\right)dx$

$= \int (x^5 + 3x^3 - 7x^2)\,dx$

$= \dfrac{x^6}{6} + \dfrac{3x^4}{4} - \dfrac{7x^3}{3} + C$

p271-272 1, 3, 9, 13, 19, 21, 25, 29, 45, 47

9) $\int \frac{1}{z^3}\,dz = \int z^{-3}\,dz = \dfrac{z^{-2}}{-2} + C = \dfrac{1}{-2z^2} + C$

13) $\int\left(t^3 - \frac{1}{2}t^2 - t\right)dt = \frac{t^4}{4} - \frac{1}{2}\cdot\frac{t^3}{3} - \frac{t^2}{2} + C$

$= \frac{t^4}{4} - \frac{t^3}{6} - \frac{t^2}{2} + C$

21) $\int\left(5x - \sqrt{x}\right)dx = \int\left(5x - x^{\frac{1}{2}}\right)dx = \frac{5x^2}{2} - \frac{x^{3/2}}{\frac{3}{2}} + C$

starts in radical form

$= \frac{5x^2}{2} - \frac{2}{3}x^{3/2} + C$

$= \frac{5}{2}x^2 - \frac{2}{3}\sqrt{x^3} + C$

ends in radical form