

WARMUP

Calculate $\|v\|$ for

$$1) v = 7i - 24j$$

$$\begin{aligned}\|v\| &= \sqrt{7^2 + (-24)^2} \\ &= \sqrt{49 + 576} = \sqrt{625} = 25\end{aligned}$$

$$2) v = 40i + 9j$$

$$\begin{aligned}\|v\| &= \sqrt{40^2 + 9^2} \\ &= \sqrt{1600 + 81} = \sqrt{1681} \\ &= 41\end{aligned}$$

$$3) v = -3i + 7j$$

$$\begin{aligned}\|v\| &= \sqrt{(-3)^2 + 7^2} = \sqrt{9 + 49} \\ &= \sqrt{58}\end{aligned}$$

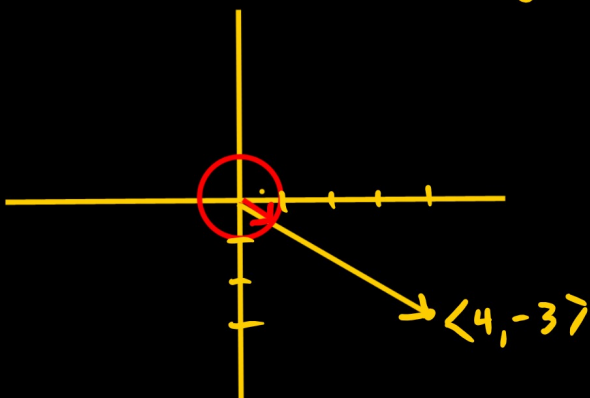
Section 8.4 (Continued and Concluded)

For any nonzero v , the vector $u = \frac{v}{\|v\|}$ is a

unit vector that has the same direction as v and a magnitude of 1.

ex: Find the unit vector in the same direction

as $v = 4i - 3j$



$$\begin{aligned}\|v\| &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

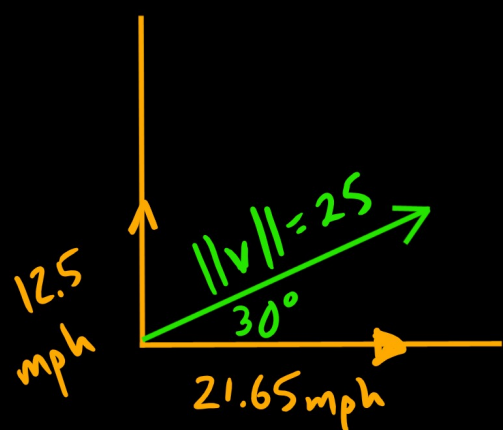
$$u = \frac{v}{\|v\|} = \frac{4i - 3j}{5} = \frac{4}{5}i - \frac{3}{5}j$$

Often the direction of vector v is given by the angle v makes with the positive x -axis.

If the angle is α , then v can be expressed by

$$v = \|v\| (\cos \alpha i + \sin \alpha j)$$

ex: You throw a ball at 25 mph so the angle is 30° with the positive x -axis.

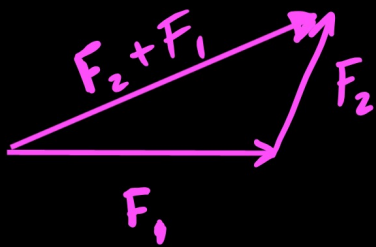


$$v = 25 (\cos 30^\circ i + \sin 30^\circ j)$$

$$v = 25 (0.866 i + 0.5 j)$$

$$v = 21.65 i + 12.5 j$$

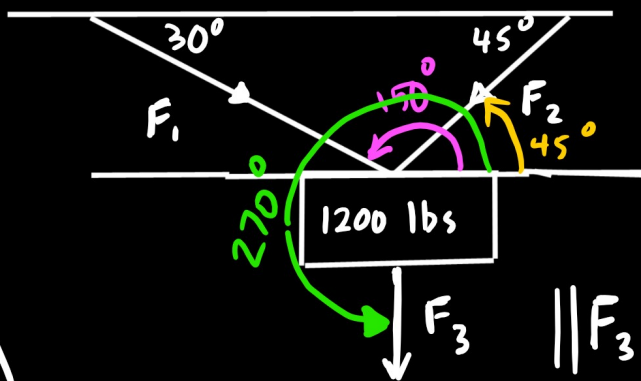
In Physics, force F is a vector. If 2 different forces F_1 and F_2 work on an object, the resultant force is $F = F_1 + F_2$



An object is in equilibrium if

- 1) the sum of all forces is 0
- 2) the object is at rest.

ex:



What are magnitudes of F_1 and F_2 ?

$$x = \|F_1\|$$

$$y = \|F_2\|$$

$$v = \|v\| (\cos \alpha i + \sin \alpha j)$$

$$\|F_3\| = 1200 \text{ lbs}$$

$$F_1 = x (\cos 150^\circ i + \sin 150^\circ j) = -0.866x i + 0.5x j$$

$$F_2 = y (\cos 45^\circ i + \sin 45^\circ j) = 0.707y i + 0.707y j$$

$$F_3 = 1200 (\cos 270^\circ i + \sin 270^\circ j) = -1200 j$$

$$F_1 + F_2 + F_3 = 0$$

$$= 0i + 0j$$

$$-0.866x + 0.707y = 0$$

$$0.5x + 0.707y - 1200 = 0$$

$$-0.866x + 0.707y = 0$$

$$- (0.5x + 0.707y = 1200)$$

subtract

$$-1.366x = -1200$$

$$x = 878.5$$

$$-0.866(878.5) + 0.707y = 0$$

$$-760.78 + 0.707y = 0$$

$$0.707y = 760.78$$

$$y = 1076.1$$

$$\|F_1\| = 878.5 \text{ lbs}$$

$$\|F_2\| = 1076.1 \text{ lbs}$$

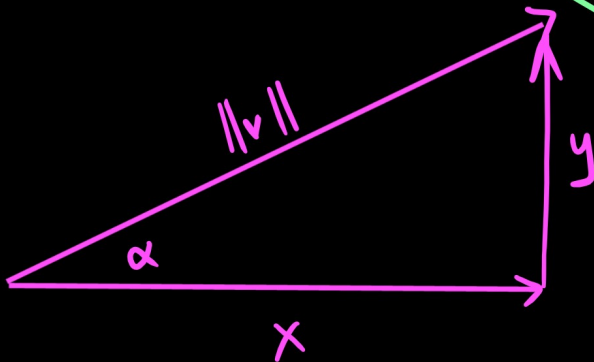
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41, 42, 53, 59

$$\begin{aligned} 59) \quad x(\cos 155^\circ i + \sin 155^\circ j) &= -.906x i + .422x j \\ y(\cos 40^\circ i + \sin 40^\circ j) &= .766y i + .643y j \\ 1000(\cos 270^\circ i + \sin 270^\circ j) &= -1000 j \end{aligned}$$

$$-0.906x + 0.766y = 0$$

$$0.422x + 0.643y - 1000 = 0$$



$$\cos \alpha = \frac{x}{\|v\|}$$

$$\|v\| \sin \alpha = y$$

$$\|v\| \cos \alpha = x$$

$$\begin{aligned} 0.766y &= 0.906x \\ y &= 1.183x \end{aligned}$$

$$y = 1.183(845.3)$$

$$y = 1000$$

$$\|F_1\| = 845.3 \text{ lbs}$$

$$\|F_2\| = 1000 \text{ lbs}$$

$$0.422x + 0.643(1.183x) = 1000$$

$$0.422x + 0.761x = 1000$$

$$1.183x = 1000$$

$$x = 845.3$$

