

Given  $\sec \alpha = \frac{25}{7}$   $270^\circ < \alpha < 360^\circ$  and  $\tan \beta = \frac{12}{5}$   $180^\circ < \beta < 270^\circ$

$r=25, x=7, y=-24$

$y=-12, x=-5, r=13$

$\sin(2\alpha)$   
 $\cos(2\alpha)$   
 $\tan(2\alpha)$

$\sin \alpha = \frac{-24}{25}$

$\sin \beta = -\frac{12}{13}$

$\cos \alpha = \frac{7}{25}$

$\cos \beta = -\frac{5}{13}$

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$= \frac{-24}{25} \cdot \left(-\frac{5}{13}\right) - \frac{7}{25} \cdot \left(-\frac{12}{13}\right) = \frac{120}{325} + \frac{84}{325} = \frac{204}{325} = \frac{y}{r}$

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$= \frac{7}{25} \cdot \left(-\frac{5}{13}\right) + \left(\frac{-24}{25}\right) \cdot \left(-\frac{12}{13}\right) = \frac{-35}{325} + \frac{288}{325} = \frac{253}{325} = \frac{x}{r}$

$\tan(\alpha - \beta) = \frac{y}{x} = \frac{204}{253}$

What quadrant is  $\alpha - \beta$  in? QI

$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$

$= 2 \left(\frac{-24}{25}\right) \left(\frac{7}{25}\right)$

$= \frac{-336}{625} = \frac{y}{r}$

$\tan(2\alpha) = \frac{-336}{-527} = \frac{336}{527}$

$2\alpha$  is in QIII

$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$

$= \left(\frac{7}{25}\right)^2 - \left(\frac{-24}{25}\right)^2$

$= \frac{49}{625} - \frac{576}{625}$

$= \frac{-527}{625} = \frac{x}{r}$

$$4) \tan\theta + \cot\theta - \sec\theta \csc\theta = 0$$

$$\tan\theta + \cot\theta - \sec\theta \csc\theta = \frac{\sin\theta}{\cos\theta} \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta} \frac{1}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 1}{\sin\theta \cos\theta}$$

$$= \frac{1 - 1}{\sin\theta \cos\theta}$$

$$= 0$$

$$\frac{5}{7} \times \frac{1}{3}$$

$$5) \frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = 4 \tan\theta \sec\theta \quad \frac{5 \cdot 3 - 1 \cdot 7}{7 \cdot 3} = \frac{15 - 7}{21}$$

$$\frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = \frac{(1 + \sin\theta)(1 + \sin\theta) - (1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} = \frac{8}{21}$$

$$= \frac{(1 + 2\sin\theta + \sin^2\theta) - (1 - 2\sin\theta + \sin^2\theta)}{1 - \sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= \frac{4 \sin \theta}{\cos^2 \theta} = 4 \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta}$$

$$= 4 \tan \theta \cdot \sec \theta$$

If  $\sin \theta = \frac{9}{41} = \frac{y}{r}$  then  $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$

$\theta$  in QI     $\frac{\theta}{2}$  in QI

$y = 9, x = 40, r = 41$

$$= \frac{\left(1 - \frac{40}{41}\right) 41}{\left(\frac{9}{41}\right) 41}$$

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$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{40}{41}}{2}} \cdot 41$$

$$\sqrt{\frac{41 - 40}{82}} = \sqrt{\frac{1}{82}} = \frac{\sqrt{82}}{82}$$

$$= \frac{41 - 40}{9}$$

$$\cos \frac{\theta}{2} =$$

$$\frac{9 \cdot \sqrt{82}}{\sqrt{82} \sqrt{82}} = \frac{9 \sqrt{82}}{82}$$

$$= \frac{1}{9}$$