PPP

1-4) $\sec\alpha = -\frac{17}{8}$ (QIII) $\pi < \alpha < \frac{3\pi}{2}$

$r = 17,\ x = -8,\ y = -15$

$\sin\alpha = -\frac{15}{17}$

$\cos\alpha = -\frac{8}{17}$

$\cot\beta = \frac{5}{12}$ $0 < \beta < \frac{\pi}{2}$

$x = 5,\ y = 12,\ r = 13$

$\sin\beta = \frac{12}{13}$

$\cos\beta = \frac{5}{13}$

1) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \left(\frac{-15}{17}\right)\left(\frac{5}{13}\right) - \left(\frac{-8}{17}\right)\left(\frac{12}{13}\right)$

$= \frac{-75}{221} + \frac{96}{221} = \frac{21}{221} = \frac{y}{r}$ $y > 0$

2) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \left(\frac{-8}{17}\right)\left(\frac{5}{13}\right) + \left(\frac{-15}{17}\right)\left(\frac{12}{13}\right)$

$= -\frac{40}{221} - \frac{180}{221} = \frac{-220}{221} = \frac{x}{r}$ $x < 0$

3) $\tan(\alpha-\beta) = \frac{21}{-220}$

4) QII

5) $\cos 105^\circ = \cos\frac{210^\circ}{2} = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{\left(1 - \sqrt{3}/2\right)}{2}} \cdot \frac{2}{2}$

105° is QII so

$= -\sqrt{\frac{2-\sqrt{3}}{4}}$

$= -\frac{\sqrt{2-\sqrt{3}}}{2}$

6) $\sin 15^\circ = \sin\frac{30^\circ}{2} = +\sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1 - \sqrt{3}/2}{2}} \cdot \frac{2}{2}$

15° is QI

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$

7-10 $\csc\theta = \frac{41}{9}$ $\quad 0 < \theta < \frac{\pi}{2}$

$r = 41$, $y = 9$, $x = 40$

$\cos\theta = \frac{40}{41}$

$\sin\theta = \frac{9}{41}$

7) $\sin(2\theta) = 2\sin\theta\cos\theta$

$= \frac{2}{1}\left(\frac{9}{41}\right)\left(\frac{40}{41}\right)$

$= \frac{720}{1681}$

8) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{40}{41}\right)^2 - \left(\frac{9}{41}\right)^2$

$= \frac{1600}{1681} - \frac{81}{1681} = \frac{1519}{1681}$

9) $\tan(2\theta) = \frac{720}{1519}$

10) $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{\left(1 - \frac{40}{41}\right)41}{\frac{9}{41}\cdot 41}$

$= \frac{41-40}{9} = \frac{1}{9}$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

11) $\sin^2\theta(1+\cot^2\theta) = 1$

$\sin^2\theta(1+\cot^2\theta) = \sin^2\theta \cdot \csc^2\theta$

$= \sin^2\theta \cdot \frac{1}{\sin^2\theta}$

$= 1$

12) $1 - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

$1 - \frac{\cos^2\theta}{1+\sin\theta} = 1 - \frac{1-\sin^2\theta}{1+\sin\theta}$

$= 1 - \frac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta}$

$= 1 - (1-\sin\theta)$

$= \sin\theta$

13) $4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

14) $2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$2\cos\theta + 1 = 0 \qquad \cos\theta - 1 = 0$

$\cos\theta = -\frac{1}{2} \qquad \cos\theta = 1$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$

15) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

16) $\sin^{-1}\left(\sin\frac{11\pi}{6}\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

17) $\sec\left(\sin^{-1}\frac{8}{17}\right) = \frac{17}{15}$ because secant is $\frac{r}{x}$

$y = 8$

$r = 17$

so $x = 15$

18) $\cot\left(\tan^{-1}\left(-\frac{24}{7}\right)\right) = -\frac{7}{24}$ because cotangent is $\frac{x}{y}$

$y = -24 \quad x = 7 \quad r = 25$

19) $\sec^{-1}(2) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

20) $\tan^{-1}(-1) = -\frac{\pi}{4}$ $\qquad \cot^{-1}(-1) = \tan^{-1}(-1) = -\frac{\pi}{4}$