

Given  $\sec \alpha = \frac{25}{7} \quad 270^\circ < \alpha < 360^\circ$  and  $\tan \beta = \frac{12}{5} \quad 180^\circ < \beta < 270^\circ$   
 $r=25, x=7, y=-24$   $y=-12 \quad x=-5 \quad r=13$

$$\begin{aligned}\sin(2\alpha) \\ \cos(2\alpha) \\ \tan(2\alpha)\end{aligned}$$

$$\sin \alpha = -\frac{24}{25}$$

$$\cos \alpha = \frac{7}{25}$$

$$\sin \beta = -\frac{12}{13}$$

$$\cos \beta = -\frac{5}{13}$$

$$\begin{aligned}\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= -\frac{24}{25} \cdot \left(-\frac{5}{13}\right) - \frac{7}{25} \cdot \left(-\frac{12}{13}\right) = \frac{120}{325} + \frac{84}{325} = \frac{204}{325} = \frac{y}{r}\end{aligned}$$

$$\begin{aligned}\cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \frac{7}{25} \cdot \left(-\frac{5}{13}\right) + \left(-\frac{24}{25}\right) \left(-\frac{12}{13}\right) = -\frac{35}{325} + \frac{288}{325} = \frac{253}{325} = \frac{x}{r}\end{aligned}$$

$$\tan(\alpha - \beta) = \frac{y}{x} = \frac{204}{253}$$

What quadrant is  $\alpha - \beta$  in? QI

$$\begin{aligned}\sin(2\alpha) &= 2 \sin \alpha \cos \alpha \\ &= 2 \left(-\frac{24}{25}\right) \left(\frac{7}{25}\right) \\ &= -\frac{336}{625} = \frac{y}{r}\end{aligned}$$

$$\tan(2\alpha) = \frac{-336}{527} = \frac{336}{527}$$

$2\alpha$  is in Q III

$$\begin{aligned}\cos(2\alpha) &= \cos^2 \alpha - \sin^2 \alpha \\ &= \left(\frac{7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2 \\ &= \frac{49}{625} - \frac{576}{625} \\ &= -\frac{527}{625} = \frac{x}{r}\end{aligned}$$

$$4) \tan\theta + \cot\theta - \sec\theta \csc\theta = 0$$

$$\begin{aligned}\tan\theta + \cot\theta - \sec\theta \csc\theta &= \frac{\overbrace{\sin\theta}^1}{\overbrace{\cos\theta}^1} + \frac{\overbrace{\cos\theta}^1 \cdot \overbrace{\cos\theta}^1}{\overbrace{\sin\theta}^1 \overbrace{\cos\theta}^1} - \frac{\overbrace{1}^1 \cdot \overbrace{1}^1}{\overbrace{\cos\theta}^1 \overbrace{\sin\theta}^1} \\&= \frac{\overbrace{\sin^2\theta}^1 + \overbrace{\cos^2\theta}^1 - 1}{\sin\theta \cos\theta} \\&= \frac{1 - 1}{\sin\theta \cos\theta} \\&= 0\end{aligned}$$

$$\frac{5}{7} \cancel{\times} \frac{1}{3}$$

$$5) \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta \sec\theta \quad \frac{5 \cdot 3 - 1 \cdot 7}{7 \cdot 3} = \frac{15-7}{21}$$

$$\begin{aligned}\frac{1+\sin\theta}{1-\sin\theta} \cancel{\times} \frac{1-\sin\theta}{1+\sin\theta} &= \frac{(1+\sin\theta)(1+\sin\theta) - (1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \\&= \frac{(1+2\sin\theta+\sin^2\theta) - (1-2\sin\theta+\sin^2\theta)}{1-\sin^2\theta} \\&= \frac{1+2\sin\theta+\sin^2\theta - 1+2\sin\theta-\sin^2\theta}{\cos^2\theta}\end{aligned}$$

$$= \frac{4 \sin \theta}{\cos^2 \theta} = 4 \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta}$$

$$= 4 \tan \theta \cdot \sec \theta$$

If  $\sin \theta = \frac{9}{41} = \frac{y}{r}$  then  $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$

$\theta$  in QI     $\frac{\theta}{2}$  in QI

$y = 9, x = 40, r = 41$

$$\tan \frac{\theta}{2} = \frac{\left(1 - \frac{40}{41}\right)41}{\left(\frac{9}{41}\right)41}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{\left(1 - \frac{40}{41}\right)41}{2 \cdot 41}}$$

$$\sqrt{\frac{41 - 40}{82}} = \sqrt{\frac{1}{82}} = \frac{\sqrt{82}}{82}$$

$$= \frac{41 - 40}{9}$$

$$\cos \frac{\theta}{2} =$$

$$\frac{9}{\sqrt{82}} \cdot \frac{\sqrt{82}}{\sqrt{82}} = \frac{9\sqrt{82}}{82}$$

$$= \frac{1}{9}$$