Section 3.7 Implicit Differentiation

Recall that $\frac{dy}{dx}$ is another way to notate the derivative. So if $y = x^2 - 5x$, then $\frac{dy}{dx} = 2x - 5$.

ex: Calculate $\frac{dy}{dx}$ if $x^2 + 3y = 5$

$$3y = 5 - x^2$$

$$y = \frac{5}{3} - \frac{1}{3}x^2$$

$$\frac{dy}{dx} = -\frac{2}{3}x$$

Implicit Differentiation

$$x^2 + 3y = 5$$

$$2x + 3\frac{dy}{dx} = 0$$

↑ we look at y as a function of x, so the derivative of 3y is 3 times the derivative of y, which is $\frac{dy}{dx}$

$$3\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{3}$$

ex: $y^3 + 2y = 5x + 6x^2$

$$3y^2 \cdot \frac{dy}{dx} + 2\frac{dy}{dx} = 5 + 12x$$

$$\frac{dy}{dx}\left(3y^2 + 2\right) = 5 + 12x$$

$$\frac{dy}{dx} = \frac{5 + 12x}{3y^2 + 2}$$

ex: $5x^3 - 7y^3 = 17x$

$$-7y^3 = 17x - 5x^3$$

$$-21y^2\frac{dy}{dx} = 17 - 15x^2$$

$$\frac{dy}{dx} = \frac{17 - 15x^2}{-21y^2}$$

ex: Find the equation of the tangent line to the graph of $3(x^2+y^2)^2 = 100xy$ at the point $(3,1)$.

SLOPE:

$$6(x^2+y^2)^1\left(2x + 2y\frac{dy}{dx}\right) = 100x \cdot 1\frac{dy}{dx} + y \cdot 100$$

$$6(3^2+1^2)\left(2\cdot 3 + 2\cdot 1\frac{dy}{dx}\right) = 100\cdot 3\cdot\frac{dy}{dx} + 1\cdot 100$$

$$6\cdot 10\left(6 + 2\frac{dy}{dx}\right) = 300\frac{dy}{dx} + 100$$

$$360 + 120\frac{dy}{dx} = 300\frac{dy}{dx} + 100$$

$$180\frac{dy}{dx} = 260$$

$$\frac{dy}{dx} = \frac{260}{180} = \frac{26}{18} = \frac{13}{9} = m$$

POINT: $(3,1)$

EQ: $1 = \frac{13}{9}\cdot 3 + b$

$\frac{3}{3} - \frac{13}{3} = 1 - \frac{13}{3} = b$

$-\frac{10}{3} = b$

$$y = \frac{13}{9}x - \frac{10}{3}$$

p 140

1, 3, 5, 6, 18, 19

3) $\sqrt{x} = 5\sqrt{y}$

18) $x^3 + 5x^2y + 2y^2 = 4y + 11$ at $(1,2)$

$$3x^2 + 5x^2\frac{dy}{dx} + y\cdot 10x + 4y\frac{dy}{dx} = 4\frac{dy}{dx}$$

$$5x^2\frac{dy}{dx} + 4y\frac{dy}{dx} - 4\frac{dy}{dx} = -3x^2 - 10xy$$

$$\frac{dy}{dx}(5x^2 + 4y - 4) = -3x^2 - 10xy$$

$$\frac{dy}{dx} = \frac{-3x^2 - 10xy}{5x^2 + 4y - 4}$$

$$\frac{dy}{dx} = \frac{-3\cdot 1^2 - 10\cdot 1\cdot 2}{5\cdot 1^2 + 4\cdot 2 - 4}$$

$$\frac{dy}{dx} = \frac{-3-20}{5+8-4} = \frac{-23}{9}$$