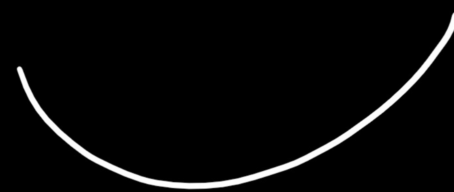
Section 3.4 Concavity and Points of Inflection

Recall:

concave down

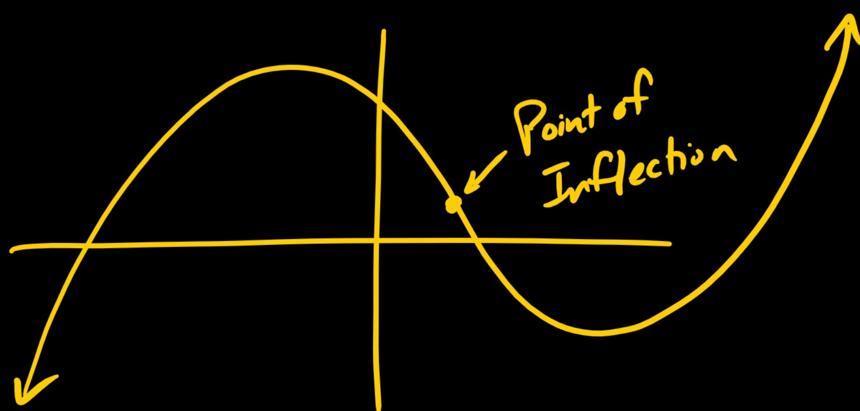
$f'' < 0$



concave up

$f'' > 0$

A point of inflection is a point where the graph changes concavity



To find points of inflection we start by finding possible inflection points (PIPs) by finding domain values for which $f''(x) = 0$ or $f''(x)$ is undefined. Then we use the same number line test that we used for first derivative.

ex: Find extrema and points of inflection for

$f(x) = x^4 - 4x^3$

Extrema: $f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$4x^2 = 0 \qquad x - 3 = 0$

$x = 0 \qquad x = 3$

-1		2		4
$+\cdot-$	0	$+\cdot-$	3	$+\cdot+$
$=-$		$=-$		$=+$

min @ $(3, f(3)) = (3, -27)$

$f(3) = 3^4 - 4\cdot 3^3$

IPs: $f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \qquad x = 2$

PIPs are $x = 0, x = 2$

-1		1		3
$-\cdot-$	0	$+\cdot-$	2	$+\cdot+$
$=+$		$=-$		$=+$

IPs: $(0, f(0)) = (0, 0)$

$(2, f(2)) = (2, -16) \leftarrow 2^4 - 4\cdot 2^3$

p194-195 use desmos to graph
or calculator

21, 23, 25, 43, 56

↑

BONUS

21) $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12 = 3(x^2-4)$

$= 3(x+2)(x-2)$

$x = -2$, $x = 2$

-3	0	3
$- \cdot -$	$+ \cdot -$	$+ \cdot +$
$+$	$= -$	$= +$

(ticks at -2 and 2)

$(-2)^3 - 12(-2)$

max @ $(-2, f(-2)) = (-2, 16)$

min @ $(2, f(2)) = (2, -16)$

$2^3 - 12 \cdot 2$

IPs: $f''(x) = 6x$

$x = 0$

$-$ | $+$

0

IP @ $(0, f(0)) = (0, 0)$

23) $f(x) = x^3 - 6x^2 + 12x - 8$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

Extrema:

$3x^2 - 12x + 12 = 0$

$3(x^2 - 4x + 4) = 0$

$\cancel{3}(x-2)(x-2) = 0$

critter is $x = 2$

0 | 4

$- \cdot -$ 2 $+ \cdot +$

$+$ $+$

no extrema

IPs:

$6x - 12 = 0$

$6x = 12$

$x = 2$

PIP $x = 2$

0 | 4

$-$ 2 $+$

$8 - 24 + 24 - 8$

IP @ $(2, f(2)) = (2, 0)$

-3 | -1 | 1 | 3

-2 0 2

$(-2,-4)$ $(0,0)$ $(2,-4)$

max

$\frac{1}{4}x^4 - 2x^2$

mins

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x = \pm\sqrt{\frac{4}{3}}$

PIPs $x = \pm 1.15$

$(-1.15, \underline{-2.21}), (1.15, \underline{2.21})$

$3x^2-4 \longrightarrow$

-2		0		2
	-1.15		1.15	
$3(-2)^2-4$		$3\cdot 0^2-4$		$3\cdot 2^2-4$
$+$		$-$		$+$
$\cup$		$\cap$		$\cup$

$$\frac{1}{4}(-1.15)^4 - 2(-1.15)^2 = -2.21$$

43) $f(2) = f(4) = 0$

$f'(x) < 0$ if $x < 3$ decreasing

$f'(3)$ is undefined vertical tangent

$f'(x) > 0$ if $x > 3$ increasing

$f''(x) < 0$ if $x \neq 3$ concave down