WARMUP

Use Desmos to find horizontal asymptotes

1) $f(x) = \frac{3x^2-4}{x^2+5x+4}$

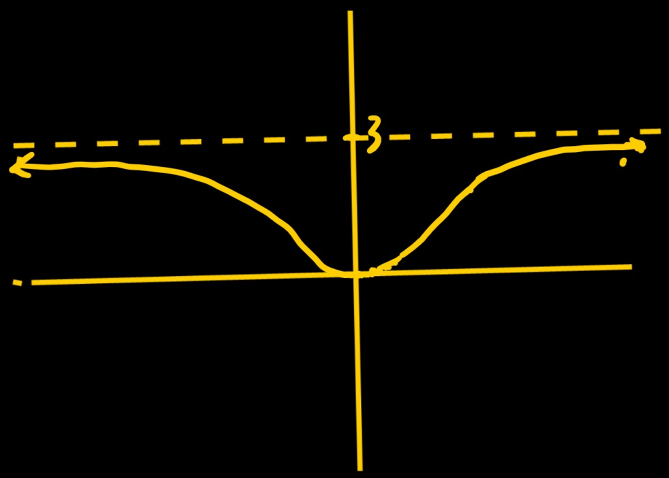
$y=3$

2) $f(x) = \frac{\sqrt{4x^2+1}}{x+2}$

$y=\pm 2$

Section 3.5 Limits at Infinity

Consider the graph of $f(x) = \frac{3x^2}{x^2+1}$



It has a horizontal asymptote at $y=3$ (Ratio of the leading coefficients)

In Calculus the H.A.s can be thought of as limits at infinity.

So $\lim_{x\to\infty} \frac{3x^2}{x^2+1} = 3$ and $\lim_{x\to-\infty} \frac{3x^2}{x^2+1} = 3$

In Calculus, instead of using the HA. rules from PreCalc, we multiply top and bottom by one over the highest power of x in the problem.

ex: $\lim_{x\to\infty} \frac{(3x^2)}{(x^2+1)} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{3}{1+\frac{1}{x^2}} = \frac{3}{1+0} = 3$

ex: $\lim_{x\to-\infty} \left(5-\frac{2}{x^2}\right) = \lim_{x\to-\infty} 5 - \lim_{x\to-\infty} \frac{2}{x^2} = 5-0 = 5$

ex: $\lim_{x\to\infty} \frac{(2x^3+5)}{(3x^2+1)} \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{2+\frac{5}{x^3}}{\frac{3}{x}+\frac{1}{x^3}} = \frac{2+0}{0+0}$

so limit does not exist $= \frac{2}{0}$

d.n.e.

(no H.A.)

ex: $\lim_{x\to-\infty} \frac{3x-2}{\sqrt{2x^2+1}} \cdot \frac{-\frac{1}{x}}{\sqrt{\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-3+\frac{2}{x}}{\sqrt{2+\frac{1}{x^2}}} = \frac{-3+0}{\sqrt{2+0}} = \frac{-3}{\sqrt{2}}$

if $x\to-\infty$ and you have $\sqrt{\ }$ put a $-$ with the term not under the square root

but $x<0$ so

$\frac{1}{x}$ and $\sqrt{\frac{1}{x^2}}$ are opposites

$\sqrt{\frac{1}{(-5)^2}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$

$\frac{1}{x} = -\frac{1}{5} \quad \sqrt{\frac{1}{x^2}} = \frac{1}{5}$

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9-15 odd, 19-23 odd

15) $\lim_{x \to \infty} \left(\frac{x^2}{x^2} \cdot \frac{2x}{1} - \frac{1}{x^2} \right) = \lim_{x \to \infty} \frac{(2x^3 - 1)}{x^2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to \infty} \frac{2 - \frac{1}{x^3}}{\frac{1}{x}}$

$= \frac{2-0}{0}$ so DNE

21) $\lim_{x \to \infty} \frac{2x+1}{\sqrt{x^2 - x}} \cdot \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2}}} = \lim_{x \to \infty} \frac{2 + \frac{1}{x}}{\sqrt{1 - \frac{1}{x}}} = \frac{2+0}{\sqrt{1-0}} = 2$

x is positive, so $\sqrt{\frac{1}{x^2}} = \frac{1}{x}$