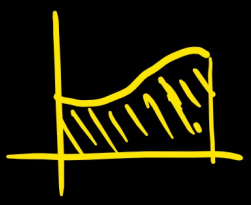
CALCULUS B

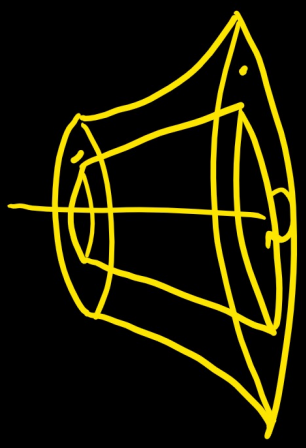
Last Semester - Differential Calculus

↳ calculating derivative
- slope of tangent line
- rate of change
- optimization

This Trimester - Integral Calculus

↳ calculating antiderivatives
- integrals (definite and indefinite)
- area under a curve
- volume



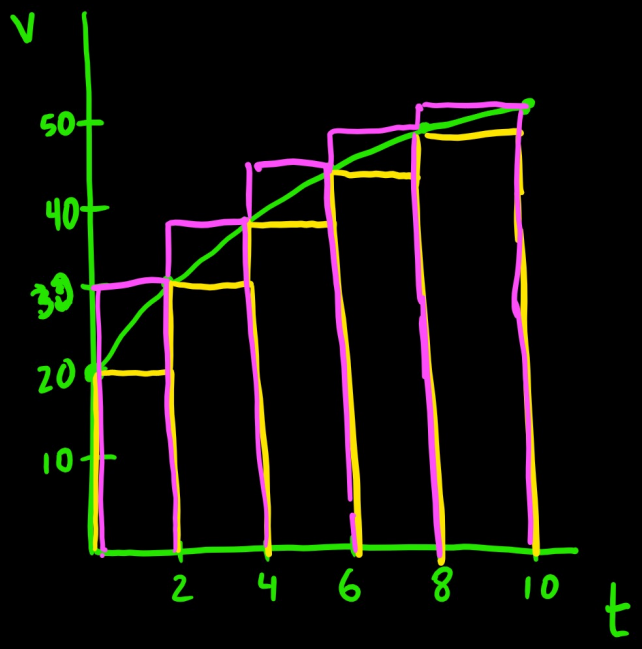


Section 5.1 How Do We Measure Distance Traveled?

distance = rate × time

Car moving with increasing velocity

Time (in sec)	0	2	4	6	8	10
Velocity (ft/sec)	20	30	38	44	48	50



Since distance = rate × time we can use area under curve to calculate total distance traveled.

The best we can do is estimate that area.

Pink Rectangles (Circumscribed)

Area = $2 \cdot 30 + 2 \cdot 38 + 2 \cdot 44 + 2 \cdot 48 \cdot 2 \cdot 50$

= 420 ft

overestimate

right-hand sum

Yellow Rectangles (Inscribed)

Area = $2 \cdot 20 + 2 \cdot 30 + 2 \cdot 38 + 2 \cdot 44 + 2 \cdot 48$

= 40 + 60 + 76 + 88 + 96

= 360 ft.

underestimate

Left-hand sum

In our example we measured velocity every 2 seconds (width of rectangle). Note that the difference between upper (420 ft) and lower (360 ft) estimates can be found by

$(50 - 20)\,2 = 60$ ft ← difference in estimates

50 ↑ greatest velocity

20 ↑ lowest velocity

2 ← width of rectangle

How frequently must we measure velocity for estimates to be within a foot?

$$(50-20)t = 1$$

$$30t = 1$$

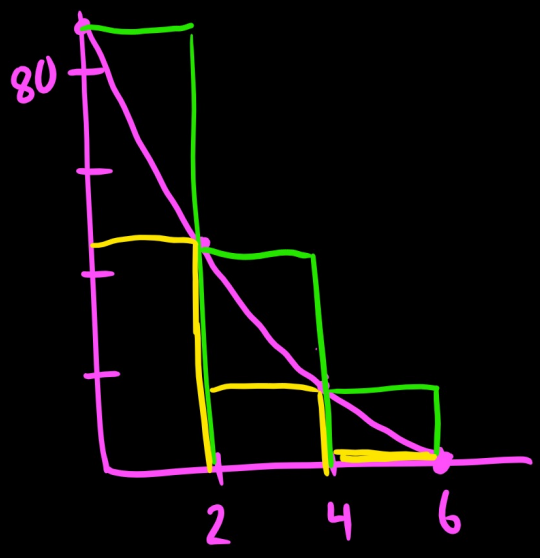
$$t = \frac{1}{30} \text{ of a second}$$

$\Rightarrow$ 300 rectangles

These examples are meant to show that if n = # of rectangles, then as $n \to \infty$ we get the true distance traveled, the true area under the curve.

p227-228 2, 3, 5, 6, 10, 11

2)



overestimate = $2 \cdot 88 + 2 \cdot 45 + 2 \cdot 16$

= 298 ft

underestimate = $2 \cdot 45 + 2 \cdot 16 + 2 \cdot 0$

= 122 ft

3)

Time	0	0.25	0.5	0.75	1	1.25	1.5
Speed	12	11	10	10	8	7	0

Left-hand over

Right-hand

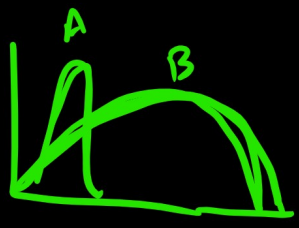
a) over : $12(0.25) + 11(0.25) =$

under : $11(0.25) + 10(0.25) =$

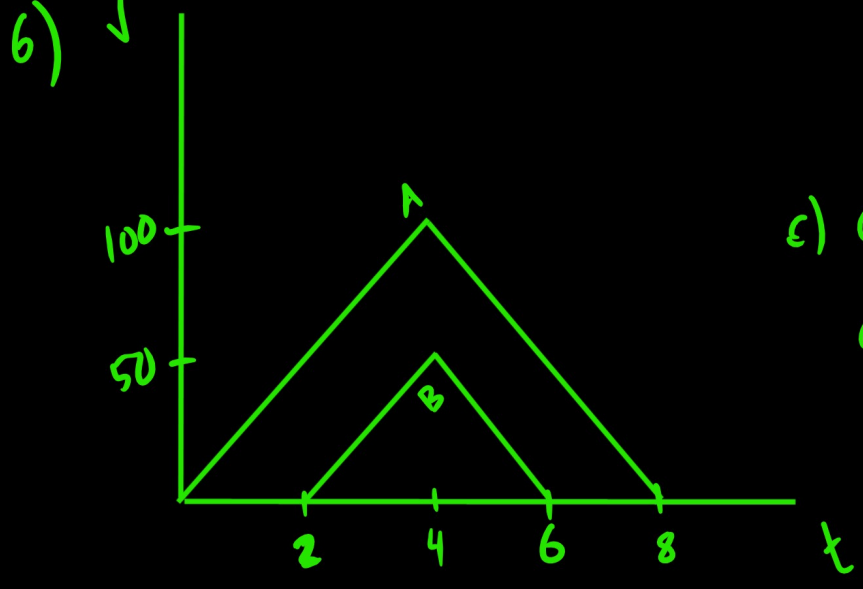
b) over : $0.25(12 + 11 + 10 + 10 + 8 + 7) =$

under : $0.25(11 + 10 + 10 + 8 + 7 + 0) =$

5)



6)



c) car A: $\frac{1}{2} \cdot 8 \cdot 100 = 400$ km

car B: $\frac{1}{2} \cdot 4 \cdot 50 = 100$ km