

## Section 8.5 The Dot Product

The product of 2 vectors is called the dot product

If  $v = a_1i + b_1j$  and  $w = a_2i + b_2j$  then  
 $v \cdot w = a_1a_2 + b_1b_2$

ex 1 p 630  $v = 2i - 3j$   $w = 5i + 3j$

a)  $v \cdot w = 2 \cdot 5 + (-3) \cdot 3 = 10 + (-9) = 1$

b)  $w \cdot v = 5 \cdot 2 + 3(-3) = 10 + (-9) = 1$

c)  $v \cdot v = 2 \cdot 2 + (-3)(-3) = 4 + 9 = 13$

d)  $\|v\| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

## Properties of Dot Product

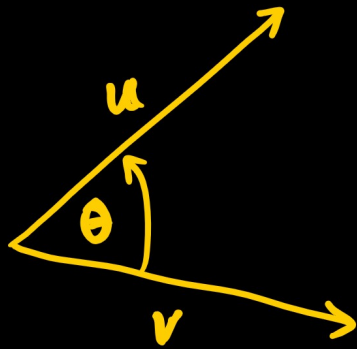
Commutative:  $v \cdot w = w \cdot v$

Distributive:  $u \cdot (v + w) = u \cdot v + u \cdot w$

$$v \cdot v = \|v\|^2$$

$$0 \cdot v = 0$$

The Dot Product can be used to find the angle between 2 vectors.



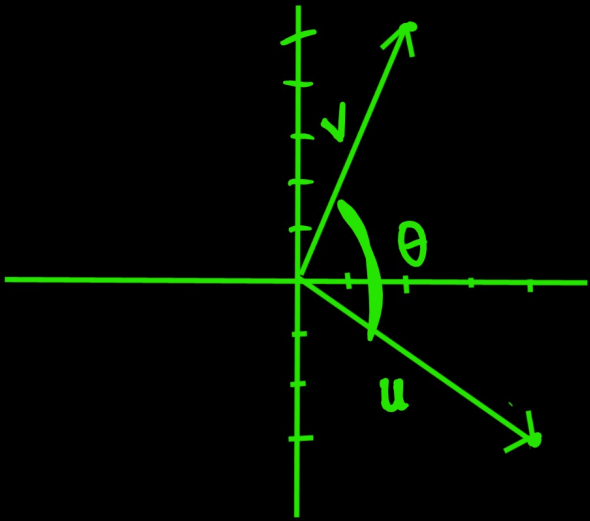
$$\cos \theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

ex:  $u = 4i - 3j$   
 $v = 2i + 5j$

$$u \cdot v = 4 \cdot 2 + (-3) \cdot 5 = -7$$

$$\|u\| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\|v\| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$$



$$\cos \theta = \frac{-7}{5\sqrt{29}}$$

$$\theta = \cos^{-1} \left( \frac{-7}{5\sqrt{29}} \right)$$

$$\theta = 105^\circ$$

Degree Mode

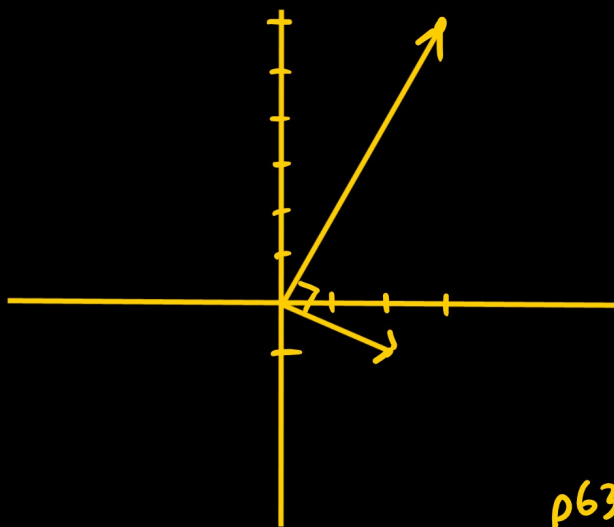
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$v$  and  $w$  are parallel if there's a scalar  $\alpha$  such that  $v = \alpha w$

ex:  $v = 3i - j$  and  $w = 6i - 2j$  are parallel because  $v = \frac{1}{2}w$

$v$  and  $w$  are orthogonal (perpendicular) if  $v \cdot w = 0$

ex:  $v = 2i - j$  and  $w = 3i + 6j$

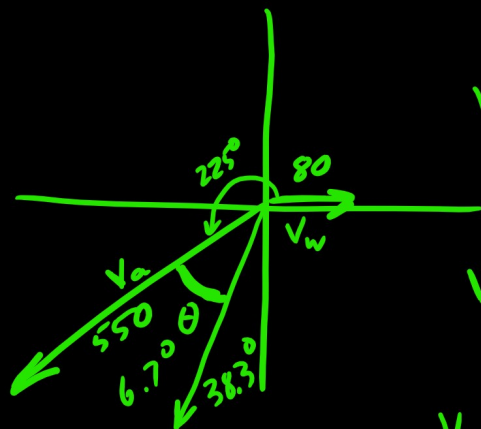


$$\begin{aligned}v \cdot w &= 2 \cdot 3 + (-1) \cdot 6 \\&= 6 + (-6) \\&= 0\end{aligned}$$

p636 - 637

1-11 odd, 19 (see ex on pg 632)

19)



$$\begin{aligned}v_a &= 550(\cos 225^\circ i + \sin 225^\circ j) \\&= -388.9 i - 388.9 j\end{aligned}$$

$$v_w = 80 i$$

$$v_s = v_a + v_w = -308.9 i - 388.9 j$$

$$\|v_s\| = \sqrt{(-308.9)^2 + (-388.9)^2} = 496.7 \text{ mph}$$

$$\cos \theta = \frac{271342.42}{550 \cdot 496.7} = \frac{271342.42}{273185}$$

$$\theta = \cos^{-1}\left(\frac{271342.42}{273185}\right) = 6.7^\circ$$

496.7 mph  
S 38.3° W