WARMUP

The John Deere company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. If the revenue R is

$$R(p) = -\frac{1}{2}p^2 + 1900p$$

parabola opening down

what unit price should be charged to maximize revenue? What is the maximum revenue?

$$R'(p) = -p + 1900 = 0$$

$$1900 = p$$

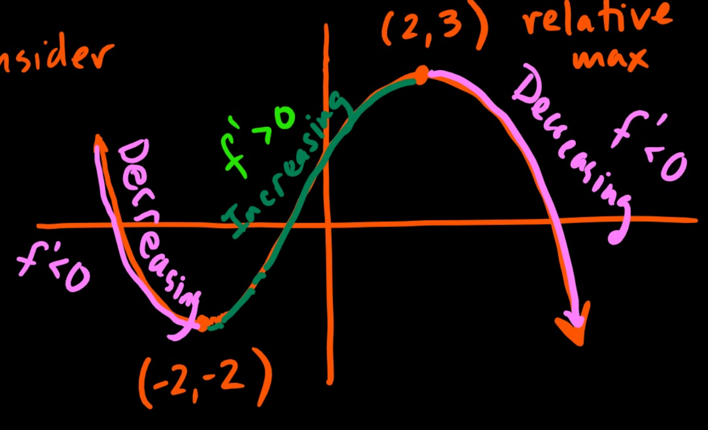
price = \$1900 = p

$$R(1900) = -\frac{1}{2}(1900)^2 + 1900(1900)$$

$$= \$1,805,000$$

Section 3.3 The First Derivative Test

Consider



when f' changes from positive to negative, we have a relative max

when f' changes from negative to positive, we have a relative min

The First Derivative Test

Let c be a critical number of f. [$f'(c) = 0$ or where $f'(c)$ is undefined]

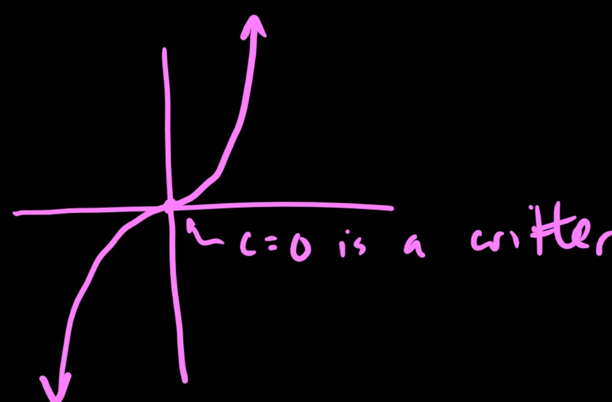
1) If f' changes from negative to positive at $x = c$ then $(c, f(c))$ is a <u>relative min.</u>

2) If f' changes from positive to negative at $x = c$, then $(c, f(c))$ is a <u>relative max</u>

There are cases when a critical number does not yield a max or min:

<u>ex</u>: $f(x) = x^3$

$f'(x) = 3x^2$



<u>ex</u>: Find the relative extrema of $f(x) = x^4 - 4x^3$

① Find critters:

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$4x^2(x-3) = 0$$

$$4x^2 = 0 \qquad x - 3 = 0$$

$$x = 0 \qquad x = 3$$

② Use a number line to determine signs of derivative on the intervals determined by critters.

Test Values $\longrightarrow$ -1 $\quad$ **0** $\quad$ 1 $\quad$ **3** $\quad$ 4

Plug test values into f' to determine the sign of f'

$x<0$	$0<x<3$	$x>3$
$4(-1)^2(-1-3)$	$4(1)^2(1-3)$	$4\cdot 4^2(4-3)$
$+\cdot+\cdot-$	$+\cdot+\cdot-$	$+\cdot+\cdot+$
$=-$	$=-$	$=+$
$\searrow$	$\searrow$	$\nearrow$

③ Use first derivative test to determine maxima and minima.

f' does not change sign at $x=0$, so $(0, f(0))$ is not a max or min, but it has a horizontal tangent.

f' changes from $-$ to $+$ at $x=3$, so $(3, f(3))$ is a relative min

$= (3, -27)$

$f(3) = 3^4 - 4\cdot 3^3 = 81 - 108$

$= -27$

ex: $f(x) = x^3 - \frac{3}{2}x^2$

① Find critters: $f'(x) = 3x^2 - 3x$

$= 3x(x-1) = 0$

$x = 0 \quad x = 1$

② Number Line:

-1	0	0.5	1	2
$-\cdot-$		$+\cdot-$		$+\cdot+$
$=+$		$=-$		$=+$

$$f'(x) = 3x(x-1)$$

③ Min and max:

$$\max @ (0, f(0)) = (0, 0)$$

$$\min @ (1, f(1)) = \left(1, -\tfrac{1}{2}\right)$$

$$f(1) = 1^3 - \frac{3}{2} \cdot 1^2 = 1 - \frac{3}{2} = -\frac{1}{2}$$

$$f(x) = -2x^2 + 4x + 3$$

$$f(x) = \frac{1}{2}x + \cos x$$

on $[0, 2\pi)$