

PRACTICE TEST SOLUTIONS

$$\tan \alpha = \frac{8}{15} \frac{y}{x} \quad \pi < \alpha < \frac{3\pi}{2}$$

$$x = -15 \quad y = -8$$

$$r = 17$$

$$\sin \alpha = \frac{-8}{17}$$

$$\cos \alpha = \frac{-15}{17}$$

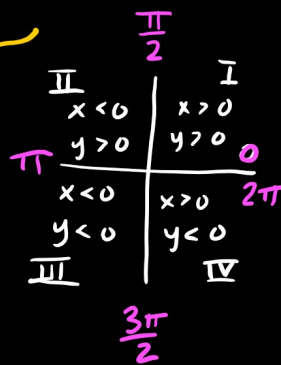
$$\sec \beta = \frac{41}{40} \frac{r}{x} \quad \frac{3\pi}{2} < \beta < 2\pi$$

$$x = 40 \quad y = -9$$

$$r = 41$$

$$\sin \beta = \frac{-9}{41}$$

$$\cos \beta = \frac{40}{41}$$



$$\begin{aligned} 1) \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \left(\frac{-8}{17}\right)\left(\frac{40}{41}\right) + \left(\frac{-15}{17}\right)\left(\frac{-9}{41}\right) \\ &= \frac{-320}{697} + \frac{135}{697} = \frac{-185}{697} = \frac{-y}{r} \end{aligned}$$

$$\begin{aligned} 2) \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(\frac{-15}{17}\right)\left(\frac{40}{41}\right) - \left(\frac{-8}{17}\right)\left(\frac{-9}{41}\right) \\ &= \frac{-600}{697} - \frac{72}{697} = \frac{-672}{697} = \frac{-x}{r} \end{aligned}$$

$$3) \tan(\alpha + \beta) = \frac{-185}{-672} = \frac{185}{672}$$

$$4) \quad y = -185 \\ y < 0$$

$$x = -672 \\ x < 0$$

Q III

PPP

$$\sec \alpha = -\frac{17}{8}$$

$$\pi < \alpha < \frac{3\pi}{2}$$

$$\cot \beta = \frac{5}{12} \quad 0 < \beta < \frac{\pi}{2}$$

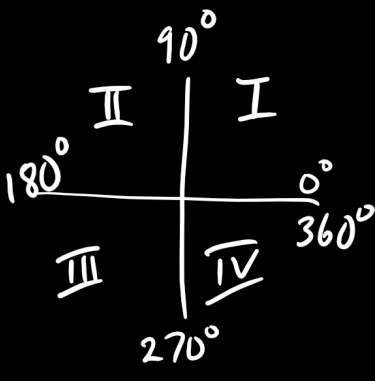
$$\sin(\alpha - \beta)$$

$$\cos(\alpha - \beta)$$

$$\tan(\alpha - \beta)$$

Quadrant $\alpha - \beta$

$$5) \sin 112.5^\circ = \sin \frac{225^\circ}{2}$$



$$= + \sqrt{\frac{1 - \cos 225^\circ}{2}}$$

$$= \sqrt{\frac{1 - (-\frac{\sqrt{2}}{2})}{2}}$$

$$= \sqrt{\frac{(1 + \frac{\sqrt{2}}{2}) \cdot \frac{2}{2}}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$= \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$6) \cos 165^\circ = \cos \frac{330^\circ}{2}$$

$$= - \sqrt{\frac{1 + \cos 330^\circ}{2}}$$

$$= - \sqrt{\frac{(1 + \frac{\sqrt{3}}{2}) \cdot \frac{2}{2}}{2}}$$

$$= - \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$= - \frac{\sqrt{2 + \sqrt{3}}}{2}$$

PPP
half-angle
5) $\cos 105^\circ$
6) $\sin 15^\circ$

$$7-10 \quad \cot \theta = \frac{24}{7} \quad 0 < \theta < \frac{\pi}{2}$$

$$x = 24 \quad y = 7$$

$$r = 25$$

$$\sin \theta = \frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$

$$7) \sin(2\theta) = 2 \sin \theta \cos \theta$$

$$= \frac{2}{1} \cdot \frac{7}{25} \cdot \frac{24}{25} = \frac{336}{625}$$

$$8) \cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$$

$$10) \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

$$= \frac{\left(1 - \frac{24}{25}\right) 25}{\frac{7}{25} \cdot 25}$$

$$= \frac{25 - 24}{7} = \frac{1}{7}$$

$$= \frac{576}{625} - \frac{49}{625} = \frac{527}{625}$$

$$9) \tan(2\theta) = \frac{336}{527}$$

PPP $\csc \theta = \frac{41}{9} \quad 0 < \theta < \frac{\pi}{2}$

$$\sin(2\theta), \cos(2\theta), \tan(2\theta)$$

$$\tan \frac{\theta}{2}$$

$$11) \cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$\cos^2 \theta (1 + \tan^2 \theta) = \cos^2 \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

$$\frac{1 - x^2}{(1-x)(1+x)}$$

$$12) \frac{\sin^2 \theta}{1 - \cos \theta} - 1 = \cos \theta$$

$$\frac{\sin^2 \theta}{1 - \cos \theta} - 1 = \frac{1 - \cos^2 \theta}{1 - \cos \theta} - 1$$

$$= \frac{\cancel{(1 - \cos \theta)}(1 + \cos \theta)}{\cancel{1 - \cos \theta}} - 1$$

$$= 1 + \cos \theta - 1 = \cos \theta$$

$$\underline{\text{PPP}} \quad \sin^2 \theta (1 + \cot^2 \theta) = 1$$

$$1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$$

$$13) \quad 4 \cos^2 \theta = 3$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$14) \quad 2 \sin^2 \theta - 3 \sin \theta + 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta - 1) = 0$$

$\underbrace{\hspace{10em}}_{\substack{-1 \sin \theta \\ -2 \sin \theta}}$

$$2 \sin \theta - 1 = 0$$

$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$

$$\sin \theta - 1 = 0$$
$$\sin \theta = 1$$

$$\underline{\text{PPP}} \quad 4 \sin^2 \theta = 3$$

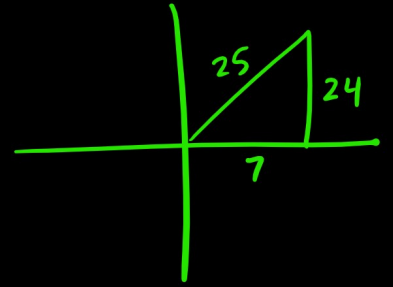
$$2 \cos^2 \theta - 3 \cos \theta - 2 = 0$$

$$15) \sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

$$16) \sin^{-1} \left(\sin \frac{5\pi}{3} \right) = \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}$$

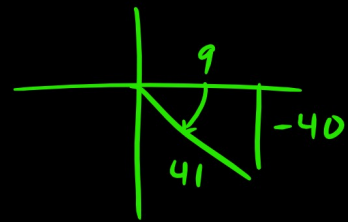
$$17) \cot \left(\sin^{-1} \frac{24}{25} \right) = \frac{7}{24}$$

← x
← y



$$18) \sec \left(\sin^{-1} \left(-\frac{40}{41} \right) \right) = \frac{41}{9}$$

← r
← x



$$\csc^{-1} \left(\frac{a}{b} \right) = \sin^{-1} \left(\frac{b}{a} \right)$$

$$19) \csc^{-1} (2) = \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6}$$

$$\sec^{-1} \left(\frac{a}{b} \right) = \cos^{-1} \left(\frac{b}{a} \right)$$

$$20) \cot^{-1} (1) = \tan^{-1} (1) = \frac{\pi}{4}$$

$$\cot^{-1} \left(\frac{a}{b} \right) = \tan^{-1} \left(\frac{b}{a} \right)$$

PPP

$$\tan^{-1} \sqrt{3}$$

$$\cot \left(\tan^{-1} \left(-\frac{24}{7} \right) \right)$$

$$\sin^{-1} \left(\sin \frac{11\pi}{6} \right)$$

$$\sec^{-1} (2)$$

$$\sec \left(\sin^{-1} \frac{8}{17} \right)$$

$$\tan^{-1} (-1)$$