

## WARMUP

What if we continued the Fibonacci sequence backwards? What are the numbers that go in the spaces?

-8, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, 13, ...

## Section 3.6 Derivative of $\ln x$

$$\frac{d}{dx} (\ln x) = \frac{1}{x} \quad \frac{d}{dx} (\ln f(x)) = \frac{1}{f(x)} \cdot f'(x)$$
$$= \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx} (\ln (f(x))) \quad \swarrow f'(x)$$

$$\text{ex: } \frac{d}{dx} (\ln (\cos x)) = \frac{-\sin x}{\cos x} = -\tan x$$

$\swarrow f(x)$

$$\text{ex: } h(x) = \underbrace{x^3}_{1^{\text{st}}} \underbrace{\ln(10x)}_{2^{\text{nd}}}$$

$$h'(x) = \underbrace{x^3 \cdot \frac{10^1}{10x}}_{f(x)} + \ln(10x) \cdot 3x^2$$

$\swarrow f'(x)$

$$h'(x) = x^2 + 3x^2 \ln(10x)$$

$$h'(x) = x^2 (1 + 3 \ln(10x))$$

ex:  $f(x) = e^{\ln(e^{2x^2+3})} = e^{2x^2+3}$   $\frac{d}{dx} [e^{f(x)}] = e^{f(x)} \cdot f'(x)$

$$f'(x) = e^{2x^2+3} \cdot 4x$$

$$f'(x) = 4x e^{2x^2+3}$$

$$\ln e^N = N$$

$$e^{\ln M} = M$$

ex:  $f(x) = \frac{\ln x}{\sin x}$

$$f'(x) = \frac{(\sin x \cdot \frac{1}{x} - \ln x \cdot \cos x) \cdot x}{(\sin x)^2 \cdot x}$$

$$f'(x) = \frac{\cancel{x \cdot \sin x} \cdot \cancel{\frac{1}{x}} - x \cdot \ln x \cdot \cos x}{x (\sin x)^2}$$

$$f'(x) = \frac{\sin x - x \ln x \cdot \cos x}{x \sin^2 x}$$

1. POWER:  $\frac{d}{dx} [x^n] = nx^{n-1}$      $\frac{d}{dx} [(f(x))^n] = n(f(x))^{n-1} \cdot f'(x)$

2. e:  $\frac{d}{dx} [e^x] = e^x$      $\frac{d}{dx} [e^{f(x)}] = e^{f(x)} \cdot f'(x)$

3. Exponential:  $\frac{d}{dx} [a^x] = a^x \cdot \ln a$      $\frac{d}{dx} [a^{f(x)}] = a^{f(x)} \cdot \ln a \cdot f'(x)$

4. TRIGS:

A.  $\frac{d}{dx} [\sin x] = \cos x$      $\frac{d}{dx} [\sin(f(x))] = \cos(f(x)) \cdot f'(x)$

B.  $\frac{d}{dx} [\cos x] = -\sin x$      $\frac{d}{dx} [\cos(f(x))] = -\sin(f(x)) \cdot f'(x)$

C.  $\frac{d}{dx} [\tan x] = \sec^2 x$      $\frac{d}{dx} [\tan(f(x))] = \sec^2(f(x)) \cdot f'(x)$

5. Natural Log:  $\frac{d}{dx} [\ln x] = \frac{1}{x}$      $\frac{d}{dx} [\ln(f(x))] = \frac{1}{f(x)} \cdot f'(x)$

6. PRODUCT:  $\frac{d}{dx} [f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$   
1st · der 2nd + 2nd · der 1st

7. QUOTIENT:  $\frac{d}{dx} \left[ \frac{\overset{\text{TOP}}{f(x)}}{\underset{\text{BOT}}{g(x)}} \right] = \frac{\text{BOT} \cdot \text{der TOP} - \text{TOP} \cdot \text{der BOT}}{\text{BOT}^2}$

8. CONSTANTS  $\frac{d}{dx} [c f(x)] = c f'(x)$      $\frac{d}{dx} [c \pm f(x)] = f'(x)$

ex:  $f(t) = \ln(t^2 + 1)$  ⑤

ex:  $f(w) = \ln(\cos(w-1))$  5, then 4B

ex:  $f(x) = \frac{x}{1 + \ln x}$  7, 5

p136 1-15 odd, 21, 29-32

3)  $f(x) = \ln e^{2x} = 2x$

$f'(x) = 2$

1) 8:30 - 9:31

2) 9:36 - 11:32

3) 11:39 - 12:43

Lunch 12:48 - 1:18

4) 1:23 - 2:24

5) 2:29 - 3:30

$\ln(f(x))$   
11)  $f(x) = \ln(e^{ax} + b)$

$$f'(x) = \frac{1}{e^{ax} + b} \cdot \frac{e^{ax} \cdot a}{1} = \frac{ae^{ax}}{e^{ax} + b}$$

$\uparrow$   $\uparrow$   
 $\frac{1}{f(x)}$   $f'(x)$

5)  $f(x) = \frac{1}{\ln x}$

$$f'(x) = \frac{\ln x \cdot 0 - 1 \cdot \frac{1}{x}}{(\ln x)^2} = \frac{-\frac{1}{x}}{(\ln x)^2} \cdot \frac{x}{x}$$
$$= \frac{-1}{x(\ln x)^2}$$

$$f(x) = (\ln x)^{-1} \quad f'(x) = -1 (\ln x)^{-2} \cdot \frac{1}{x}$$

$$(f(x))^n \quad n (f(x))^{n-1} \cdot f'(x)$$

$$= \frac{-1}{x (\ln x)^2}$$

30)  $y = \overbrace{2x (\ln x + \ln 2)}^{\text{PRODUCT}} - 2x + e$

$$y' = 2x \left( \frac{1}{x} \right) + (\ln x + \ln 2) \cdot 2 - 2 + 0$$

$$y' = \cancel{2x} + (\ln x + \ln 2) 2 - \cancel{2}$$

$$y' = 2 (\ln x + \ln 2)$$

7)  $f(x) = \ln(1 - e^{-x}) = \frac{1}{1 - e^{-x}} \cdot \frac{+e^{-x} \cdot (-1)}{1}$

5, (2)  $= \frac{e^{-x}}{1 - e^{-x}}$

32)  $f(t) = \overbrace{\ln(\ln t)}^{\text{Rule 5}} + \overbrace{\ln(\ln 2)}^{\text{constant}}$

$$f'(x) = \frac{1}{\ln t} \cdot \frac{1}{t} + 0 = \frac{1}{t \ln t}$$

$$\frac{d}{dt} (\ln f(t)) = \frac{1}{f(t)} \cdot f'(t)$$