STEPS FOR GRAPHING

STEP 1: Determine intercepts using calculator

STEP 2: Determine asymptotes if the function is a fraction

V.A.s set denominator = 0

H.A.s find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

STEP 3: Determine extrema

- critters - domain values for which $f'(x)=0$ or $f'(x)$ is undefined
- number line: include critters and VAs
- determine where max or min

STEP 4: Determine IPs:

- PIPs: domain values for which $f''(x)=0$ or $f''(x)$ is undefined
- number line - include PIPs and VAs
- determine IPs.

STEP 5: Draw a beautiful graph!

ex: Graph $f(x)=\dfrac{2(x^2-9)}{x^2-4}$

1) intercepts: x-ints set top = 0

$2(x^2-9)=0$

$2(x+3)(x-3)=0$

$x=-3 \quad x=3$

$(-3,0), (3,0)$

y-int: set $x=0$

$\dfrac{2(0^2-9)}{0^2-4}=\dfrac{2(-9)}{-4}=\dfrac{-18}{-4}=\dfrac{9}{2}$

$\left(0, \frac{9}{2}\right)$

2) Asymptotes: V.A.s set bottom = 0

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2 \quad x=2$

H.A. $\lim_{x\to\infty} \dfrac{(2x^2-18)\frac{1}{x^2}}{(x^2-4)\frac{1}{x^2}}$

$=\lim_{x\to\infty} \dfrac{2-\frac{18}{x^2}}{1-\frac{4}{x^2}}=\dfrac{2-0}{1-0}$

$=2$

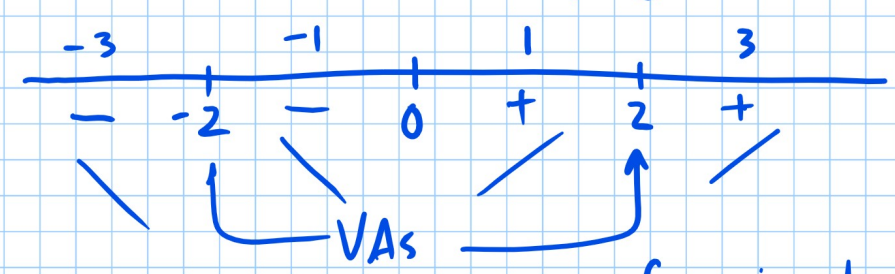
$y=2$ is H.A.

3) Extrema: $f'(x)=\dfrac{(x^2-4)4x-(2x^2-18)2x}{(x^2-4)^2}$

$$f'(x) = \frac{4x^3 - 16x - 4x^3 + 36x}{(x^2-4)^2} = \frac{20x}{(x^2-4)^2}$$

$f'(x) = 0$ when $20x = 0$

$x = 0$



graph may change from inc. to dec. or vice versa around VAs.

relative min @ $(0, f(0)) = \left(0, \frac{9}{2}\right)$

IPs: $f''(x) = \dfrac{(x^2-4)^2 20 - 20x \cdot 2(x^2-4)' \cdot 2x}{(x^2-4)^4}$

$$f''(x) = \frac{\frac{20}{20}(x^2-4)^2 - \frac{80}{20}x^2\frac{(x^2-4)}{(x^2-4)}}{(x^2-4)^4}$$

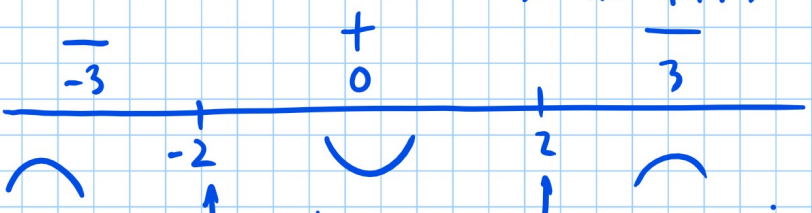
$$\frac{20(x^2-4)\left[(x^2-4) - 4x^2\right]}{(x^2-4)^{4}{}^{3}}$$

$$\frac{20(-3x^2-4)}{(x^2-4)^3}$$

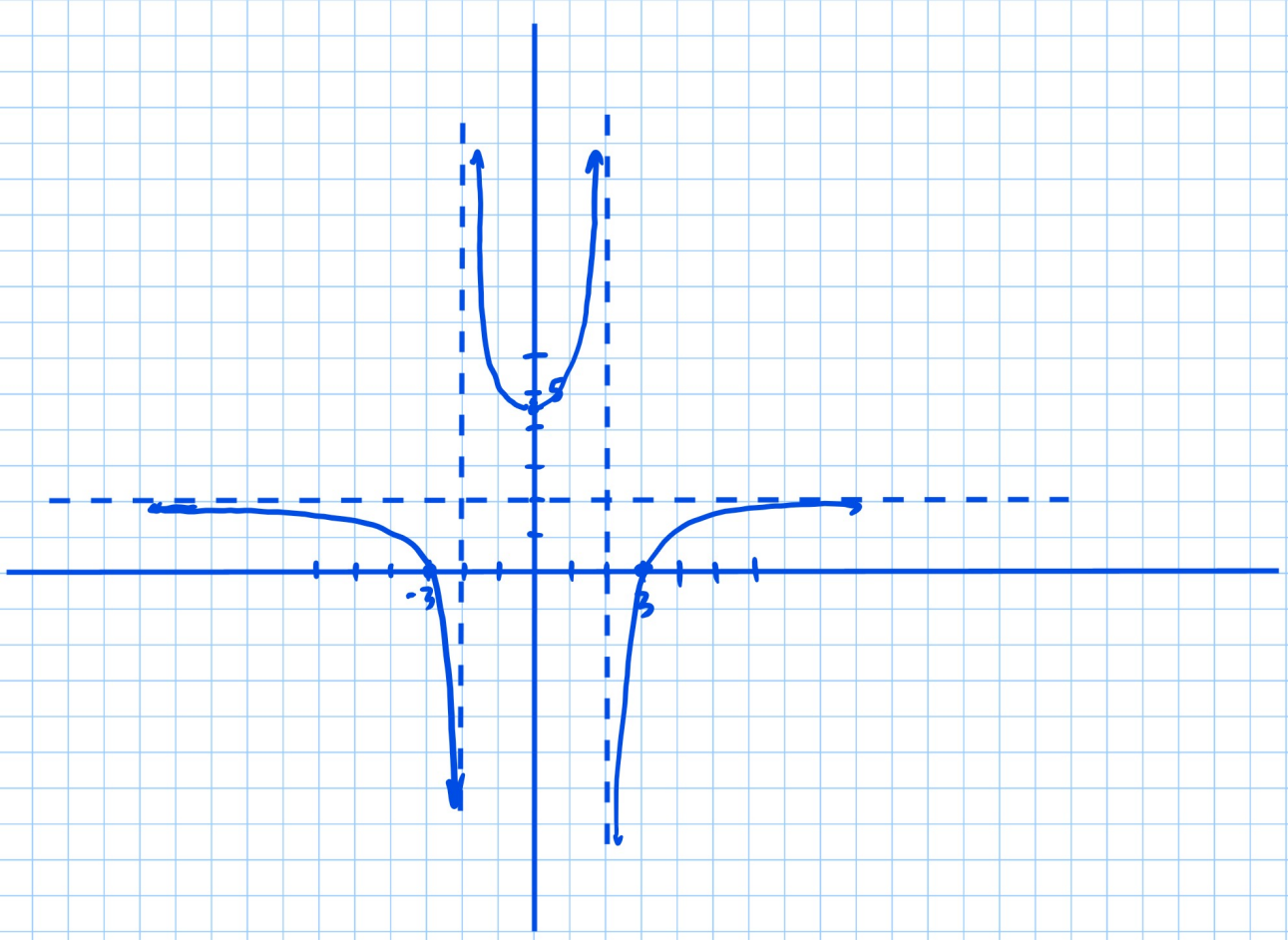
$20(-3x^2-4) = 0$

$-3x^2 = 4$

$x^2 = -\frac{4}{3}$ no real solutions so no PIPs



graph can change concavity around asymptotes



Now Do:

$$f(x) = \frac{x^2-4}{2x^2-32}$$

$$f'(x) = \frac{(2x^2-32)2x - (x^2-4)(4x)}{(2x^2-32)^2}$$

$$f'(x) = \frac{4x^3 - 64x - 4x^3 + 16x}{(2x^2-32)^2} = \frac{-48x}{(2x^2-32)^2}$$

$x = 0$

+ + | − | −

-4 0 4

VAs

max @ $(0, \frac{1}{8})$