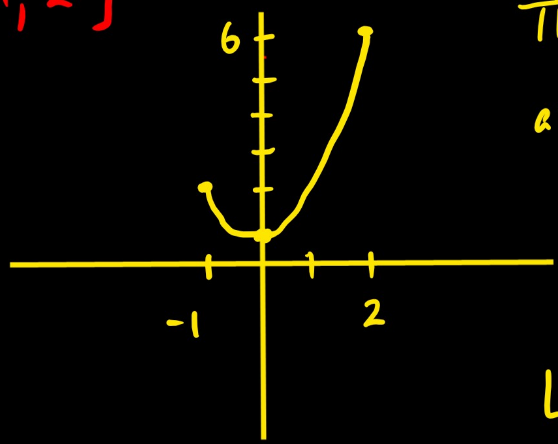
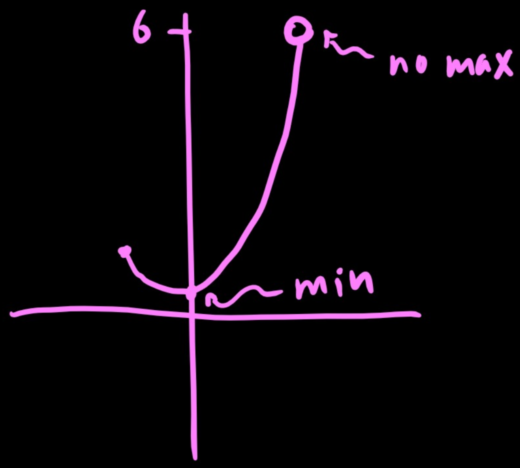
Section 3.1 Extrema on an Interval

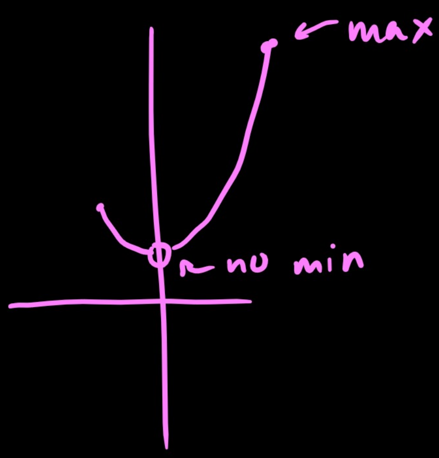
Consider the following graph on the interval $[-1, 2]$



This graph has a minimum at $(0,1)$ since it's the lowest y-coordinate of any point.

Likewise, the graph has a maximum at $(2,6)$ since it has the highest y-coordinate of any point.

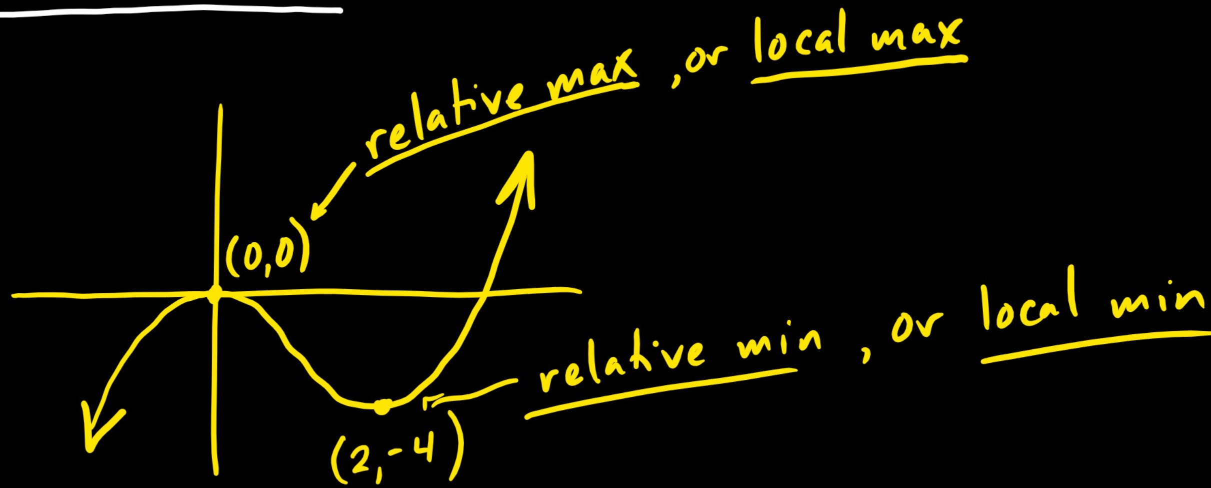




A max or a min is called an <u>extremum</u>. The lowest point on an interval is the <u>absolute min</u>. The highest point is the <u>absolute max</u>.

If $f(x)$ is continuous on the closed interval $[a,b]$ then f has a min and a max on that interval.

Extrema on a closed interval must happen at an endpoint, or at $x = c$ such that $f'(c) = 0$ or $f'(c)$ is undefined. Such a c is called a <u>critical number</u> of f.



<u>To find Absolute Extrema on a $[a, b]$</u>

1) Find the critical numbers in (a, b)
2) Evaluate f at those critters.
3) Evaluate f at each endpoint
4) The greatest of these numbers is the max
   The least of these numbers is the min.

<u>ex 2 p168</u> $f(x) = 3x^4 - 4x^3$ on $[-1, 2]$

1) $f'(x) = 12x^3 - 12x^2$

$$= 12x^2(x - 1) = 0$$

$$12x^2 = 0 \qquad x - 1 = 0$$

$$x = 0 \qquad x = 1$$

4) $f(1) = -1$ is min

$f(2) = 16$ is max

2) $f(0) = 0$

$f(1) = 3 \cdot 1^4 - 4 \cdot 1^3 = -1$

3) $f(-1) = 3(-1)^4 - 4(-1)^3 = 7$

$f(2) = 3 \cdot 2^4 - 4 \cdot 2^3 = 48 - 32 = 16$

ex 3 p169 $\quad f(x) = 2x - 3x^{2/3}$ on $[-1, 3]$

$f'(x) = 2 - 2x^{-1/3} = 2 - \frac{2}{x^{1/3}}$ undefined when $x = 0$

use calc.

$f(0) = 0 \leftarrow$ max

$f(1) = -1$

$f(-1) = -5 \leftarrow$ min

$f(3) = -0.24$

$2 - \frac{2}{x^{1/3}} = 0$

$+\frac{2}{x^{1/3}} = +2$

$2 = 2x^{1/3}$

$1^3 = (x^{1/3})^3$

$1 = x$

p170-171 $\quad$ 7, 9, 13, 17, 29, 31, 44

7) $f(x) = x^2(x-3) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

$x = 0$, $x = 2$ are critters

9) $f(t) = t(4-t)^{1/2}$

$f'(t) = t \cdot \frac{1}{2}(4-t)^{-1/2} + (4-t)^{1/2} \cdot 1$

$f'(t) = \underbrace{\frac{-t}{2\sqrt{4-t}}}_{\text{undefined when } t=4} + \sqrt{4-t} = 0$

$\frac{-t}{2\sqrt{4-t}} = -\sqrt{4-t}$

$t = 2(4-t)$

$3t = 8$

$t = 8/3$

44) $C = 2x + \frac{300000}{x} = 2x + 300000x^{-1}$

$C' = 2 - 300000x^{-2} = 0$

$1 \leq x \leq 300$

$\frac{-300000}{x^2} = -2$

$-300000 = -2x^2$

$x^2 = 150000$

$x = 387$

$C(1) = 2 \cdot 1 + \frac{300000}{1} = 300{,}002$

$C(300) = 2 \cdot 300 + \frac{3000\cancel{00}}{3\cancel{00}} = 600 + 1000$

$= \$1600$