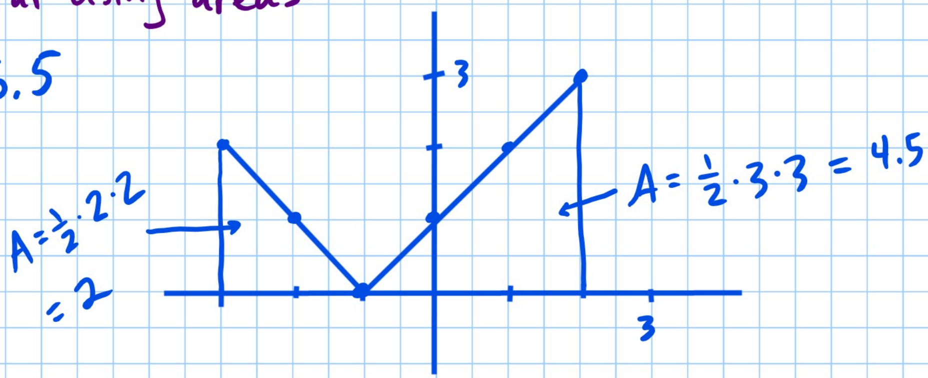
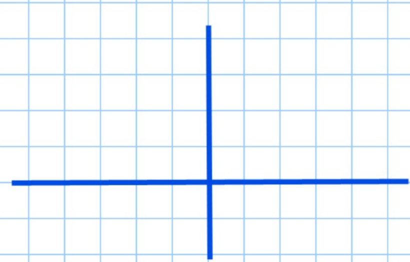
WARMUP

Calculate each integral using areas:

1) $\int_{-3}^{2} |x+1|\, dx = 6.5$

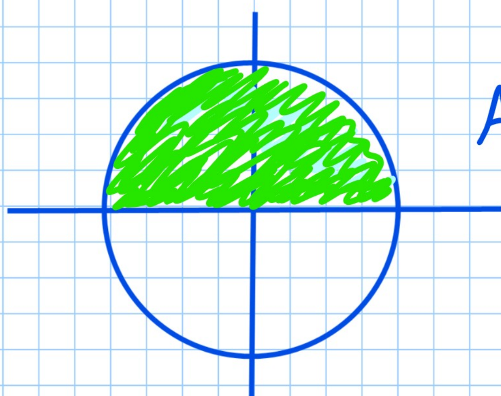


2) $\int_{-4}^{4} \sqrt{16 - x^2}\, dx = 8\pi$



Hint: $x^2 + y^2 = r^2$ is a circle centered at the origin, so $y^2 = r^2 - x^2$ or $y = \sqrt{r^2 - x^2}$ is a semicircle of radius r centered at origin since it's the positive square root.

Circle: $A = \pi r^2$



$A = \frac{\pi 4^2}{2} = \frac{16\pi}{2} = 8\pi$

Section 5.3 Interpretations of the Definite Integral

Definite Integral of a Rate Gives the Total Change

If $v(t)$ is the velocity in ft/sec then

$\int_{1}^{2} v(t)\underline{dt}$ is the total change in distance from $t=1$ to $t=2$ seconds

$\lim_{n\to\infty} \left[\sum_{i=1}^{n} f(t_i)\Delta t\right]$

Units $\int_{1}^{2} v(t)\, dt = \frac{ft}{\cancel{sec}} \cdot \cancel{sec} = ft$

$$\underbrace{F(b) - F(a)} = \text{total change in } F \text{ from } t=a \text{ to } t=b = \underbrace{\int_a^b F'(t)\,dt}$$

p237 #1

$F(0) = 5$ million

Rate is 2^t million bacteria per hour

$$F'(t) = 2^t$$

Increase in first hour

$$F(1) - F(0) = \int_0^1 F'(t)\,dt = \int_0^1 2^t\,dt$$

We use a calculator to compute $\int_0^1 2^t\,dt$

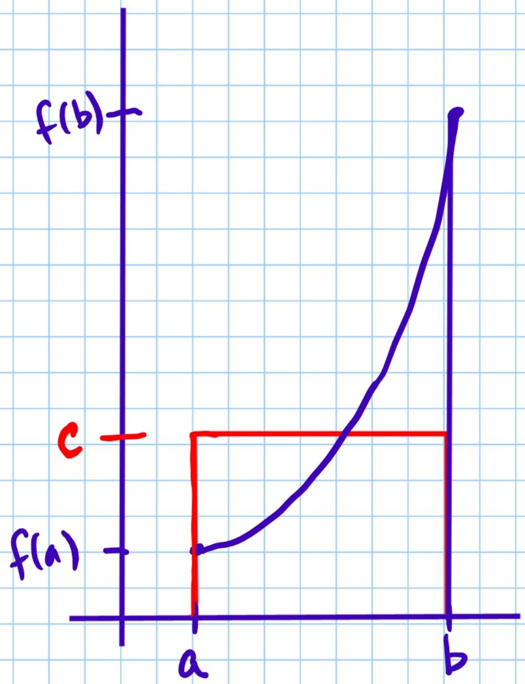
MATH fnInt enter 2^X, X, 0, 1) enter

$$\text{fnInt}(\underset{\text{function}}{2\text{^}X}, \underset{\text{variable}}{X}, \underset{\text{lower limit}}{0}, \underset{\text{upper limit}}{1})$$

$$\int_0^1 2^t\,dt = \underbrace{1.44 \text{ million}}_{\text{total change from } t=0 \text{ to } t=1}$$

so we have $5 + 1.44 = 6.44$ million $= F(1)$

Average Value of a Function



Average value, c, is the y-value that makes the area under the curve equal to the area of the rectangle.

Area Under the Curve — Area of rectangle

$$\frac{1}{b-a}\cdot\int_a^b f(x)\,dx = \cancel{(b-a)}\,c\cdot\frac{1}{\cancel{b-a}}$$

$$\boxed{\text{Avg. value of } f \text{ on } [a,b] = c = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

ex 3 p240

$$P = f(t) = 67.38(1.026)^t$$

t = # of yrs since 1980.

avg. population from 2000 to 2020

$\uparrow$ $t = 20$ $\qquad$ $\uparrow$ $t = 40$

$$\text{avg. pop. on } [20,40] = \frac{1}{40-20}\underbrace{\int_{20}^{40} 67.38(1.026)^t\,dt}_{\text{fnInt}(67.38*1.026^\wedge X,X,20,40)}$$

$$= \frac{1}{20}(2942.66)$$

$$= 147.1 \text{ million}$$