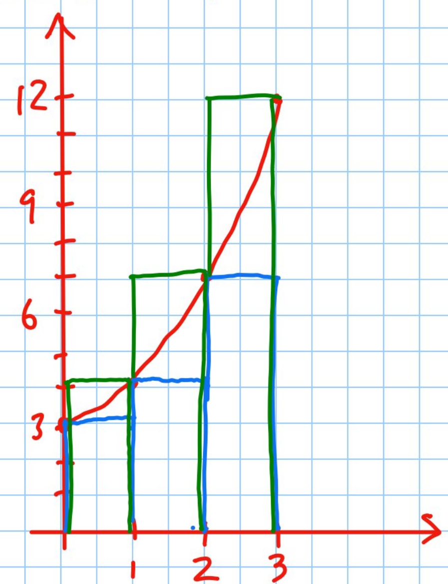
WARMUP

Graph $f(t) = t^2 + 3$ on the interval $[0, 3]$.

Calculate the left-hand and right-hand sums using 3 intervals.

STEP 1: Graph it.



left-hand sums:

$= 1 \cdot 3 + 1 \cdot 4 + 1 \cdot 7 = 14$

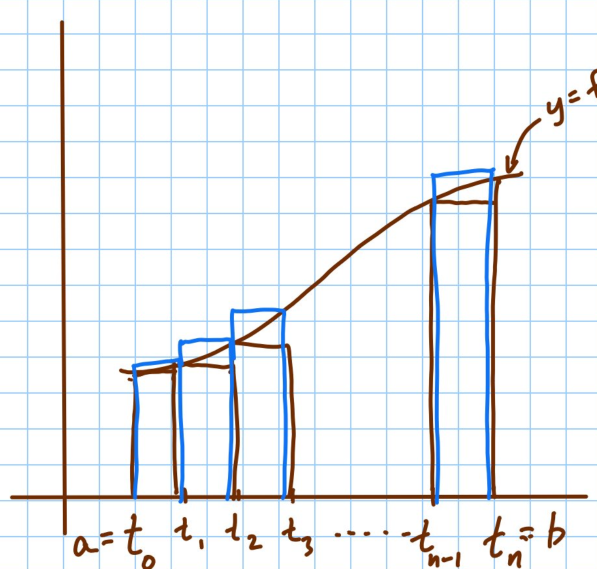
under

right-hand sums:

$= 1 \cdot 4 + 1 \cdot 7 + 1 \cdot 12$

$= 23$ over

Section 5.2 The Definite Integral



$\Delta t = t_i - t_{i-1}$

Left-hand sum:

$= \lim_{n\to\infty} \left[f(t_0)\Delta t + f(t_1)\Delta t + \ldots + f(t_{n-1})\Delta t \right]$

$= \lim_{n\to\infty} \left[\sum_{i=0}^{n-1} f(t_i)\Delta t \right]$

Right-hand sum:

$= \lim_{n\to\infty} \left[f(t_1)\Delta t + f(t_2)\Delta t + \ldots + f(t_n)\Delta t \right]$

$= \lim_{n\to\infty} \left[\sum_{i=1}^{n} f(t_i)\Delta t \right]$

The definite integral is $\int_a^b f(t)\,dt$. f is continuous on $[a, b]$. $\int$ is the integral sign. a is the <u>lower limit</u>

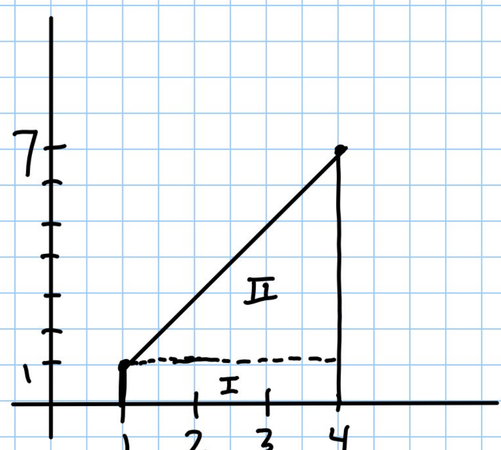
b is the <u>upper limit of integration</u>. If $f(t) \geq 0$ on $[a, b]$ then $\int_a^b f(t)\,dt$ is the <u>area under the curve</u>

from $t = a$ to $t = b$.

$$\text{So} \quad \int_a^b f(t)\,dt = \underbrace{\lim_{n\to\infty}\left[\sum_{i=0}^{n-1} f(t_i)\,\Delta t\right] = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(t_i)\,\Delta t\right]}_{\text{Riemann}}$$

ex: $\int_1^4 (2x-1)\,dx \Rightarrow$ Area under $f(x) = 2x-1$ from $x = 1$ to $x = 4$

① STEP 1: Graph the function on the interval



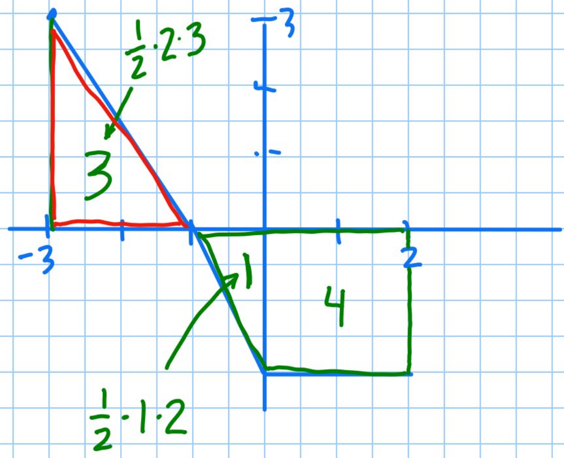
② Calculate the area between $f(x) = 2x-1$, $x = 1$, $x = 4$ and the x-axis.

Area of I: $A = 1 \cdot 3 = 3$

Area of II: $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 3 \cdot 6 = 9$

$$\text{So} \quad \int_1^4 f(x)\,dx = 12$$

If $f(x) \le 0$ on $[a,b]$ then $\int_a^b f(x)\,dx$ is the negative of the area between the curve and the x-axis.



$$\int_{-3}^{2} f(x)\,dx = -2$$

$$\int_{-3}^{-1} f(x) = 3$$

$$\int_{-1}^{2} f(x) = -5$$

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sums, find the difference between the right- and left-hand Riemann sums.

30. Use Figure 5.26 to find the values of

(a) $\int_a^b f(x)\,dx$ = 13 (b) $\int_b^c f(x)\,dx$ = −2

(c) $\int_a^c f(x)\,dx$ = 11 (d) $\int_a^c |f(x)|\,dx$ = 15

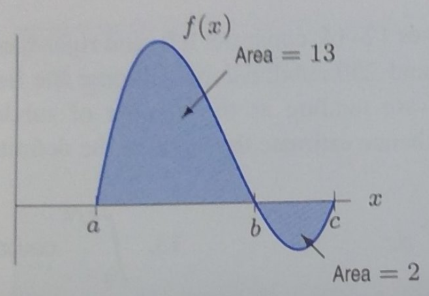

Figure 5.26

31. Given $\int_{-2}^{0} f(x)dx = 4$ and Figure 5.27, estimate:

(a) $\int_0^2 f(x)dx$ = −4 (b) $\int_{-2}^{2} f(x)dx$ = 0

(c) The total shaded area = 8

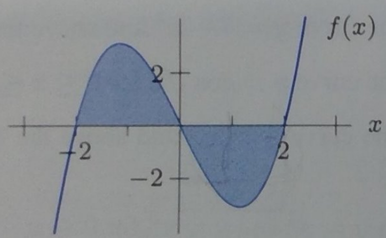

Figure 5.27

32. (a) Using Figure 5.28, find $\int_{-3}^{0} f(x)\,dx$. = −2

(b) If the area of the shaded region is A, estimate $\int_{-3}^{4} f(x)\,dx$. = $-\frac{1}{2}A$

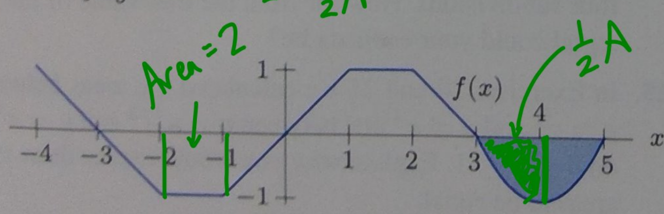

Figure 5.28