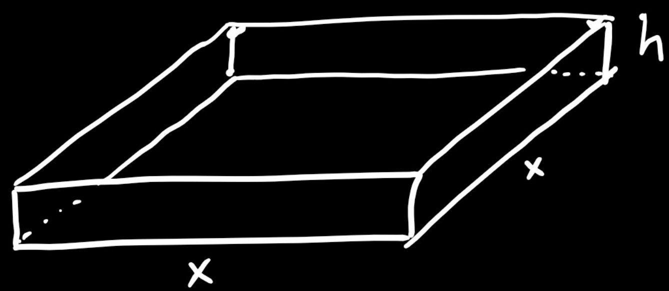
Section 3.7 Optimization Application Problems

ex: A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box of maximum volume?

STEP 1: If applicable, draw a picture. Define variables.



STEP 2: Write a primary equation - equation that uses what we're trying to optimize.

$$V = x^2 h$$ trying to maximize volume.

STEP 3: Write a constraint equation - an equation that relates the quantities that we know with the variables.

We know surface area is 108 in^2

$$x^2 + 4xh = 108$$

STEP 4: Solve the constraint equation for one of the variables and substitute for that variable in the primary equation.

Solve for h: $4xh = 108 - x^2$

$$h = \frac{108 - x^2}{4x}$$

$$h = \frac{108}{4x} - \frac{x^2}{4x}$$

$$h = \frac{27}{x} - \frac{x}{4}$$

$$V = \frac{x^2}{1}\left(\frac{27}{x} - \frac{x}{4}\right)$$

STEP 5: Simplify your primary equation and find its derivative

$$V = \frac{27x^2}{x} - \frac{x^3}{4}$$

$$V = 27x - \frac{1}{4}x^3$$

$$V' = 27 - \frac{3}{4}x^2$$

STEP 6: Set derivative = 0 to find critters

$$V' = 27 - \frac{3}{4}x^2 = 0$$

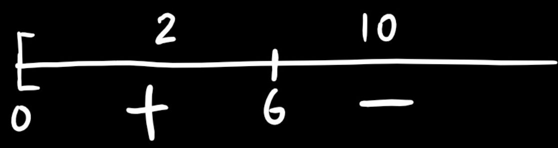
$$\frac{4}{3}\left(+\frac{3}{4}\right)x^2 = +\cancel{27}^{9} \cdot \frac{4}{\cancel{3}}$$

$$x^2 = 36$$

$$x = \pm\sqrt{36} = \pm 6$$

$$x = 6 \quad (x \neq -6)$$

STEP 7: Prove the critter yields a max (or min) using the number line.



so $x = 6$ yields a max

STEP 8: Answer the question.

$x = 6 \qquad h = \frac{27}{x} - \frac{1}{4}x = \frac{27}{6} - \frac{1}{4} \cdot 6$

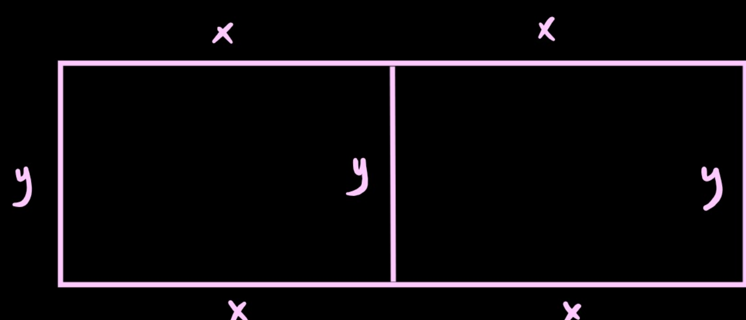
$= 4.5 - 1.5$

$= 3$

$\boxed{\text{So dimensions are } 6'' \times 6'' \times 3''}$

ex: A rancher has 200 ft of fencing with which to enclose 2 congruent adjacent rectangular corrals. What dimensions will maximize the area enclosed?

1)



2) Primary: $A = 2xy$

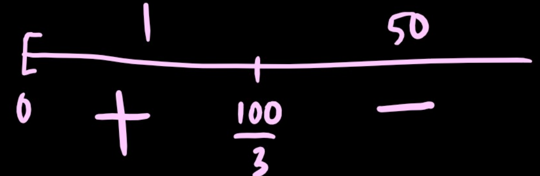
3) Constraint: $4x + 3y = 200$

4)
$$\frac{4x}{4} = \frac{200}{4} - \frac{3y}{4}$$
$$x = 50 - \frac{3}{4}y$$

$$A = 2\left(50 - \frac{3}{4}y\right)y$$

5)
$$A = 2y\left(50 - \frac{3}{4}y\right)$$
$$A = 100y - \frac{3}{2}y^2$$
$$A' = 100 - 3y$$

6)
$$100 - 3y = 0$$
$$-3y = -100$$
$$y = \frac{100}{3}$$

7) 

8) $y = 33\frac{1}{3}'$

$$x = 50 - \frac{3}{4}y$$
$$= 50 - \frac{\cancel{3}}{\cancel{4}} \cdot \frac{\overset{25}{\cancel{100}}}{\cancel{3}}$$
$$= 50 - 25$$
$$= 25$$

each corral is $25' \times 33\frac{1}{3}'$