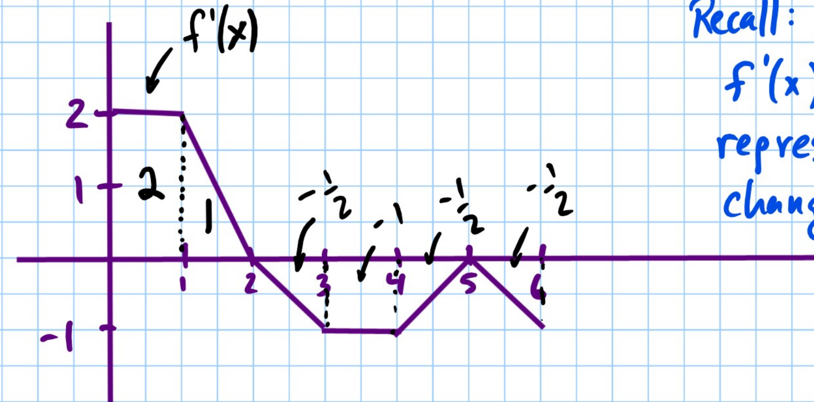
## WARMUP

Fill in the table given the graph of $f'(x)$

| $x$ | $y=f(x)$ |
|---|---|
| 0 | 3 |
| 1 | 5 |
| 2 | 6 |
| 3 | 5.5 |
| 4 | 4.5 |
| 5 | 4 |
| 6 | 3.5 |




Recall: Area under $f'(x)$ on $[a,b]$ represents total change on $[a,b]$.

$$f(1)-f(0)=\underbrace{\int_0^1 f'(x)\,dx}_{\text{Area}}$$

$$f(1)-3=2$$

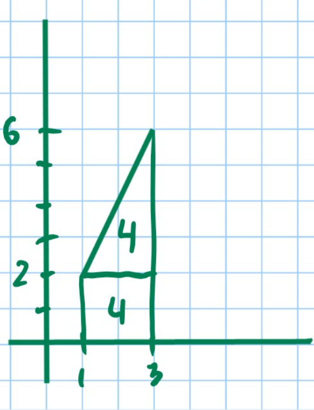
## Section 5.4 Theorems About Definite Integrals

### Theorem 5.1 The Fundamental Theorem of Calculus

If $f$ is continuous on $[a,b]$ and $f(t)=F'(t)$ meaning $F$ is an antiderivative of $f$, then

$$\int_a^b f(t)\,dt = F(b)-F(a)$$

ex: $\int_1^3 2x\,dx = 8$



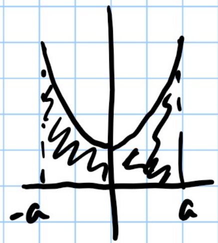

$$\int_1^3 2x\,dx = \left[x^2\right]_1^3 = 3^2-1^2$$

$$=9-1$$

$$=8$$

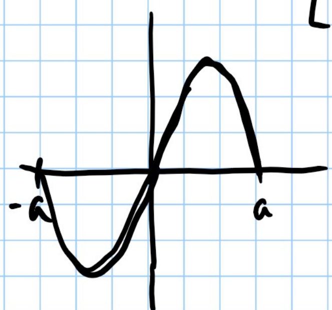
If $f$ is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

y-axis symmetry




If $f$ is odd, then $\int_{-a}^{a} f(x)\,dx = 0$

origin



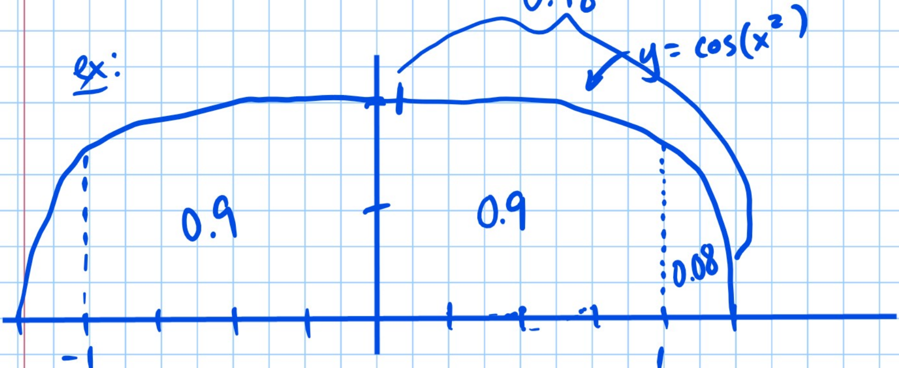

$$\int_{-100}^{100} \sin x = 0$$

**Theorem 5.2** If $a$, $b$, and $c$ are any numbers and $f$ is continuous, then

1) $\displaystyle\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$

2) $\displaystyle\int_{a}^{c} f(x)\,dx = \int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx$

ex:




$$\int_{0}^{1.25} \cos(x^2) = 0.98$$

$$\int_{0}^{1} \cos(x^2) = 0.9$$

a) $\displaystyle\int_{1}^{1.25} \cos(x^2)\,dx = 0.08$

b) $\int_{-1.25}^{1.25} \cos(x^2)\,dx = 2\int_{0}^{1.25} \cos(x^2)\,dx = 2(0.98) = 1.96$

c) $\int_{1.25}^{-1} \cos(x^2)\,dx = -\int_{-1}^{1.25} \cos(x^2)\,dx = -(0.9+0.9+0.08) = -1.88$

Theorem 5.3 Let f and g be continuous and let c be a constant

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

ex: $\int_a^b f(x)\,dx = 8 \qquad \int_a^b g(x) = -2$

$$\int_a^b (f(x) - g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$
$$= 8 - (-2) = 10$$

$$\int_b^a (5g(x))\,dx = -5\int_a^b g(x)\,dx$$
$$= -5(-2) = 10$$

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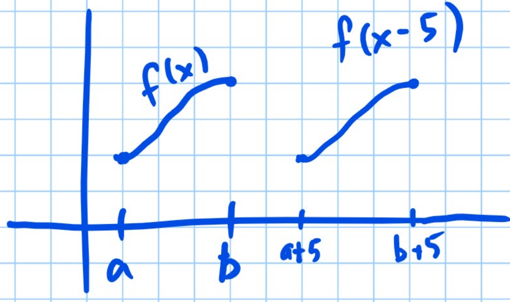
$$\int_{-\pi}^{\pi} x^{1/3}\,dx = 0$$

8) $\int_a^b (c_1 g(x) + (c_2 f(x))^2)\,dx$

$2c_1 + c_2^2 (f(x))^2 = 2c_1 + 12c_2^2$

9)




$$\int_{a+5}^{b+5} f(x-5)\,dx = \int_a^b f(x)\,dx$$

8

22) $$\int_a^a f(x)\,dx = -\int_a^a f(x)\,dx$$

$$2\int_a^a f(x)\,dx = 0$$

$$\int_a^a f(x)\,dx = 0$$