WARMUP

Find $f'(x)$ if $f(x) = (3x^2+2)^2 = (3x^2+2)(3x^2+2)$

$$= 9x^4 + 12x^2 + 4$$

$$f'(x) = 36x^3 + 24x$$

$$= 12x(3x^2+2)$$

Now do $f(x) = (3x^2+2)^{19}$

Section 3.4 The Chain Rule

Consider the function $f(x) = (3x^2+2)^2$.

We could calculate $f'(x)$ like the warmup or we could use the Chain Rule.

ex: $f(x) = (3x^2+2)^2$

$$f'(x) = \underbrace{2(3x^2+2)^1}_{\text{POWER RULE}} \cdot \underbrace{6x}_{\text{deriv. of what's in parentheses}}$$

$$f'(x) = 12x(3x^2+2)$$

ex: $f(x) = (3x^2+2)^{19}$

$$f'(x) = 19(3x^2+2)^{18} \cdot 6x$$

$$f'(x) = 114x(3x^2+2)^{18}$$

CHAIN RULE: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

ex: $w = (x^3+1)^{100}$

$$w' = 100(x^3+1)^{99} \cdot 3x^2 = 300x^2(x^3+1)^{99}$$

BASIC	CHAIN RULE
$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$	$\frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)$
$\frac{d}{dx}\left[e^x\right] = e^x$	$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$
$\frac{d}{dx}\left[a^x\right] = a^x \ln a$	$\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \cdot \ln a \cdot f'(x)$

ex: $f(x) = \sqrt{x^4+1} = (x^4+1)^{\frac{1}{2}}$

$$f'(x) = \underbrace{\frac{1}{2}(x^4+1)^{-\frac{1}{2}} \cdot 4x^3} = \frac{2x^3}{(x^4+1)^{1/2}}$$

$$= \frac{2x^3}{\sqrt{x^4+1}}$$

ex: $f(x) = e^{2x}$

$f'(x) = e^{2x} \cdot 2$

$f'(x) = 2e^{2x}$

In general

$\frac{d}{dx}\left[e^{kx}\right] = ke^{kx}$

ex: $g(x) = 3^{7x^3+2x}$

$g'(x) = 3^{7x^3+2x} \cdot \ln 3 \cdot (21x^2 + 2)$

ex: $f(x) = xe^{5-2x}$

$f'(x) = \underset{\uparrow \atop \text{1st}}{x} \underbrace{e^{5-2x}(-2)}_{\text{der 2nd}} + e^{5-2x} \cdot 1$

$= e^{5-2x}(-2x + 1)$

p 126 1-45 eoo

17) $g(t) = e^{(1+3t)^2}$

$g'(x) = e^{(1+3t)^2} \cdot 2(1+3t)^1 \cdot 3$

$g'(x) = 6(1+3t)e^{(1+3t)^2}$

21) $y = e^{3w/2} = e^{\frac{3}{2}w}$

$$y' = \frac{3}{2}e^{\frac{3}{2}w}$$

41) $f(y) = \sqrt{10^{(5-y)}} = \left(10^{(5-y)}\right)^{\frac{1}{2}}$

$$= 10^{\frac{5}{2} - \frac{1}{2}y}$$

$$f'(y) = \underbrace{10^{\frac{5}{2} - \frac{1}{2}y}} \cdot \ln 10 \cdot \left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2}\sqrt{10^{(5-y)}} \cdot \ln 10$$